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Abstract— The development of ambulatory arterial pulse oximetry is key to longer term monitoring and treatment of cardiovascular and respiratory conditions. The investigation presented in this paper will assist the designer of an ambulatory pulse oximetry monitor in minimizing the overall LED power consumption ( $P_{LED,TOT}$ ) levels by analyzing the lowest achievable limit as constrained by the optical components, circuitry implementation and final SpO2 reading accuracy required. LED duty cycle ( $D_{LED}$ ) reduction and light power ( $P_{LED,ON}$ ) minimization are proposed as methods to reduce  $P_{LED,TOT}$ . Bandwidth and signal quality calculations are carried out in order to determine the required  $P_{LED,TOT}$  as a function of the different noise sources.

#### I. INTRODUCTION

hotoplethysmographic (PPG) cardiovascular monitoring is widely used in hospitals as a means to obtain readings on heart rate (HR), arterial oxygen saturation (SpO2) and pulse wave analysis. The aforementioned physiological readings are important to the medical profession as they help evaluate the instantaneous wellbeing of a patient. It is also useful to observe the fluctuations of SpO2 over time as they reflect both general cardiovascular and respiratory health [1], [2]. Since a SpO2 monitor provides information from both cardiovascular and respiratory systems the focus of this paper will be put on this application of photoplethysmography. The current SpO2 devices used in hospitals rely on power supplied by the mains and therefore their mobility is limited to the extension of the power lead.

Continuous ambulatory monitoring of SpO2 would prove helpful to the medical profession as it would enable care to be sustained at the patient's home. SpO2 monitoring devices are beneficial to patients with dysfunctional respiratory systems such as part of the elderly population [2] and subjects suffering from high risk of apnea [1]. In these cases the patient would wear an ambulatory SpO2 monitoring device logging the parameter fluctuations over time and if necessary triggering an instantaneous treatment such as supply of oxygen or an alarm.

Photoplethysmography as a technique consists in observing the changes in light attenuation (LA) across a bulk of tissue where one or more near skin arteries can be found. In order to obtain a reading, light is shone through the skin onto an artery and is observed at either the other side

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(transmittance) or near the light source (reflectance) [3]. In both cases the arterial volumetric changes caused by the heart beats generate variations in LA which then are used for SpO2 calculations. This technique can be implemented on any part of the body having near skin arteries and only requires two LEDs of different peak wavelengths, red (R) and infrared (IR), and one shared photodiode.

Two different wavelengths are necessary because SpO2 measurements are based on hemoglobin's variable spectrum characteristics which vary depending on whether the molecule is carrying oxygen or not. Many different wavelengths can be used but generally 660nm and 940nm for R and IR respectively are suitable for this application [4]. In order to distinguish the *LA* caused by arterial blood from that caused by tissue and venous blood, the pulsating *AC* component of the received signal needs to be observed - see Fig.1. Normally the magnitude of the *AC* component is about 3% that of the *DC* [4].



Fig. 1. Non-scaled graphical representation of the AC and DC components of light attenuation in photoplethysmography caused by pulsating arterial blood (AC) and sum of non-pulsating arterial blood, venous blood and tissue (DC) [3], [4].

SpO2 is calculated as a function of the ratio (*Q*) between the two AC components of *LA* for the *R* and *IR* lights ( $AC_{660}$  and  $AC_{940}$ ). Since the scaling of the *R* and *IR* signals could be different at their arrival to the photodiode, *Q* is calculated in a normalized form in order for the magnitude of each *AC* component to be comparable [4].

$$Q = \frac{AC_{660}}{AC_{940}}$$
(1)

### II. AMBULATORY SYSTEM DESIGN CONSIDERATIONS

In the case of ambulatory photoplethysmographic HR and SpO2 monitoring, the main goals of the design are to reduce power consumption, size and weight to make the device more comfortable to wear while maintaining a satisfactory signal quality. The LED power consumption  $(P_{LED})$  can represent the highest power consumption in ambulatory PPG system if not optimized [5].

The benefits of LED switching and duty cycle  $(D_{LED})$  reduction in  $P_{LED,TOT}$  diminution have been mentioned in previous research [6] but research on the implications of  $D_{LED}$  changes in bandwidth and noise have not been found. Sampling frequency (*fs*) and  $D_{LED}$  are defined as

$$D_{LED} = \frac{t_{on}}{T_s} \quad \text{where} \quad T_s = \frac{1}{f_s} \tag{2, 3}$$

Light power reduction has also been mentioned as another mean for  $P_{LED}$  reduction [7] but no studies relating it to the aimed signal to noise ratio (*SNR*) for pulse oximetry have been found either. Therefore we have carried out a study taking into account these two power reducing techniques to define the trade-offs and limitations of low power ambulatory SpO2 monitoring system.

