

AN EFFICIENT ANALYSIS METHOD TO DESIGN OPTIMUM SPARSE ARRAY SCHEMES FOR PHASED ARRAY TRANSDUCERS

Gi-Duck Kim, Tai-Kyong Song

Dept. of Electronic Engineering, Sogang University, Seoul, South Korea

tksong@sogang.ac.kr

Abstract: An efficient analysis method to design optimum sparse array scheme is presented. Using this method, we could find optimum periodic sparse array patterns, in which dominant grating lobes due to receive sparse array are eliminated by transmit sparse array beam pattern. We also found that effective apertures of these patterns have a smooth shape. The proposed method was verified through computer simulation results for a simple case.

Introduction

Currently, most medical ultrasound scanners are employing receive dynamic focusing (RDF) with array transducers. The main-lobe width of ultrasound beams by RDF is inversely proportional to aperture size. A most general way to improve the lateral resolution is therefore using a large number of elements. However, this results in the accompanying increase in hardware complexity.

On the other hand, for size and cost issues, small portable scanners should use a limited number of active channels, which reduces the aperture size and consequently degrades the spatial resolution. To minimize the sacrifice in image quality, a customary method is to use sparse array techniques[2-4]. Sparse array techniques have been used to increase the aperture size effectively without using more elements. Since the array elements are sparsely distributed, however, sparse array patterns should be properly designed so as to suppress the elevation of unwanted high grating lobes.

In this paper, we present an efficient analysis method to design optimum sparse array. For this analysis, we define a periodic sparse array model, and describe a method for designing optimum sparse array patterns for sector scanning with 1D phased array. Our method is based on the far field CW beam pattern of receive and transmit arrays. First, grating lobe positions of sparse arrays with equal element spacing are analyzed. Then, for a given receive sparse array pattern, an optimum transmit sparse array pattern is determined, which can eliminate the most dominant grating lobes (usually the nearest one to the main lobe).

In this way, optimum sparse array schemes for any receive sparse array patterns can be found. We also found that when the receive and transmit sparse arrays of such optimum sparse arrays are convolved, we

always get smooth effective apertures, of which the beam pattern does not show high grating lobes.

Computer simulations were performed to verify the analysis method and some optimum sparse array patterns are presented.

Analysis Methods

Figure 1 illustrates the geometry of conventional receive dynamic focusing using a fully sampled array in sector phased array scanning, where $\theta_0 = \sin^{-1}u_0$ denote the steering angle, and d is element pitch.

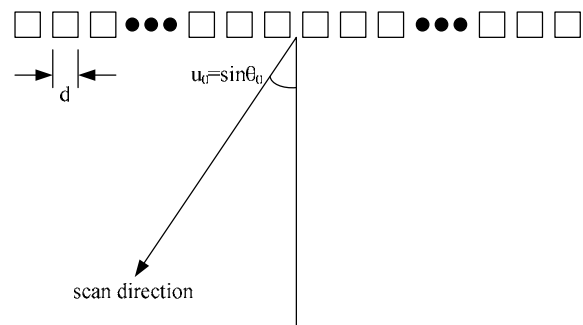


Figure 1: Geometry for receive dynamic focusing with fully sampled arrays.

The continuous wave lateral beam pattern produced by this array can be expressed by [1]

$$\Psi(u) = \Psi_{N_T}(u) \cdot \Psi_{N_R}(u) \quad (1)$$

where,

$$\begin{aligned} \Psi_N(u) &= \Phi_e(u) \cdot \sum_{n=0}^{N-1} \exp[-jk(u-u_0)nd] \\ &\approx \Phi_e(u) \cdot \frac{\sin(\pi Ndu'/\lambda)}{\sin(\pi du'/\lambda)}, u' = u - u_0 \end{aligned} \quad (2)$$

In equations (1) and (2), N_T and N_R represent the number of active transmit and receive elements, respectively, k wave number, and d element spacing of a phased array to be used. $\Phi_e(u)$ in equation (2) represents the far-field beam pattern of a single element, given by $\text{sinc}(du/\lambda)$. Equation (2) shows that as the

aperture size Nd increases, one can get better lateral resolution.

It is also well known that the first grating lobe is observed at

$$u = u_0 \pm \lambda / d. \quad (3)$$

Since u and u_0 have magnitude not greater than 1, grating lobe will not exist $d < \lambda/2$. Our analysis is limited to the case where the array to be used satisfies this condition.

The fully sampled array in figure 1 has been successfully used to provide a required image resolution by using a proper number of active elements. However, for small scale ultrasound scanners, the number of active elements must be limited to meet size and cost requirements. It results in degradation of image resolution because of the reduced aperture size.

