COMPARISON OF PHASE COUPLING PARAMETERS FOR THE ANALYSIS OF NEURONAL SYNCHRONIZATION

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Abstract: The investigation of large-scale neural integration by synchronization is essential for the understanding of the brain organization. Up to the present, transient phase synchronization is one of most suitable concepts for a neural interaction process explanation. This paper deals with the quantification and detection of the bivariate phase coupling. Here the most applied phase synchronization parameters in neuroscience i.e. phase locking value (PLV) as well as Shannon entropy parameters are compared. The features of these approaches can directly be tested on generated circular data without estimating instantaneous phases. For this purpose, instantaneous phase sets with defined probability distributions are generated. The results of the study show that the PLV represents the most suitable analysis method for detecting phase synchronization in noisy signals.

Introduction

The investigation of electrophysiological brain signals, acquired during the execution of cognitive processes revealed that transient cooperation of numerous functional brain regions are realized by dynamic interactions of widely distributed neural assemblies separated by large distances that transmit signals constantly to each other. In the literature, such interactions have also been addressed as "large-scale synchronization" [1] and can be investigated by bivariate analyses (relation between two localisations). Hypothesised that during cognitive acts the interactions of neuronal populations may be realized as phase synchronous activities present for a certain time window, the information about the instantaneous phases of the (scalp acquired) brain signals provides an important index to such large-scale synchronization phenomena. For this reason, such interactions are also known as phase synchronizations [1]. Additionally, integration of nerve cells to an assembly is called local synchronization which can be observed by univariate analyses like ERD/ERS [2]. It is to be emphasized that several neurological diseases such as epilepsy or Parkinson manifest a pathological form of the phase synchronization processes.

The mathematical approaches to quantify such large-scale interactions constitute a bivariate phase coupling analysis, which is generally implemented by a methodically linear way. After data acquisition and pre-processing not only the extraction of instantaneous phase information, but also the phase quantification is performed for each channel in order to compute events of phase coupling. Finally, a significance test follows. Respecting the great variety of existing phase quantification parameters, the gist of the study presented is the comparison of phase quantification parameter and the selection of the most suitable one for the succeeding bivariate phase coupling detection.

Materials and Methods

General Procedure

The single steps in the signal processing depend on the problem to be investigated and on the quality of the recorded data. In general pre-processing, phase extraction, phase quantification and detection steps are considered. In case of EEG signals, pre-processing usually consists of a filtering of the raw signals (e.g. Laplacian, second spatial derivation) as well as of a band-pass filtering. The second step, the extraction of instantaneous phase information, takes place by Short-Time Fourier-Transform (STFT), Hilbert-Transform or Wavelet-Approach. The differences of the mentioned fundamental methods to extract instantaneous phases were already discussed extensively by Bruns [3]. The author of this study concluded that these approaches constitute no difference on a certain level, and thus all these approaches should be formally and effectively equivalent. The current paper can be understood as developing the consequence of the Bruns study further. The extracted instantaneous phases of two channels are analyzed in terms of phase coupling by using either entropy parameters or, more generally, by using phase coupling indexes. To test the statistical significance of such coupling indexes, a nonparametric binary test is commonly used, since in this case no a-priori hypotheses concerning the distribution of the phase data are required. Permutation methods or methods of surrogate data are used to test the coupling indexes against the background fluctuations [4], while bootstrap methods deliver a confidence that can be contrasted with the one of another interval [4].

In particular, this work focuses on the third step following the methodically order, namely the possible opportunities to quantify phase coupling values. Adequate data acquisition, pre-processing and extraction of the instantaneous phases were assumed to be already performed successfully. Furthermore, the final statistical step could be a special topic of further works.

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Phase synchronization

The transformations of the real valued data in step two provide complex coefficients C_x and C_y, while the instantaneous phase of each channel corresponds to the angle of their complex coefficients. Phase synchronization is realized, if the difference of these two instantaneous phases φ_x and φ_y, the so called relative phase θ, is equal to zero. This definition means a very strong constraint and is hardly applicable to real data. For this reason, a weaker definition was postulated for irregular or non static oscillation systems as exemplified by the brain, where events of phase synchronization are found, since the relative phase may stay constant over several epochs (trials) or within selected time intervals, generally over a representative sample.

