A SIMULATION PROGRAM FOR ULTRASOUND TOMOGRAPHY IMAGING BASED ON FIELD II

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Abstract: Pulse-echo ultrasound signal formation can be simulated by numerical emulation of the process chain: emit signal - electromechanical emit transformation - wave (e. g. pressure) propagation and scattering - electromechanical receive transformation - receive signal. The simulation software Field II has been successfully used for this task. We present an extension to Field II for the simulation of the signal formation in tomographic ultrasound imaging. Signals due to scattered and incident field components are taken into consideration. An example is given for an ultrasound tomography system with eight transducers.

Introduction

Ultrasound tomography can provide information that is missing in conventional B-mode imaging and so is of interest for several clinical applications. However there are still numerous practical problems associated with most tomographic ultrasound approaches, making further research necessary. Contrary to classical tomography algorithms straight line propagation of energy can generally not be assumed with ultrasound pressure waves but diffraction effects have to be taken into account. A simulation program for the signal formation in ultrasound tomographs which allows for diffraction is presented.

Numerous articles have been published regarding the calculation of transient pressure fields of acoustic transducers. A review has been given by Harris [1]. The most common method relies on Green's function solutions of the homogeneous wave equation under appropriate boundary conditions in the time domain [2]. This method has also has been extended to compute the pressure impinging onto a transducer from a reflecting body due to acoustic reciprocity [3]. Lhémery pointed out several assumptions that are made often only implicitly in previous papers, thus providing an important insight into the limitations of the method [4]. However the results derived are generally in good agreement with experimental findings [3, 5].

Jensen applied the method to calculate the reception signals in pulse echo ultrasound imaging due to scattering processes in human tissue under the Born approximation [5]. Based on this theory, he implemented the software package Field II, which has become a standard tool in the simulation of ultrasonic pulse echo imaging. However, Field II is not capable to handle ultrasound systems with separately located transducers for transmission and reception and can therefore not be used to simulate ultrasound tomography systems. Recently Bloomfield published an extension of the Field II formalism to separately located transmit and receive apertures [6].

We implemented a simulation program for the signal formation in ultrasonic diffraction tomography with an arbitrary number of transmitting and receiving transducers on the basis of Field II and Bloomfields extension theory in Matlab®. Our program includes the calculation of signal components from the incident field which are missing in Bloomfields paper.

Theory

The geometry of the ultrasound tomography system with an exemplary transmitting transducer m, an exemplary receiver n and a point scatterer k inside the irradiated object is depicted in Figure 1. A complete tomography system consists of a number of transducers which are generally used both as transmitters and receivers. Usually, although not necessarily, the transducers are situated in a circle around the object which is going to be probed by ultrasound waves.

Wave Equation: Assuming linear conditions, propagation of sound pressure waves $p(\mathbf{r},t)$ inside the acoustically irradiated object is described by the inhomogeneous wave equation [7]

$$\nabla\left(\frac{1}{\rho(\mathbf{r})}\nabla p(\mathbf{r},t)\right)-\kappa(\mathbf{r})\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2}=0. \qquad (1)$$

The equilibrium density and compressibility of the medium at position $\mathbf{r}$ are denoted by $\rho(\mathbf{r})$ and $\kappa(\mathbf{r})$ respectively.

By applying the product rule onto (1):

$$\nabla\left(\frac{1}{\rho(\mathbf{r})}\right)\nabla p(\mathbf{r},t)+\frac{1}{\rho(\mathbf{r})}\nabla^2 p(\mathbf{r},t) - \kappa(\mathbf{r})\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2}=0 \qquad (2)$$

and using the relation between speed of sound $c(\mathbf{r})$, den-

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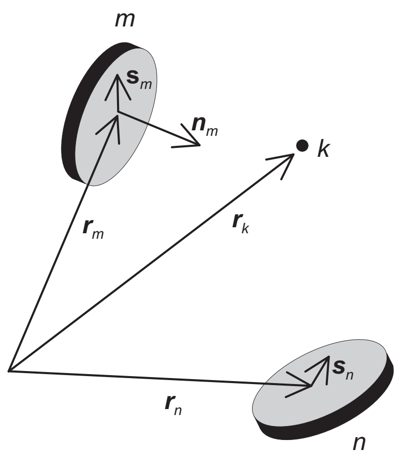