A simple way to effectively increase the aperture size is to use sparsely distribute active elements, so called sparse array techniques. However, uniform sparse arrays produce the unwanted grating lobes, because the effective element spacing would be greater than $\lambda/2$.

We will find optimum conditions of designing transmit and receive array that can eliminate or reduce the dominant grating lobes, by means of analysis method for periodic sparse array patterns.

Figure 2 illustrates the model of periodic sparse array patterns used in the proposed analysis method, where L consecutive elements are used in the interval of P elements. We will define P/L as sparseness factor.

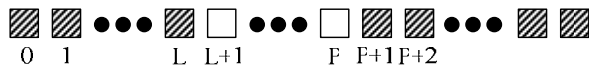


Figure 2: A model to represent periodic sparse array patterns.

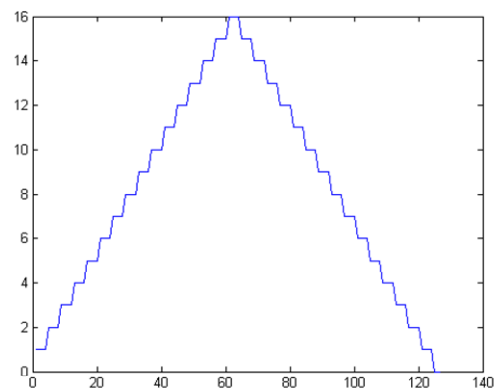
The lateral one-way beam pattern of this model can be written as

$$\begin{aligned} \Psi_N(u) &= \Phi_e(u) \cdot \sum_{n=0}^{N_p-1} \exp(-jku' Pdn) \cdot \sum_{l=0}^{L-1} \exp(-jku' dl) \\ &\approx \Phi_e(u) \cdot \frac{\sin(\pi N_p Pdu / \lambda)}{\sin(\pi Pdu / \lambda)} \cdot \frac{\sin(\pi dLu' / \lambda)}{\sin(\pi du' / \lambda)} \quad (4) \\ &= \Phi_e(u) \cdot \Psi_p(u') \cdot W_L(u') \end{aligned}$$

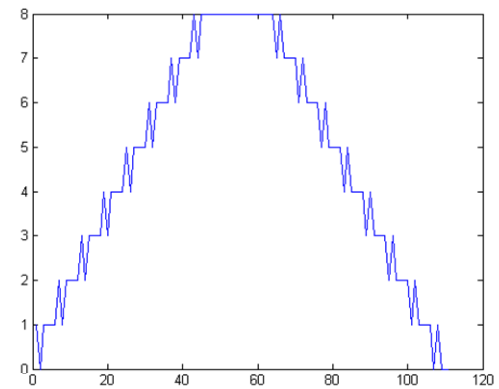
Where N_p represents the number of P element intervals so that $N = N_p L$ is the number of total active elements. Equation (4) is identical to equation (2) except for the weighting function $W_L(u')$ which is the beam pattern of the L active elements in each interval. In equation (4), $\Psi_p(u')$ governs the main beam pattern, of which half main lobe width is $\lambda/(N_p P d)$. Since $N_p P$ is greater than $N = N_p L$ if the

sparseness factor P/L is greater than 1, it can be said that sparse arrays provide improved resolution than fully sampled arrays ($P/L=1$) when the same number of active elements are used in both schemes. However, $\Psi_p(u')$ has grating lobes at $u' = n\lambda/(Pd)$ which do not exist in case of the fully sampled array.

$W_L(u')$ has a main response whose half width is given by $\lambda/(Ld)$ which is much greater than $\lambda/(N_p Pd)$. Therefore, $W_L(u')$ can be ignored when the main response of the overall beam pattern is concerned. However, it is interesting to note that $W_L(u')$ has null positions at $u' = m\lambda/(Ld)$. This property will be used to suppress the most dominant grating lobe of sparse arrays.



(a)



(b)

Figure 3: Effective aperture plots for (a) $P_T=4, L_T=P_R=2, L_R=1$ and (b) $P_T=3, L_T=P_R=2, L_R=1$.

