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Two-dimensional least-squares estimation for motion tracking in ultrasound elastography

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***Abstract*—This paper proposes a method of 2-D translations estimation using an *a priori* signal model. Two analytical signals defined with multidimensional Hilbert transform are considered and shown to have linear phases with respect to the translations to estimate. A least squares estimator (LSE) is then developed to adjust the measured phases of the complex signals to their theoretical forms. Moreover, the LSE provides an analytical solution to the 2-D translation estimation problem. The estimator is then included in a block matching method for motion tracking with ultrasound images. We compared our results with those obtained with a classical sum of absolute differences (SAD) cost function. We show that with our method there is no need of interpolating the images. Thus, for images at the original resolution level, the results obtained with the proposed estimator are largely more accurate than with SAD. Moreover, we show that using SAD on images with resolution five times higher provide roughly the same results as with our method, but the processing time is ten times higher in this case.**

I. Introduction

Ultrasound elastography [1] is a technique of characterizing the elasticity of tissues. It consists in acquiring two or more images of the same medium under different levels of compression. In most static elastography applications the compression is applied directly with the ultrasound probe. Strain images are usually calculated by derivation of the estimated motion between the acquired images. Many conventional 2-D tracking techniques use block matching based methods to estimate this motion. Under small deformation, the local displacements to estimate are usually small compared to ultrasound images resolution. Thus, one of the main challenges in motion tracking for elastography is to find local translations estimators capable to estimate such displacements. Among these estimators, we find in the literature methods based on cost functions [2] such as normalized and non-normalized cross-correlation, sum of absolute differences, sum of squared differences. A comparison between these estimators applied to ultrasound images can be found in [3]. It can be easily seen that the accuracy of this type of estimator is directly depending on the images sampling frequency. Thus, interpolation is generally used to obtain sub-pixel estimation precision [4].

Moreover, estimators using the phase of the complex cross-correlation function were proposed. Thus, we can find in [5] a 1-D iterative phase zero estimation. Extensions to two-dimensions were proposed by Sumi in [6] and [7] and by Ebbini in [8].

The aim of this study is to propose a new technique of analytically estimating sub-sample local 2-D translations based on an *a priori* model of images. The proposed estimator is applied directly on the signals. Further, the final estimation, which is the relative delay between the two signals, is calculated as the difference between the estimation found for each signal. Thus, with our method there is no need of processing the cross-correlation function.

As shown by Liebgott *et al.* in [9], non-conventional beamforming techniques enable access to ultrasound radio-frequency (RF) images with lateral modulations. This allows us to consider that images can locally follow the 2-D signal model presented in this paper, based on a product of two sinusoids. Phase adjustment between measured phases of two single-orthant analytical signals and their theoretical forms is then achieved using least squares method. Thus, a system of two equations is found and allows us to analytically calculate the estimation of the 2-D translations. We show how our estimator can be used to estimate 2-D translations with a block matching method. The results presented are considered with experimental images on phantom. The performances of the proposed estimator are compared to classical SAD cost function. We show that with images at the original resolution our estimator is more accurate. Moreover, motion estimation with SAD applied on images interpolated by a factor of five in both directions was processed. In this case, the estimation accuracy is roughly the same as with our estimator applied on the original images, but the computation time becomes ten times higher.

II. Signal Model

In this work, the proposed method estimates the 2-D translations vector $\boldsymbol{d} = (d_1, d_2)^T$ considering the 2-D function $r : \mathbb{N}^2 \to \mathbb{R}$ given in (1).

$$r(m,n) = \cos(2\pi f_1 (m - d_1)) \cdot \cos(2\pi f_2 (n - d_2)) \cdot w(m,n) \qquad (1)$$

where f_1 and f_2 are the normalized frequencies on the two main directions of the signal r, m and n are the variables of each dimension and $w(m,n)$ is a 2-D window defined arbitrarily and having its Fourier spectrum disjoint from the spectrum of the 2-D cosinusoid.