### **III.** POWER REDUCTION CALCULATIONS

A - LED duty cycle ( $D_{LED}$ ) and bandwidth (BW) relationship

It is important to understand how the duty cycle of the LED switching would affect the overall system BW and noise.

Fig. 2 is a representation of the signal arriving at the photodiode from the two *R* and *IR* LEDs. Every period  $(T_s)$  is composed of two light pulses of duration  $(t_{on})$  as defined by (2). These light pulses would appear amplitude modulated by the volumetric changes of the artery *LA* changes.



Fig. 2. Representation of the light pulses at the photodiode [4].

Heart beat waves have fairly low frequency components, the highest being below 5Hz ( $BW_{BaseBand}$ ) [5] therefore the frequency at which they should be sampled would be the Nyquist frequency ( $f_{Nyquist}$ ) defined below

$$f_S = f_{Nyauist} = 2 \cdot BW_{BaseBand} = 10Hz \tag{4}$$

Assuming that the DC components of the two arriving lights are comparable in magnitude and that there is no motion artifact or fast changes in background light, it would be reasonable to say that the bandwidth of the photodiode  $(BW_{Phot})$  should be the same as the one of the signal sent by a single LED.

Fig. 3 shows the square light pulse signal  $s_{LED}(t)$  of amplitude  $P_{LED,ON}$  emitted by an individual LED. In the figure and the forthcoming calculations the signal pulse time duration  $t_{cm}$  is redefined as  $2\alpha$  and Ts as  $2\pi$ 



Fig. 3.Square wave signal with varying duty cycle emitted by LED.

and the total signal power being

$$P_{LED,TOT} = P_{LED,ON} \frac{\alpha}{\pi} = P_{LED,ON} D_{LED}, [W]$$
(6)

For BW calculation purposes,  $S_{LED}(t)$  can also be expressed as a sum of its Fourier series harmonics x(t)times its amplitude  $P_{LED,ON}$ 

$$s_{LED}(t) = P_{LED,ON} \cdot x(t), [W]$$
<sup>(7)</sup>

where a unitary magnitude square wave x(t) is defined as

$$x(t) = \left\{ \frac{\alpha}{\pi} + \frac{2}{\pi} \sum_{n=1}^{+\infty} \left( \frac{1}{n} \sin(n\alpha) \cos(n\theta) \right) \right\}$$
(8)

and its power spectral density defined as

$$P_{LED,TOT} = P_{LED,ON} \cdot \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(\omega)|^2 d\omega, [W]$$
(9)

In (9)  $X(\omega)$  is the Fourier transform of x(t)

$$X(\omega) = \frac{\alpha}{\pi} + \frac{2}{\pi} \sum_{n=1}^{+\infty} \left( \frac{1}{n} \sin(n\alpha) \frac{\sqrt{2\pi}}{2} \left( \delta(\omega - n\omega_s) + \delta(\omega + n\omega_s) \right) \right)^{(10)}$$

Signal bandwidth is defined as the difference between the highest significant frequency and the lowest significant frequency in a signal spectrum. As the spectrum of the square wave is infinite, the best definition of BW will have to be decided on. Because of the inherent properties of the received signal, its bandwidth will here be defined as starting from zero Hz to maintain the DC component and expanding to the frequency of its highest harmonic (k) required to achieve 95% of the total signal power.

$$P_{BW} = 0.95 \cdot P_{LED,TOT}, [W] \tag{11}$$

$$P_{BW} = P_{LED,ON} \cdot \left\{ \frac{\alpha^2}{\pi^2} + \frac{2}{\pi^2} \sum_{n=1}^{k} \left( \frac{1}{n^2} \sin^2(n\alpha) \right) \right\}, [W] \quad (12)$$

Fig. 4 shows how k increases as  $D_{LED}$  is reduced. These results were obtained from a MATLAB simulation calculating k following (12). As could be expected, k=0 when  $D_{LED} = 100\%$  meaning that no harmonics are required and therefore does not appear on the logarithmic curve. In the range of  $D_{LED}$  {100-10%} a ripple can be perceived as a

result of the sinusoid magnitude factor in the harmonics (12). Further on when  $D_{LED} < 10\%$ , k can be expressed approximately as

$$k \cong 2 \cdot \frac{1}{D_{LED}} \tag{13}$$



Fig. 4. Curve showing the increasing number of harmonics required to reach 95% of  $P_{LED,TOT}$  as duty cycle  $(D_{LED})$  is decreased.