Phase quantification parameters

In quite diverse fields, long history efforts of developing phase quantification methods can be found. The underlying objective of such attempts was the quantification of coupling phenomena either taking place in natural subsystems or nonlinear dynamics of technical systems. In the field of neuroscience, phase synchronization has been defined just for the bivariate case so far, a constraint possibly done for the sake of simplification. The most frequently used fundamental parameters in order to detect instances of phase synchronization are Shannon's entropy as well as phase locking value PLV, phase coherence or phase consistency respectively. The application of Shannon entropy as phase coupling measure was introduced in form of a standardized and slightly modified version by Bhattacharya et al. [5]. Accordingly, perfect synchrony should be realized (index equal to one), if the relative phases are δ-distributed. In addition, the signals are regarded to be independent (index equal to zero), if the relative phases are uniformly distributed onto the circle indicating the maximal entropy:

$$H_{\theta,\max} = \log_2\left(M\right). \tag{1}$$

Thus, the strength of phase synchronization is expressed as deviation of circular uniform distribution and is quantified by the index called modified standardized Shannon entropy of the relative phase:

$$\rho = \frac{H_{\theta,\max} - H_\theta}{H_{\theta,\max}}, \tag{2}$$

with the entropy defined by:

$$H_\theta = -\sum_{i}^{M} p_i \log_2\left(p_i\right), \tag{3}$$

where M represents the number of bins, and p_i the probability of the relative phases within the i^{th} bin. According to the investigations of R. Otnes and L. Enochson [6], the optimal number of bins should be $e^{0.626+0.4\ln(N-1)}$, where N is the number of samples. Furthermore, a modified application of Shannon's entropy is known as joint entropy [7], which considers the both phases:

$$H_{\varphi_x\varphi_y} = -\sum_{i}^{M}\sum_{j}^{M} p_{i,j} \log_2\left(p_{i,j}\right). \tag{4}$$

$P_{i,j}$ denotes here the joint distribution of the both instantaneous phases. We assume that the investigated instantaneous phases are equal in their number of samples. The corresponding phase coupling index is defined in the same way as the Shannon entropy with the only exception that the impact of the number of samples is double for maximal joint entropy:

$$H_{\varphi_x\varphi_y,\max} = \log_2\left(M^2\right) \tag{5}$$

Another approach to detect phase coupling is the so called PLV, also known as phase consistency or phase coherence that was started to get most prominently introduced in the field of neuroscience by Lachaux [8]. The PLV index is defined as an averaged value that measures the variability of the relative phase within a representative sample (over a set of experimental trials or epochs for example):

$$\chi = \left|\frac{1}{M}\sum_{i=1}^{M}\frac{C_x \cdot C_y^*}{\left|C_x\right|\left|C_y\right|}\right| = \left|\frac{1}{M}\sum_{i=1}^{M} e^{i\cdot\theta}\right|. \tag{6}$$

More precisely, this phase coupling index evaluates the relative phases in the form of their angles onto the unit circle in the complex (Gaussian) plane. PLV is similar to the definition of coherence, except that amplitude contributions of the complex coefficients are eliminated. So if the phase difference varies a little across the sample, the index is close to one, otherwise the index results in a value close to zero.

Formal Comparison

Phase coupling indexes based on entropy required the estimation of discrete probabilities or their corresponding histograms, respectively. Hence, the continuous and closed-phase space [0, 2π) is subdivided into finite elements. As found out by R. Otnes and L. Enochson an optimal number of elements in terms of linear histogram can be approximated based on the fact that different numbers of bins result in different estimation of histograms that differ in accuracy. In addition to the proposed bin estimation, there is still another adjustable parameter affecting the uncertainty of a given estimation. Circular data provides an angular histogram. And, in contrast to linear histograms, angular histograms involve an arbitrary choice of the starting point. A non appropriate choice of the starting point for the elements (0°, 10° etc.) can result in serious distortion of probability information in the sample about the localizations of modal groups. Finally, entropy parameters depend on probability estimation and consequently on the number of finite elements (discretization), and more importantly, in case of circular data on the starting point of circular division. Applying entropy parameters to circular data delivers a bimodal phenomenon due to the linearization of circular data. I.e., if the phase sample is grouped a modal way around a mean direction of 0°, a linearization at starting point 0° cause a false estimation of bimodal probability distribution. In contrast to the entropy based phase-coupling measures, the PLV-index requires no additional estimation and, thus, no additional parameters must be adjusted. In this consideration, the PLV represents advantageously phase coupling index than entropy based indexes.