Figure 1: Geometry of the ultrasound tomography system with exemplary transmitter m, exemplary receiver n and point scatterer k.

sity $\rho(\mathbf{r})$, and compressibility $\kappa(\mathbf{r})$ of the medium

$$c(\mathbf{r}) = \frac{1}{\sqrt{\rho(\mathbf{r})\kappa(\mathbf{r})}} \tag{3}$$

the wave equation can be rewritten as

$$\nabla^2 p(\mathbf{r},t) - \frac{1}{c^2(\mathbf{r})}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2} = \frac{\nabla\left(\rho(\mathbf{r})\right)\nabla p(\mathbf{r},t)}{\rho(\mathbf{r})}. \tag{4}$$

Differentiation rules and introduction of an arbitrary reference density ρ_{ref} lead to an alternative formulation of the inhomogeneous wave equation:

$$\nabla^2 p(\mathbf{r},t) - \frac{1}{c^2(\mathbf{r})}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2} = \nabla\left(\ln\left(\frac{\rho(\mathbf{r})}{\rho_{\mathrm{ref}}}\right)\right)\nabla p(\mathbf{r},t), \tag{5}$$

which was used by Chernov [8, p. 37] and Jensen [5].

To solve the inhomogeneous wave equation (1) or (5) respectively for scattering media with non-trivial functions $\rho(\mathbf{r})$ and $\kappa(\mathbf{r})$ the pressure field $p(\mathbf{r},t)$ is decomposed into an incident field $p_0(\mathbf{r},t)$ which would be present in a homogeneous i. e. scatterer free medium and an additional scattered field $p_{\mathrm{s}}(\mathbf{r},t)$ [7]:

$$p(\mathbf{r},t) = p_0(\mathbf{r},t) + p_{\mathrm{s}}(\mathbf{r},t). \tag{6}$$

For the fictitious homogeneous medium with a density $\rho(\mathbf{r}) = \rho_{\mathrm{h}}\ \forall\mathbf{r}$, compressibility $\kappa(\mathbf{r}) = \kappa_{\mathrm{h}}\ \forall\mathbf{r}$ and speed of sound $c(\mathbf{r}) = c_{\mathrm{h}}\ \forall\mathbf{r}$, the wave equation eq. (1) or eq. (5) respectively reduces to

$$\frac{1}{\rho_{\mathrm{h}}}\nabla^2 p(\mathbf{r},t) - \kappa_{\mathrm{h}}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2} = 0 \tag{7}$$

or to the equivalent equation

$$\nabla^2 p(\mathbf{r},t) - \frac{1}{{c_{\mathrm{h}}}^2}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2} = 0. \tag{8}$$

Incident Pressure Field: The incident pressure $p_{0m}(\mathbf{r},t)$ from transmitting transducer m is calculated by utilizing a Green's function solution to the homogeneous wave equation for the velocity potential $\psi_{0m}(\mathbf{r},t)$

$$\nabla^2\psi_{0m}(\mathbf{r},t) - \frac{1}{{c_{\mathrm{h}}}^2}\frac{\partial^2\psi_{0m}(\mathbf{r},t)}{\partial t^2} = 0, \tag{9}$$

which is related to the pressure $p_{0m}(\mathbf{r},t)$ by

$$p_{0m}(\mathbf{r},t) = \rho_{\mathrm{h}}\frac{\partial\psi_{0m}(\mathbf{r},t)}{\partial t}. \tag{10}$$