Following on from (13), the required bandwidth of the photodiode  $(BW_{Phot})$  can also be expressed following this approximation.

$$BW_{Phot} \cong f_S \cdot \frac{2}{D_{LED}} \tag{14}$$

In this section of the design considerations, the parameters that would limit the system are; maximum LED bandwidth  $(BW_{LED,max})$ , the maximum photodiode bandwidth  $(BW_{Phot,max})$  and the circuitry bandwidth  $(BW_{circuit})$  resulting in an overall maximum system bandwidth  $(BW_{SYS max})$  over which noise would be integrated.  $BW_{LED,max}$  is defined as a function of the LED rise  $(t_r)$  and fall  $(t_f)$  times following (2, 3, 4)

$$BW_{LED,\max} = \frac{2}{t_r + t_f} \tag{15}$$

$$BW_{SYS\,\text{max}} = \min\{BW_{LED,\text{max}}, BW_{Phot,\text{max}}, BW_{circuit}\}$$
(16)

## *B* – *Light power and noise relationship*

When calculating SpO2, a certain maximum uncertainty margin  $\Delta SpO2$  needs to be guaranteed in order to consider the measurement accurate enough. This uncertainty margin directly reflects as a minimum light power signal to noise ratio (*SNR*) for the received light signal. Studying the required *SNR* will compliment the duty cycle and *BW* study by defining the required light power emitted by the LED ( $P_{LED,ON}$ ) as a function of noise, making the minimum accurate enough to accurate enough the maximum distribution.

the minimum power consumption possible to estimate.

SpO2 is calculated as a function of ratio Q [4] where Q itself is as previously defined in (1).

The propagation of uncertainty of Q can be expressed as:

$$\left(\frac{\Delta Q}{Q}\right)^2 = \left(\frac{\Delta A C_{660}}{A C_{660}}\right)^2 + \left(\frac{\Delta A C_{940}}{A C_{940}}\right)^2 + \left(\frac{\Delta D C_{660}}{D C_{660}}\right)^2 + \left(\frac{\Delta D C_{940}}{D C_{940}}\right)^2$$
(17)

For situations where no motion artifact is present, the uncertainty affects the measurements of all components equally. Since the magnitudes of the different signal components are not equivalent the uncertainty contribution of the DC components can be ignored, giving a simplified expression of the uncertainty of Q.

$$\left(\frac{\Delta Q}{Q}\right)^2 = \left(\frac{\Delta A C_{660}}{A C_{660}}\right)^2 + \left(\frac{\Delta A C_{940}}{A C_{940}}\right)^2 \tag{18}$$

As an example, the maximum acceptable uncertainty  $(\Delta SpO2)$  can be chosen to be 1point of a percent in the range between {100-90%}. Given the required uncertainty margin the calculations for the minimum *SNR* required of the *AC* magnitude measurement would be as follow.

$$\left(\frac{\Delta Q_{100-90\%}}{Q_{100-90\%}}\right)^2 = \left(\frac{1\% \cdot dQ}{max Q_{100-90\%}}\right)^2 = \left(\frac{0.03}{0.8}\right)^2 = \left(\frac{1}{26.66}\right)^2$$
(19)

Assuming that both signals (660nm and 940nm) are performing equally at their lowest *SNR*, the calculation can be simplified to

$$\left(\frac{\Delta Q}{Q}\right)^2 = \left(SNR_{AC660}^{-1}\right)^2 + \left(SNR_{AC940}^{-1}\right)^2 = 2\left(SNR_{AC}^{-1}\right)^2$$
(20)

which then can be rearranged as

$$SNR \ge \sqrt{2 \cdot 26.66} = 37.71$$
 (21)

Pulse oximetry systems have several sources of noise along the signal path and those are, starting from the light source and excluding motion artifact; LED noise  $(N_{LED})$ , photodiode noise equivalent power  $(NEP_{PD})$  and circuit input referred noise  $(N_{CIRC})$ .