Data

To compare the selected phase quantification parameters that determine the phase coupling, sets of circular data are simulated. Due to the fact that the phase coupling parameters are computed only along one dimension of the representative sample (i.e. parameters represent one-dimensional operators) the features of each approach could be tested directly on circular data without estimating instantaneous phases. Hence, supposed that the fundamental approaches are the same, it is not necessary to generate multi-trial circular data as well as real valued single-trial and multi-trial data sets. Therefore, bivariate data sets of (instantaneous) phases with defined probability distributions, in order to engender phase synchronization, are created by a random generator, which are based on certain statistical models. For the purpose of detecting phase synchronization, the following postulate could be formulated. The distribution of representative samples like a set of trials is unimodal. Hence, zero valued PLV-indices are excluded following the reason of symmetric multimodal phase distribution (higher order).

The most common unimodal distribution model for circular data, the von Mises Distribution [9] was used to generate random phases:

$$f(\varphi) = \frac{1}{2\pi \cdot I_0(\kappa)} e^{\kappa \cdot \cos(\varphi - \mu)}, \qquad (7)$$

where $I_0(\kappa)$ describes the modified Bessel function of order one. This distribution converges to circular uniformity, if the concentration factor κ is close to zero and otherwise the distribution shows a tendency towards the δ-distribution concentrated around the mean direction μ. The simulation of von Mises distributed random phases was performed in accordance to an algorithm of Best and Fisher [9]. Another unimodal circular distribution model called Wrapped Normal Distribution is derived from the Normal or Gaussian Distribution in case of linear data, respectively. Both distributions are symmetric and satisfy the postulated unimodality. The Wrapped Normal Distribution and the von Mises Distribution are similar in appearance and differ only by a high order of sample sizes, in which the peak of the von Mises Distribution is sharper [9]. Furthermore, it is possible to transfer the parameters of both distributions to each other. These circumstances enabled us to represent also by means of the ordinary parameters of the Normal Distribution. Finally, two discrete vectors of M=1000 circular pseudo-random phases were generated from the von Mises Distribution. Γ=1000 realizations were produced in order to simulate different distributions. The mean direction for each data set was thereby fixed, i.e. adjusted to a radian of $\mu_x=1/4\pi$ for channel x and $\mu_y=2/3\pi$ for channel y. In contrast to the mentioned mean direction, the concentration factor was indirect specified in a nonlinear way based on linear adjusted ordinary (cycle) standard deviation σ for the sake of improved illustration. If either one or both concentration factors are low, no phase coupling is present and thus a minor phase coupling index will be expected. Otherwise, if both concentration factors are large (circular standard deviation is close to zero), a phase coupling index close to one will be expected.

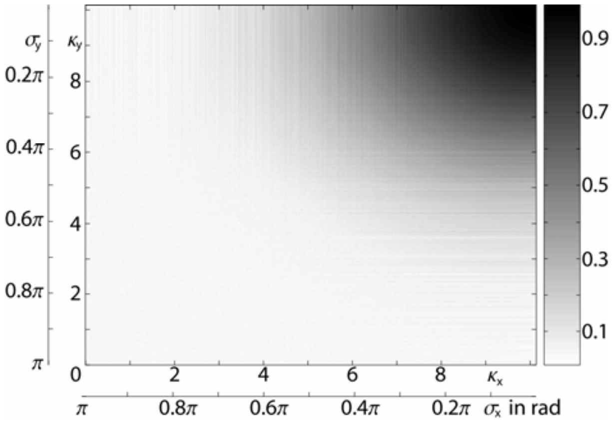

Figure 1: PLV-index dependence on concentration factor κ (or cycle standard deviation σ). The axes describe the parameters of two synthetic phase sets x and y.