Let us define by small letters signals in the spatial domain and by capital letters their 2-D Fourier transform. Thus, we can define two analytical signals [10], noted $r_1(m,n)$ and $r_2(m,n)$, and which 2-D Fourier transforms are calculated as follows.

$$R_1(u_1,u_2) = (1+\mathrm{sgn}(u_1))(1+\mathrm{sgn}(u_2))R(u_1,u_2) \tag{2}$$

$$R_2(u_1,u_2) = (1-\mathrm{sgn}(u_1))(1+\mathrm{sgn}(u_2))R(u_1,u_2), \tag{3}$$

where the couple (u_1,u_2) is the 2-D variable in frequency domain.

Analytical calculations show that, given the form of signal in (1), the phases of the two analytical signals r_1 and r_2 have the form:

$$\Phi_1(m,n) = 2\pi f_1(m-d_1) + 2\pi f_2(n-d_2) \tag{4}$$

$$\Phi_2(m,n) = -2\pi f_1(m-d_1) + 2\pi f_2(n-d_2) \tag{5}$$

III. Least Squares Estimation

Given the linear phases in (4) and (5), we use a least square method to estimate d_1 and d_2. The two expressions in (4) and (5) lead to the data model in (6).

$$\boldsymbol{y} = H\boldsymbol{d}, \tag{6}$$

with

$$\boldsymbol{y} = \begin{pmatrix} 2\pi f_1 m + 2\pi f_2 n - \phi_1(m,n) \\ -2\pi f_1 m + 2\pi f_2 n - \phi_2(m,n) \end{pmatrix}$$
$$H = \begin{pmatrix} 2\pi f_1 & 2\pi f_2 \\ -2\pi f_1 & 2\pi f_2 \end{pmatrix}, \tag{7}$$

where ϕ_1 and ϕ_2 are the measured phases of the two defined analytical signals r_1 and r_2.

In the linear model in (6), $\boldsymbol{y}$ and H are known exactly and our objective is to minimize the square error between $\boldsymbol{d}$ and the estimate of $\boldsymbol{d}$, noted $\hat{\boldsymbol{d}} = \left(\hat{d}_1, \hat{d}_2\right)^T$. In the literature, it is shown that among all possible unbiased estimators the least squares estimator (LSE) minimizes the variance [11]. The solution given by the LSE is [12]:

$$\hat{\boldsymbol{d}} = \left(H^T H\right)^{-1} H^T \boldsymbol{y}, \tag{8}$$

where by $(\cdot)^T$ we denote the transpose of a matrix.

We can easily show that in our case the matrices H and H^T are invertible, which let us simplify the expression in (8).

$$\hat{\boldsymbol{d}} = H^{-1}\boldsymbol{y} \tag{9}$$

The result in (9) gives the form of our estimations:

$$\begin{pmatrix} \hat{d}_1 \\ \hat{d}_2 \end{pmatrix} = \begin{pmatrix} m - \dfrac{1}{4\pi f_1}\left(\phi_1(m,n) - \phi_2(m,n)\right) \\ n - \dfrac{1}{4\pi f_2}\left(\phi_1(m,n) + \phi_2(m,n)\right) \end{pmatrix} \tag{10}$$

In practical applications we do not use only one measure of phases $\phi_1(m,n)$ and $\phi_2(m,n)$, but a series of measures which lead us to consider $m \in [M_1, M_2]$ and $n \in [N_1, N_2]$. In this case, the final estimation is the mean value of the estimations corresponding to each measure. Estimations in (10) become:

$$\begin{aligned} \begin{pmatrix} \hat{d}_1 \\ \hat{d}_2 \end{pmatrix} &= \frac{1}{(M_2 - M_1 + 1)(N_2 - N_1 + 1)} \begin{pmatrix} \displaystyle\sum_{m=M_1}^{M_2}\sum_{n=N_1}^{N_2} m - \frac{1}{4\pi f_1}\left(\phi_1(m,n) - \phi_2(m,n)\right) \\ \displaystyle\sum_{m=M_1}^{M_2}\sum_{n=N_1}^{N_2} n - \frac{1}{4\pi f_2}\left(\phi_1(m,n) + \phi_2(m,n)\right) \end{pmatrix} \\ &= \begin{pmatrix} \dfrac{M_1 + M_2}{2} \\ \dfrac{N_1 + N_2}{2} \end{pmatrix} - \frac{1}{(M_2 - M_1 + 1)(N_2 - N_1 + 1)} \begin{pmatrix} \dfrac{1}{4\pi f_1} \displaystyle\sum_{m=M_1}^{M_2}\sum_{n=N_1}^{N_2}\left(\phi_1(m,n) - \phi_2(m,n)\right) \\ \dfrac{1}{4\pi f_2} \displaystyle\sum_{m=M_1}^{M_2}\sum_{n=N_1}^{N_2}\left(\phi_1(m,n) + \phi_2(m,n)\right) \end{pmatrix} \end{aligned} \tag{11}$$

A statistical study of our 2-D estimator can be found in [13].

IV. APPLICATION

A. RF image formation

We presented in the previous section a method of 2-D translations estimation using the signal model in (1). In order to apply our estimator to ultrasound RF images, we use a non conventional beamforming technique which allows images to follow our signal model. Note that conventional images present oscillations only in the direction of ultrasound wave propagation, which we call axial direction. Therefore, oscillations in the lateral direction of the images are necessary to allow the 2-D local model approximation in (1). Lateral oscillations are obtained by the beamforming method discussed by Liebgott *et al.* in [9]. This method is based on the Fourier relation between the lateral profile of the acoustic pressure field and the weighting function applied to the active part of the ultrasound array in receive. The beamforming method was implemented on the research scanner Sonix RP by Ultrasonix Medical Corporation, Canada, with a 8-MHz linear.

Figure 1 shows the experimental point spread function (PSF) that we get using this beamforming technique and its corresponding profiles. The lateral and axial profiles of the PSF confirm the interest of using this beamforming method in obtaining images following controlled models in both spatial directions. Note that the frequencies in (1) are depending on the characteristics of the PSF, which allows us to consider them known in the estimation method ($f_1 = 5.5mm^{-1}$, $f_2 = 0.57mm^{-1}$).