If time and space variables are separable at the surface S_m of the transducer the boundary conditions are

$$\left.\frac{\partial\psi_{0m}(\mathbf{r},t)}{\partial\mathbf{n}_m}\right|_{\mathbf{r}=\mathbf{r}_m+\mathbf{s}_m} = \begin{cases}\Gamma_m(\mathbf{s}_m)v_m(t) & ,\mathbf{s}_m \in S_m\\ 0 & ,\mathbf{s}_m \notin S_m\end{cases} \tag{11}$$

in case the transducer is assumed to be placed in an infinite rigid baffle. Here $\mathbf{n}_m$ denotes the normal vector onto the transducer surface, $\mathbf{r}_m$ is the position of the transducer and $\mathbf{s}_m$ is a vector pointing from $\mathbf{r}_m$ towards any position on the transducer surface or the baffle. The velocity $v_m(t)$ of the moving transducer surface can be calculated from the exciting voltage $u_m(t)$ by means of the convolution

$$v_m(t) = \int_0^t h_m^{\mathrm{Trm}}(t-\tau)u_m(\tau)\mathrm{d}\tau \tag{12}$$

with the electromechanical impulse response $h_m^{\mathrm{Trm}}(t)$ of the transducer. Position dependency of the transducer displacement e. g. due to clamping of its edges is taken into account by the surface velocity distribution function $\Gamma_m(\mathbf{s}_m)$. The Green's function solution to this problem is

$$\psi_{0m}(\mathbf{r},t) = 2\iint_{S_m}\int_0^t \Gamma_m(\mathbf{s}_m)v(\tau)g(\mathbf{r},t|\mathbf{r}_m+\mathbf{s}_m,\tau)\mathrm{d}\tau\mathrm{d}\mathbf{s}_m \tag{13}$$

with the Green's function [5]

$$g(\mathbf{r},t|\mathbf{r}_0,\tau) = \frac{\delta(t-\tau-|\mathbf{r}-\mathbf{r}_0|/c_{\mathrm{h}})}{4\pi|\mathbf{r}-\mathbf{r}_0|}. \tag{14}$$

Substituting the electromechanical relation (12) and the relation between velocity potential and pressure (10) into (13) and defining

$$h_m^{\mathrm{Sp}}(\mathbf{r},\mathbf{r}_m,t) = \iint_{S_m}\Gamma_m(\mathbf{s}_m)g(\mathbf{r},t|\mathbf{r}_m+\mathbf{s}_m,\tau)\mathrm{d}\mathbf{s}_m \tag{15}$$

as the spatial impulse response [2, 4, 5], the pressure due to transmitter m is

$$p_{0m}(\mathbf{r},\mathbf{r}_m,t) = 2\rho u_m(t) * h_m^{\mathrm{Trm}} * \frac{\partial}{\partial t}h_m^{\mathrm{Sp}}(\mathbf{r},\mathbf{r}_m,t), \tag{16}$$

where $*$ abbreviates the convolution integrals. Since linear relations are assumed, the incident pressure field $p_0(\mathbf{r},t)$ of all transmitters $m = 1\ldots M$ is the sum of the individual fields:

$$p_0(\mathbf{r},t) = \sum_{m=1}^{M} p_{0m}(\mathbf{r},t). \tag{17}$$

Scattered Pressure Field: To calculate the scattered sound the scatterers are assumed to be the source of a pressure field $p_s(\mathbf{r},t)$ which is radiated into a homogeneous and otherwise source free medium. This is expressed by the homogeneous wave equation

$$\nabla^2 p_s(\mathbf{r},t) - \frac{1}{c_h{}^2}\frac{\partial^2 p_s(\mathbf{r},t)}{\partial t^2} = T\,p(\mathbf{r},t) \qquad (18)$$

where the right hand side is the source term with the scattering operator $T(\cdot)$. The Green's function solution is [5]

$$p_s(\mathbf{r},t) = \iiint\limits_V \int\limits_0^t T(p(\mathbf{r}_0,\tau))g(\mathbf{r},t|\mathbf{r}_0,\tau)\mathrm{d}\tau\mathrm{d}\mathbf{r}_0, \qquad (19)$$

where the Green's function $g(\mathbf{r},t|\mathbf{r}_0,\tau)$ has already been defined in (14) and V denotes the irradiated volume.