Results

In the case of phase coupling index based on standardized joint entropy leakage effects were observed. The resulting index expected to be close to one, if both sample distributions are adjusted by a high concentration factor. But if one of the considered channels is closely δ-distributed and the second one deviated from the δ-distribution (is e.g. circular uniform distributed) then the channels are not phase synchronized, but the modified joint entropy index takes values higher than zero. Summarizing, this phase quantification index is not suitable for detecting phase coupling. The PLV-index as well as the index based on standardized Shannon entropy evinces (in figure 1) spread punctually behaviour within the two dimensional channel parameter-plane. Therefore, both indexes are expected to be close to one, if both channels are δ-distributed. And if either one or both phase sample distributions slightly deviate from the δ-distribution, the indexes decrease towards zero with different gradient. In general, both phase quantification parameters are suitable to detect bivariate phase coupling. The gradient for phase coupling index based on Shannon entropy is higher than the PLV gradient (see figure 2). Thus, the modified standardized Shannon entropy index is more sensitive to noise and probability deviations and, consequently qualitatively less suitable than the PLV-index.

Discussion

Principally two phase quantification approaches, the PLV (phase consistency or phase coherency respectively) and Shannon's entropy were under consideration. In case of Shannon’s entropy, the standardized as well as the standardized joint entropy were explicitly investigated. The formal comparison yielding that the PLV is more accurate than Shannon's entropy parameters, since the Shannon entropy required an additional factor to be estimated (i.e. histogram)

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necessitating the determination of two parameters. In contrast to Shannon's entropy, the PLV- index is parameter free and, thus, generally easier to manage. PLV-index, phase consistency or phase coherence is a special parameter to quantify phases but Shannon's entropy is defined for linear data in terms of statistics and thus only applicable to circular data. The simulation provided evidence that the joint entropy is not an eligible parameter to test phase samples on synchronization, since the probabilities of both phase samples made a separate and specific contribution. The other examined phase quantification parameters based on standardized Shannon entropy and the PLV obtained results indicating their suitability of detecting phase synchrony. The simulation results alowed the conclusion that the standardized Shannon entropy is more sensitive to noise than the PLV. Finally, the PLV is the most suitable phase quantification technique of the examined parameters in order to detect bivariate phase coupling. This index corresponds to the traditionally known definition of the mean resultant length and consequently to the definition of the first kind trigonometric moment (centred as well as uncentred) in statistical theory of circular data [9]. Thus, with our novel view in term of phase synchronization it is firstly possible to establish parametric tests.

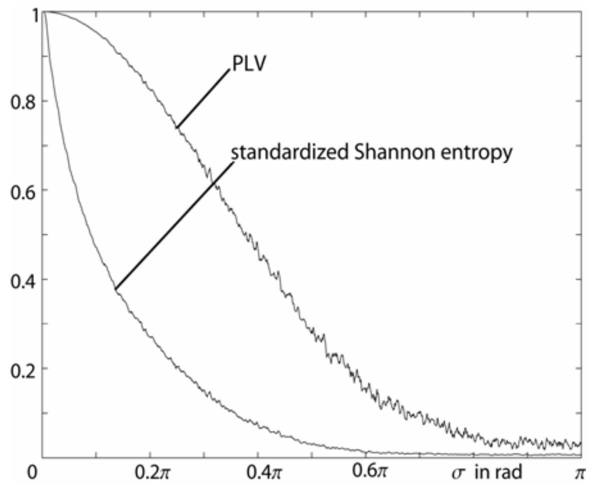

Figure 2: PLV-index and standardized Shannon entropy (y-axis) dependence on cycle standard deviation σ (x-axis).

Conclusions

The main concern of this study was the comparison of the most common phase quantification parameters: PLV and Shannon's entropy measures (standardized and joint entropy) that are applied to determine instances of phase coupling or synchronization. For this purpose, the fundamental techniques were described and compared in terms of their formal functionality and their parameters. Furthermore, synthetic data sets of phases were produced to investigate the phase quantification parameters suitability for the succeeding detecting aims. The data was produced by a random generator and the coupling was adjusted by parameters of the underlying statistical model.

As result of the study can be concluded that the PLV is the most suitable phase quantification method of the examined parameters in order to detect bivariate phase coupling. The equalization of the PLV index and the first trigonometric moment may enable more and eventually better statistics for testing the PLV on significance. Additionally the comparison of parametric and nonparametric statistical tests for the purposes of phase synchronization detection should be investigated in further studies.

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