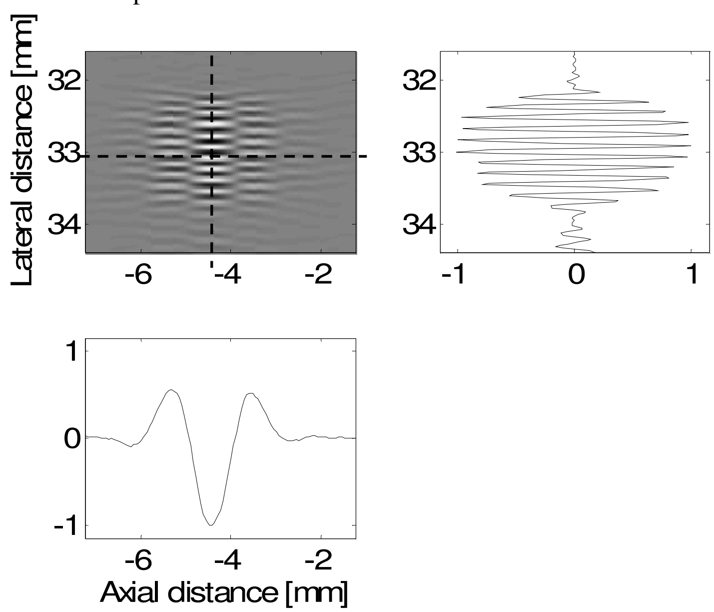


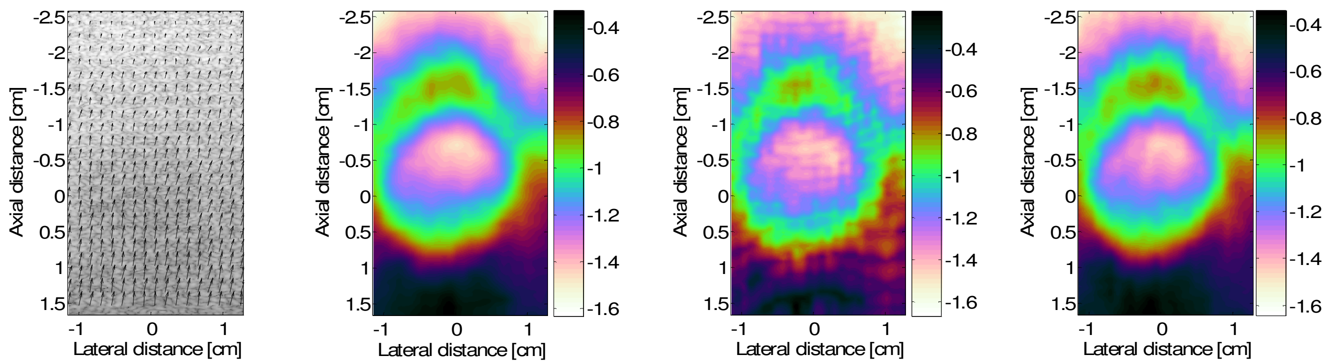
Figure 1. Experimental point spread function corresponding to ultrasound RF images with lateral oscillations and two axial and lateral profiles, corresponding to the two dashed lines

B. Results

With ultrasound elastography, strain images are usually calculated by derivation of the estimated displacement induced by a compressive force applied to the tissue surface. The most widely used technique of 2-D motion estimation for ultrasound elastography is speckle tracking, known as block matching in video applications [14]. 2-D translations are then locally estimated using classical research criteria as normalized cross-correlation (NCC) or sum of absolute differences (SAD). In ultrasound elastography, images resolution and the way of applying the deformation directly with the ultrasound probe make the local displacements small and often smaller than the pixel size. Thus, in order to estimate sub-pixel displacements, classical block matching requires that images be interpolated.

We use here the estimator described in the previous sections to estimate sub-pixel 2-D translations without interpolating the 2-D signals. The results are compared to those obtained using SAD cost function.

The experimental result we present in this paper is considered with phantom data. The phantom (Elasticity QA Phantom, model 049, by CIRS Tissue Simulation & Phantom Technology, USA) was designed for ultrasound elastography and presented a spherical 20-mm diameter inclusion of 6 kPa for a surrounding medium of 29 kPa. The ultrasound RF images were acquired and formed using the beamforming method described previously. Note that the inclusion is clearly visible on the strain images, figures 2 (b), (c) and (d), whereas it is not the case on the ultrasound image. On strain images the inclusion is the region which is the most deformed, as it is softer that the other parts of the phantom. The displacement corresponding to the strain image in figure 2(b) was estimated using the proposed estimator applied on the images at the original resolution. Figures 2(c) and 2(d) are obtained using a classical estimation with SAD cost function applied on the original images and respectively on the images interpolated by factors of five in both directions. Note that if the computation time is roughly the same to obtain results in figures 2(b) and 2(c), the estimation time for the result in figure 2(d) was ten times higher. The ultrasound B-mode image in figure 2 (a) was calculated by axial demodulation of the RF image.



(a) (b) (c) (d)

Figure 2. Estimated motion vectors with the proposed estimator superimposed on the experimental ultrasound B-mode image (a) and axial strain image in % (b) with our estimator using the original images, (b) with SAD using the original images, (d) with SAD after interpolation of the original images by a factor of 5 in both directions.

V. Conclusion

A new method of 2-D translation estimation based on a given signal model is presented. The linear phases of two analytical signals and a least squares estimator allow us to analytically estimate the local displacement. Further, the estimator is integrated in a block matching method and used for motion tracking with non-conventional ultrasound RF images. Indeed, a special beamforming technique gives the possibility to have RF images locally following the proposed 2-D signal model.

An application on motion tracking for ultrasound elastography is considered. It shows that our estimator gives also good results for low resolution images. Further, we compare our results with a conventional SAD cost function applied on original data and on images oversampled by factors of five in both directions. Thus, we show that for the original resolution level our estimator is largely more accurate than the classical SAD. To achieve roughly the same accuracy with SAD as with our estimator applied on the original ultrasound data, an interpolation of the images by a factor of five was processed, but in this case the computation time becomes ten times higher.

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