It is possible to simplify the calculation for the scattered field under the assumption, that the inhomogeneities are lumped into a discrete number of point scatterers at positions $\mathbf{r}_k$, $k = 1\ldots K$. This can mathematically be expressed by convolving the integrand in (19) with a weighted sum over three-dimensional Dirac distributions:

$$p_s(\mathbf{r},t) = \iiint\limits_V \int\limits_0^t \sum_{k=1}^{K} \frac{V_k}{V}\delta(\mathbf{r}_k-\mathbf{r}_0) * (T(p(\mathbf{r}_0,\tau))g(\mathbf{r},t|\mathbf{r}_0,\tau))\,\mathrm{d}\tau\mathrm{d}\mathbf{r}_0. \qquad (20)$$

The expression

$$\delta(\mathbf{r}_k-\mathbf{r}_0) * (T(p(\mathbf{r}_0,\tau))g(\mathbf{r},t|\mathbf{r}_0,\tau)) = \iiint\limits_V \delta(\mathbf{r}_k-\mathbf{r}_0-\mathbf{r}_1)T(p(\mathbf{r}_1,\tau))g(\mathbf{r},t|\mathbf{r}_1,\tau)\mathrm{d}\mathbf{r}_1$$

has finite amplitudes at $\mathbf{r}_0 = \mathbf{r}_k$ and equals zero elsewhere. Thus (20) can be rewritten as

$$\sum_{k=1}^{K} V_k \int\limits_0^t T(p(\mathbf{r}_k,\tau))g(\mathbf{r},t|\mathbf{r}_k,\tau)\mathrm{d}\tau, \qquad (21)$$

where the scattering strength of each point scatterer is determined by its volume V_k.

As (6) states, the pressure field $p(\mathbf{r},t)$ is the sum of incident and scattered pressure field. Analysis of (21) however reveals that the pressure $p(\mathbf{r},t)$ must be known to calculate the scattered field. To derive an explicit equation, $p(\mathbf{r},t)$ in (21) is replaced by the incident pressure $p_0(\mathbf{r},t)$, which is the first order Born expansion [5]:

$$p_1(\mathbf{r},t) = p_0(\mathbf{r},t) + \sum_{k=1}^{K} V_k \int\limits_0^t T(p_0(\mathbf{r}_k,\tau))g(\mathbf{r},t|\mathbf{r}_k,\tau)\mathrm{d}\tau. \qquad (22)$$

The Born approximation

$$p(\mathbf{r},t) \approx p_1(\mathbf{r},t) \qquad (23)$$

is valid if the scattering is weak and the scatterers are small compared to the wavelength of sound [7].

Receive Signal: An ultrasound transducer in receive mode is sensitive to the total pressure impinging onto its surface. The spatial dependency of the transducer's receive sensitivity is generally equal to the surface velocity distribution in transmit mode [4] and is thus denoted by $\Gamma_n(\mathbf{s}_n)$. The total weighted pressure $P_n(t)$ onto the surface S_n is

$$P_n(t) = \iint\limits_{S_n} \Gamma_n(\mathbf{s}_n)p(\mathbf{r}_n+\mathbf{s}_n,t)\mathrm{d}\mathbf{s}_n \qquad (24)$$

and the output voltage is related to the total pressure by a convolution

$$y_n(t) = \int\limits_0^t h_n^{\mathrm{Rec}}(t-\tau)P_n(\tau)\mathrm{d}\tau \qquad (25)$$

with the electromechanical impulse response $h_n^{\mathrm{Rec}}(t)$. By substitution of the pressure field under the Born approximation (23) into (24) one gets

$$P_n(t) \approx \iint\limits_{S_n} \Gamma_n(\mathbf{s}_n)\Bigg(p_0(\mathbf{r}_n+\mathbf{s}_n,t) + \sum_{k=1}^{K} V_k \int\limits_0^t T(p_0(\mathbf{r}_k,\tau))g(\mathbf{r}_n+\mathbf{s}_n,t|\mathbf{r}_k,\tau)\mathrm{d}\tau\Bigg)\mathrm{d}\mathbf{s}_n. \qquad (26)$$