The first rule to avoid the elevation of grating lobes in sparse array imaging is not to use sparse array patterns that produce grating lobes at the same positions on both transmit and receive. In case that the P and L parameters of transmit and receive arrays are given by (P_T, L_T) and (P_R, L_R) , respectively, this is possible if P_T and P_R are co-prime. We found that the better way is to select (P_T, L_T) and (P_R, L_R) such that the null positions

of $W_L(u')$ of transmit or receive beam pattern are located where dominant grating lobes, representing ones located at the same position in both transmit and receive, occur. Based on this method, the optimum conditions of designing periodic sparse array can be written as

$$P_T = 2L_T, P_R = L_T, L_R = 1 \quad (5)$$

Sparse array has also been designed using so called effective aperture approach [2-4]. Figure 3 shows two effective aperture shapes, one satisfying the conditions in (5) (panel (a)) and the other not (panel (b)). Specifically, $P_T=4, L_T=P_R=2, L_R=1$ for panel (a) and $P_T=3, L_T=P_R=2,$ and $L_R=1$ for panel (b). We can see from figure 3(b) that the effective aperture is not smooth although P_T and P_R are co-prime. However, figure 3(a) shows that if the conditions in (5) are met, the resulting effective aperture has a smooth shape. Note that on transmit sparseness factor for figure 3(b) is larger than that for figure 3(a).

Results and Discussion

Figure 4 shows the beam pattern of the sparse array pattern used for figure 3(a) where the beam patterns of the transmit and receive arrays are plotted by a dotted line and dashed line, respectively, and their overall beam pattern is plotted by a solid line. In this simulation, 32 element phased array with center frequency of 3MHz was used..

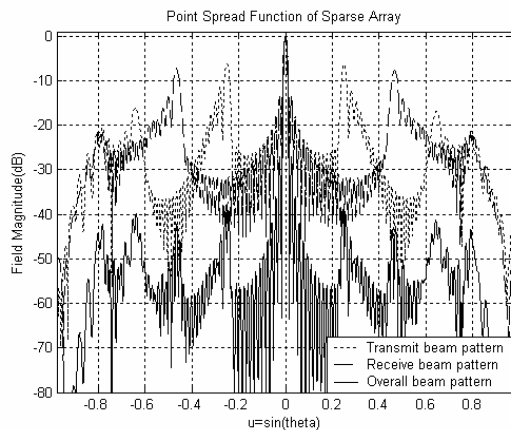


Figure 4: Spatial beam pattern of a sparse array scheme for the case where $P_T=4, L_T=P_R=2,$ and $L_R=1$.

The transmit and receive sparse arrays produce grating lobes at different locations because $P_T=4$ and $P_R=2$. The first grating lobe of the transmit beam pattern is significantly reduced due to the low side lobe levels of the receive beam pattern in its vicinity. In addition, the transmit beam pattern does not have grating lobe at $u'd/\lambda = 1/2$ (second one) because it is

cancelled by its $W_L(u')$. As a result, grating lobes are successfully suppressed in the overall beam pattern. This result agrees well with the theoretical analysis.

Figure 5 shows the computer generated phantom images for the conventional beamforming technique (a) and the sparse array scheme in figure 4(b). The conventional beamforming was performed using 32 elements of a phased array with the center frequency of 3MHz and only 16 elements were used in sparse array imaging. Due to the successful grating lobe reduction, method produces almost the same image in terms of spatial resolution as the conventional beamforming method.

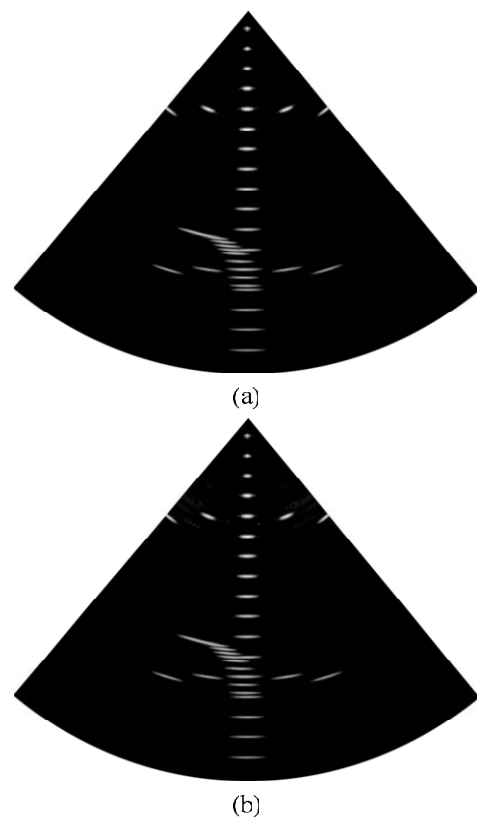


Figure 5 : The simulation images by (a) conventional beamforming method and (b) sparse array method with $P_T=4, L_T=P_R=2, L_R=1$.

Conclusions

In this paper, we present an analysis method for the design of optimum periodic sparse array patterns, which was verified through computer simulation results for a simple case. Other sparse array patterns for various sparseness factors (L/P) can be easily found using the same method. We believe that such schemes can be effectively used in small portable imaging systems. Further work will consider the solution for the other array transducers like as linear and curved linear array.

References

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