With the aim of calculating the LED power consumption tradeoff, all the noise contributions will be referred to the LED. Firstly, the photodiode input referred noise  $(N_{PD,TOT})$  can be expressed as a function of the circuit input referred noise  $(N_{CIRC})$ , photodiode sensitivity (*S*) and the photodiode noise equivalent power  $(NEP_{PD})$ .

$$N_{PD,TOT} = \frac{N_{CIRC}}{S} + NEP_{PD}, \left[W/Hz^{\frac{1}{2}}\right]$$
(22)

Lastly,  $N_{PD,TOT}$  can then be referred as LED driving current noise ( $N_{LED,TOT}$ ) as a function of LA, noise emitted by the LED ( $N_{LED}$ ), LED efficiency ( $\eta$ ), AC/DC ratios for each wavelength. In this expression LA accounts for the ratio between  $P_{LED,ON}$  and the power of the pulses arriving at the photodiode.

$$N_{LED,TOT} = \frac{1}{\eta} \cdot \left( \frac{N_{PD,TOT}}{LA} \cdot \frac{DC}{AC} + N_{LED} \right), \left[ W / Hz^{\frac{1}{2}} \right]$$
(23)

From previous experiments with a self build prototype performing for reflectance PPG on a finger base [9] *LA* was found to be in the rage of  $10^{-3}$ . Considering  $N_{LED}$  to be of similar magnitude as  $NEP_{PD}$ , it would become insignificant due to the *LA* and *AC/DC* factors (23).

In this section of the design considerations, the parameter that would limit the system is the maximum LED light flux.

# $C - P_{IED TOT}$ as a function of LED duty cycle and noise

The generic expression for total minimum LED power consumption ( $P_{LED,TOT,min}$ ) of each LED can be expressed as a function of  $P_{LED,ON,min}$  and  $D_{LED,min}$ .

$$P_{LED,TOT,\min} = P_{LED,ON,\min} \cdot D_{LED,\min}, [W]$$
(24)

where  $P_{LED,ON,\min}$  is itself a function of SNR,  $BW_{SYS\max}$  and  $N_{LED,TOT}$ .

$$P_{LED,ON,\min} = \left(\frac{1}{\eta}SNR \cdot \sqrt{k \cdot f_{S}} \cdot \left(\frac{N_{CIRC}}{S} + NEP_{PD}\right)\frac{DC}{LA \cdot AC}\right)D_{LED}$$
(25)

In order to provide the reader with an evaluation of the expected power consumption results  $P_{LED,TOT,min}$  has been calculated for the following values from the literature across the interesting range of  $D_{LED}$  and SNR values;  $N_{CIRC} = 4.2 \cdot 10^{-12}$   $\begin{bmatrix} pA/Hz^{1/2} \end{bmatrix}$  [8],  $LA = 10^{-3}$  [9],  $BW_{circuit} = 3.5 \cdot 10^{6} [Hz]$  [8],  $NEP_{PD} = 9.6 \cdot 10^{-16}$   $\begin{bmatrix} w/Hz^{1/2} \end{bmatrix}$  [10], S = 0.6 [A/W] [10],  $\eta = 0.25$  [11], AC/DC=0.03 [4].

The numerical values of the limiting factors in this design are; maximum LED light flux 120mW for 660nm device and the  $BW_{SYS \max} = 1GHz$  due to the LED speed of the 940nm device. LED speed is ruled by  $t_r = t_f = 10^{-6}$  sec respectively. Following (2, 3, 4) the minimum duty cycle ( $D_{LED,\min}$ ) can be calculated to be

$$D_{LED,\min} = 2 \cdot 10^{-5} = 0.002\% \tag{26}$$

Fig. 5 presents the 
$$P_{IED TOT min}$$
 results of the

aforementioned calculations following (25), for the literature values as a function of D and *SNR* displaying the full possible range of D values.

At this stage, the designer might want to use a different set of components or circuit and therefore the overall design values changing but this study will serve as a key tool in optimizing LED power consumption.

## IV. CONCLUSION

Graphs and derivations have been presented which aid the designer in determining when it is advantageous to vary the LED duty cycle and desired SNR in order to attain the minimum possible LED power consumption, depending on maximum system bandwidth ( $BW_{SYSmax}$ ), maximum LED

light flux and overall system noise ( $N_{LED,TOT}$ ). An example has been shown for a possible system with real specifications. As a result, it has been demonstrated that by reducing to the minimum possible  $D_{LED}$  and for the desired SNR=37.71,  $P_{LED,TOT}$  can be reduced by up to three orders of magnitude down to the microwatt range.



Fig. 5. Curve representing  $P_{LED,TOT,min}$  values as a function of the desired  $D_{LED}$  and *SNR* given the aforementioned system characteristics.

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