Exchanging the order of integration and summation and using the symmetry of the Green's function $g(\mathbf{r},t|\mathbf{r}_0,\tau) = g(\mathbf{r}_0,t|\mathbf{r},\tau)$ which follows directly from its definition (14), the spatial impulse response as defined for transmission (15) can be employed for the calculation of the receive signal as well. For receiver n and scatterer k the spatial impulse response is thus

$$h_n^{\mathrm{Sp}}(\mathbf{r}_k,\mathbf{r}_n,t) = \iint\limits_{S_n} \Gamma_n(\mathbf{s}_n)g(\mathbf{r}_k,t|\mathbf{r}_n+\mathbf{s}_n,\tau)\mathrm{d}\mathbf{s}_n$$

and $P_{0n}(t)$ can be calculated by

$$P_n(t) \approx P_{0n}(t) + \sum_{k=1}^{K} V_k h_n^{\mathrm{Sp}}(\mathbf{r}_k,t) * T(p_0(\mathbf{r}_k,t)), \qquad (27)$$

with

$$P_{0n}(t) = \iint\limits_{S_n} \Gamma_n(\mathbf{s}_n)p_0(\mathbf{r}_n+\mathbf{s}_n,t)\mathrm{d}\mathbf{s}_n \qquad (28)$$

being the total weighted pressure due to the incident field. Incorporation of the electromechanical receive impulse response (25) into (27) finally enables calculation of the receive voltage signal:

$$y_n(t) \approx h_n^{\mathrm{Rec}}(t) * \left(P_{0n}(t) + \sum_{k=1}^{K} V_k h_n^{\mathrm{Sp}}(\mathbf{r}_k,t) * T(p_0(\mathbf{r}_k,t))\right). \qquad (29)$$

Equations (16), (17), (28), (29) and an adequate scattering operator $T(\cdot)$ provide a mathematical formulation for the whole signal chain (emit voltage trace - electromechanical emit transformation - wave propagation and scattering - electromechanical receive transformation - receive voltage trance) in ultrasound tomogaphy.

Scattering Operator: To find the scattering operator $T(\cdot)$ which operates onto the pressure $p(\mathbf{r}_k,t)$, e. g. onto the incident pressure $p_0(\mathbf{r}_k,t)$ under the Born approximation, the inhomogeneous wave equation (1) or (5) respectively is transformed to the form of the wave equation for the scattered field (18).

Subtraction of the left hand side of the wave equation for the fictitious homogeneous medium (7) on both sides of the inhomogeneous wave equation (1) yields

$$-\frac{1}{\rho_\mathrm{h}}\nabla^2 p(\mathbf{r},t)+\kappa_\mathrm{h}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2}=\nabla\left(\frac{1}{\rho(\mathbf{r})}\nabla p(\mathbf{r},t)\right)-\kappa(\mathbf{r})\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2}-\frac{1}{\rho_\mathrm{h}}\nabla^2 p(\mathbf{r},t)+\kappa_\mathrm{h}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2} \quad (30)$$

Using equivalent transformations and the relation between density, compressibility and sound speed (3) this can be rewritten as

$$\nabla^2 p(\mathbf{r},t)-\frac{1}{c_\mathrm{h}{}^2}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2}=\frac{1}{c_\mathrm{h}{}^2}\left(\frac{\kappa(\mathbf{r})}{\kappa_\mathrm{h}}-1\right)\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2}-\nabla\left(\left(\frac{\rho_\mathrm{h}}{\rho(\mathbf{r})}-1\right)\nabla p(\mathbf{r},t)\right). \quad (31)$$

That is the formulation which Bloomfield [6] calls Morse Ingard formulation with reference to [9]. It has also been derived by Gore and Leeman [10]. Comparison with (18) shows that the scattering operator operating onto pressure $p(\mathbf{r},t)$ is

$$T_\mathrm{MI}p(\mathbf{r},t)=\gamma_{\kappa\,\mathrm{MI}}\frac{1}{c_\mathrm{h}{}^2}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2}+\nabla\left(\gamma_{\rho\,\mathrm{MI}}\nabla p(\mathbf{r},t)\right) \quad (32)$$

with the monopole scattering coefficient

$$\gamma_{\kappa\,\mathrm{MI}}=\frac{\kappa(\mathbf{r})}{\kappa_\mathrm{h}}-1 \quad (33)$$

and the dipole scattering coefficient

$$\gamma_{\rho\,\mathrm{MI}}=1-\frac{\rho_\mathrm{h}}{\rho(\mathbf{r})}. \quad (34)$$

An alternative solution for the scattering operator can be derived by subtracting the term

$$\left(\frac{1}{c_\mathrm{h}{}^2}-\frac{1}{c^2(\mathbf{r})}\right)\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2}$$

on both sides of the Chernov wave equation (5). After some equivalent transformations the wave equation

$$\nabla^2 p(\mathbf{r},t)-\frac{1}{c_\mathrm{h}{}^2}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2}=\nabla\left(\ln\left(\frac{\rho(\mathbf{r})}{\rho_\mathrm{ref}}\right)\right)\nabla p(\mathbf{r},t)-\left(1-\frac{c_\mathrm{h}{}^2}{c^2(\mathbf{r})}\right)\frac{1}{c_\mathrm{h}{}^2}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2} \quad (35)$$

is derived. This is the equation which Bloomfield calls Chernov formulation with respect to [8]. Chernov himself however used an approximation to this equation as shown in (39) and (40). Comparison with (18) shows that the scattering operator operating onto $p(\mathbf{r},t)$ is

$$T_\mathrm{Ch}p(\mathbf{r},t)=\gamma_{c\,\mathrm{Ch}}\frac{1}{c_\mathrm{h}{}^2}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2}+\nabla(\gamma_{\rho\,\mathrm{Ch}})\nabla p(\mathbf{r},t) \quad (36)$$

with the monopole and dipole scattering coefficients

$$\gamma_{c\,\mathrm{Ch}}=\frac{c_\mathrm{h}{}^2}{c^2(\mathbf{r})}-1, \quad (37)$$

and

$$\gamma_{\rho\,\mathrm{Ch}}=\ln\left(\frac{\rho(\mathbf{r})}{\rho_\mathrm{h}}\right). \quad (38)$$

respectively. As reference density the density of the surrounding medium has been used.

If the acoustic parameters of the scatterers and the surrounding medium differ only slightly, the Chernov scattering coefficients $\gamma_{c\,\mathrm{Ch}}$ and $\gamma_{\rho\,\mathrm{Ch}}$ can be approximated by

$$\gamma_{c\,\mathrm{ax}}=2\frac{c(\mathbf{r})-c_\mathrm{h}}{c_\mathrm{h}} \quad (39)$$

and

$$\gamma_{\rho\,\mathrm{ax}}=\frac{\rho(\mathbf{r})-\rho_\mathrm{h}}{\rho_\mathrm{h}}, \quad (40)$$

which are the coefficients Chernov [8] originally used. The scattering operator onto $p(\mathbf{r},t)$ is in this case

$$T_\mathrm{ax}p(\mathbf{r},t)=\gamma_{c\,\mathrm{ax}}\frac{1}{c_\mathrm{h}{}^2}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2}+\nabla(\gamma_{\rho\,\mathrm{ax}})\nabla p(\mathbf{r},t). \quad (41)$$

When a scattering operator is substituted into the equation for the output signal (29), the medium properties $\rho(\mathbf{r})$, $\kappa(\mathbf{r})$ and $c(\mathbf{r})$ have to be evaluated at the scatterers positions $\mathbf{r}_k$. If the distance between the transducers and the scatterer $|\mathbf{r}_n-\mathbf{r}_k|$ and $|\mathbf{r}_k-\mathbf{r}_m|$ is large compared to the radius of the scatterer and the dimensions of the transducers, the approximation

$$T_\mathrm{MI}p(\mathbf{r},t)\approx(\gamma_{\kappa\,\mathrm{MI}}+\gamma_{\rho\mathrm{MI}}\cos\Theta)\frac{1}{c_\mathrm{h}{}^2}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2} \quad (42)$$

holds for the Morse Ingard scattering operator [6]. The angle $\Theta=\angle(\mathbf{r}_k-\mathbf{r}_m,\mathbf{r}_k-\mathbf{r}_n)$ is measured between transmitter, scatterer and receiver. For the Chernov scattering operators the approximations

$$T_\mathrm{Ch}p(\mathbf{r},t)\approx\left(\gamma_{c\,\mathrm{Ch}}+\gamma_{\rho\,\mathrm{Ch}}(\cos\Theta-1)\right)\frac{1}{c_\mathrm{h}{}^2}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2} \quad (43)$$

and

$$T_\mathrm{ax}p(\mathbf{r},t)\approx\left(\gamma_{c\,\mathrm{ax}}+\gamma_{\rho\,\mathrm{ax}}(\cos\Theta-1)\right)\frac{1}{c_\mathrm{h}{}^2}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2} \quad (44)$$

respectively are valid under the same conditions [6].

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Implementation

The ultrasound tomography simulation program which computes the signal processing chain from the exciting voltage signal $u(t)$ to the receive voltage trace $y(t)$ as described by (16), (17), (28) and (29) has been implemented in Matlab®. The program allows for any number of arbitrarily positioned transducers and point scatterers. Each point scatterer may have individual physical properties and is represented by one of the scattering operators $T_{\mathrm{MI}}(\cdot)$, $T_{\mathrm{Ch}}(\cdot)$, $T_{\mathrm{ax}}(\cdot)$ using (42), (43) or (44).

For most transducer types and surface velocity distributions analytic solutions for the spatial impulse responses are unknown. To compute (16) and (29) our program employs Field II, which provides a powerful numerical method to calculate impulse responses for numerous transducer geometries and surface velocity distributions [11]. An adaptive Simpson's algorithm is used to calculate the surface integral in (28).

Results

As an example to our program we simulated an ultrasound tomography system consisting of eight circular piston transducers with a radius of 2 mm which have been situated equidistantly in a circle of 20 cm diameter around the irradiated object. The electromechanical impulse response for transmission and reception has been a Gaussian modulated sinusoid of 5 MHz and 3.5 cycles. Transducer 1 was excited with a 5 MHz burst of one cycle. The receive signals $y_n(t)$ of the receiving transducers due to the incident field are plotted in Figure 2. Signal distortion resulting from diffraction effects between the transmitter and the receivers can clearly be seen. Note especially the low amplitude of those transducers lying oblique to the transmitter as opposed to signal $y_5(t)$ of the transducer opposing the transmitter.

The output signals $y_n(t)$ due to the scattered field of a single point scatterers positioned 2 cm off the tomograph's centre and irradiated by transducer 1 is depicted in Figure 3. The point scatterer had an equilibrium compressibility $\kappa(\mathbf{r}_1) = 3.84 \cdot 10^{-10}\,1/\mathrm{Pa}$ and a equilibrium density $\rho(\mathbf{r}_1) = 1085\,\mathrm{kg/m^3}$. It was modelled by the Morse Ingard scattering operator. For the surrounding medium a compressibility $\kappa(\mathbf{r}_1) = 4.06 \cdot 10^{-10}\,1/\mathrm{Pa}$ and a density of $\rho(\mathbf{r}_1) = 1000\,\mathrm{kg/m^3}$ has been assumed. Diffraction effects in combination with the spatial dependency of the scattering operator result in various signal shapes, depending on the receiver's relative position to scatterer and transmitter.

Discussion

We have shown that the Green's function approach can be successfully extended to simulation of the incident and the scattered field in ultrasound tomography imaging systems. Our program offers the possibility to study the effects of diffraction on tomography receive signals and can therefore be used to develop enhanced image reconstruction algorithms.

Different scattering operators have been derived from the inhomogeneous wave equation and employed to compute the scattered field under the Born approximation. Although the Born approximation is widely used in ultrasound simulation programs, there is currently no definite evidence that it approximates the circumstances in biological tissue well [7]. Thus further research and possibly an extension of the simulation program to higher order scattering is necessary.

Acknowledgement

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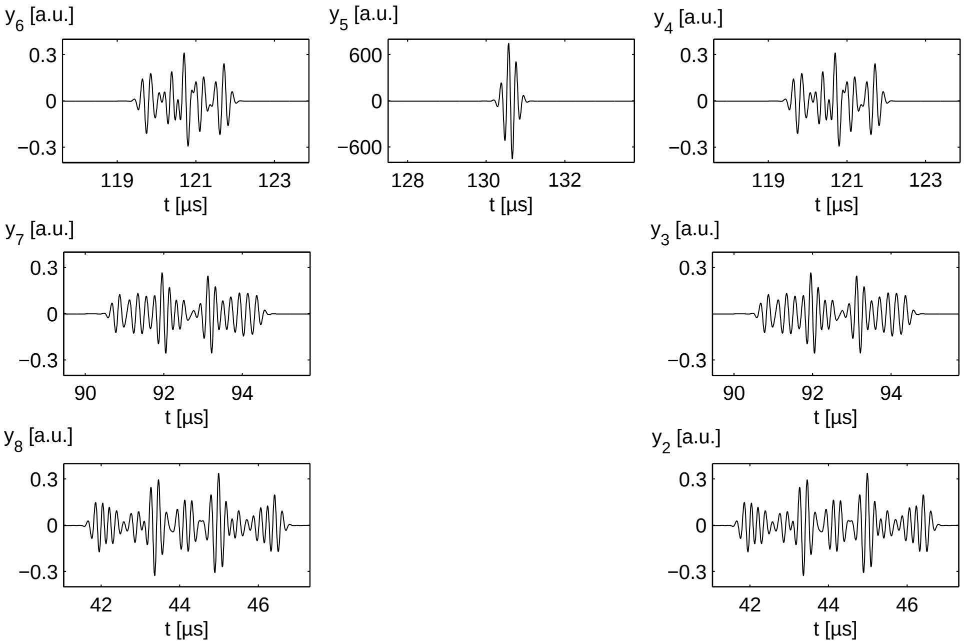

Figure 2: Output signals y_n of receivers $n = 2 \ldots 8$ equidistantly arranged in a circle at angles $(n-3)\pi/4$ due to the incident field of transmitter $m = 1$.

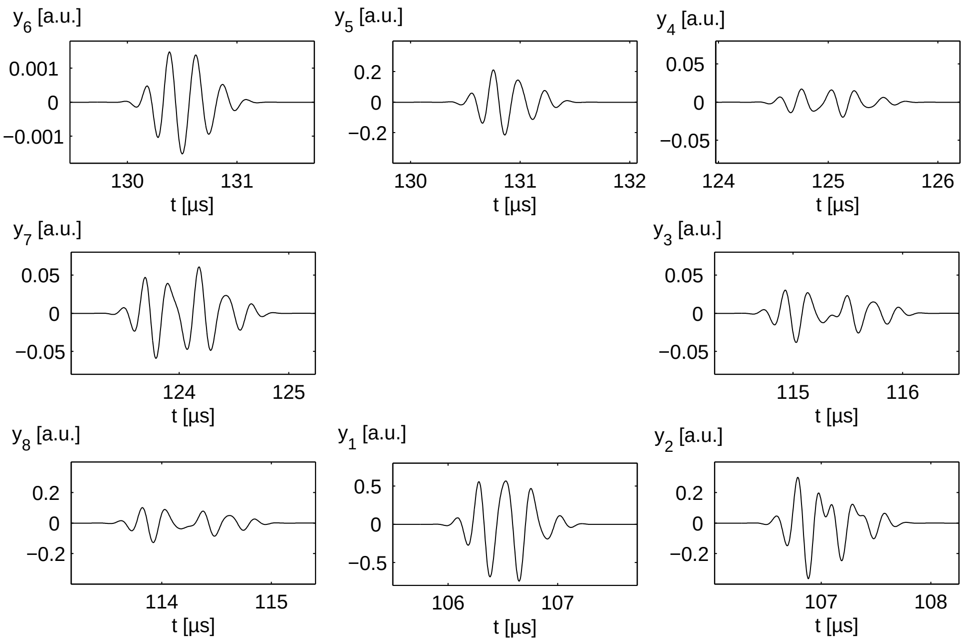

Figure 3: Output signals y_n of receivers $n = 1 \ldots 8$ equidistantly arranged in a circle at angles $(n-3)\pi/4$ due to the scattered field of a point scatterer irradiated by transmitter $m = 1$.

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