COUPLING SKIN TO INTERNAL BREAST TISSUE FOR FINITE ELEMENT MODELLING OF BREAST BIOMECHANICS

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Abstract: Non-rigid-body registration techniques, that constrain the set of possible soft tissue deformations to be consistent with the basic laws of physics, offer a means of providing realistic and accurate estimates of breast tissue movement under mammographic compression. Such constraints can be imposed by the use of anatomically accurate finite element models that predict soft tissue deformations. Our overarching aim is to develop tools that use such finite element models to track regions of tissue across multiple images (different views taken at different times) for image-guided surgeries and reliable diagnostic and theraputic monitoring of breast cancer. In this paper, we propose a method of modelling the breast as a coupled system of skin as a twodimensional membrane coupled to three dimensional underlying breast tissue.

Introduction

Automatic non-rigid image registration can be a very useful method to combine various pieces of information that different imaging modalities provide in a breast cancer diagnostic procedure. A number of finite element (FE) models of the breast have been proposed to provide physical constraints to heuristic algorithms for accurate image registration [11]. However, due to the complex structure of breast tissue and the complex loading and boundary conditions, it is hard to discern the sources of error in the proposed models.

We have adopted a systematic approach to modelling breast mechanics by performing experiments on silicon gel phantoms to validate each modelling component. The purpose of these validation studies is to identify and quantify the sources of error due to the underlying modelling assumptions. This paper shows results of a validation study we performed on modelling skin as a twodimensional membrane coupled to underlying volume elements representing internal breast tissue.

Modelling Theory

We model the deformation of breast tissues using finite deformation theory ([7]), which can accurately capture soft tissue deformation. Here, we outline the important aspects of finite deformation theory.

Finite Deformation Theory

Kinematics:

The problem is to find the coordinates, (\mathbf{x}) , of the deformed body, v, given the coordinates, (\mathbf{X}) , of the undeformed body (V). The deformation gradient tensor **F** provides the relationship to map between the undeformed and the deformed states, and is defined as

$$\mathbf{F} = \left\{ \frac{\partial x_i}{\partial X_M} \right\} \tag{1}$$

The Lagrangian Green strain tensor ${\bf E}$ is calculated using:

$$\mathbf{E} = \frac{1}{2} \left(\mathbf{F}^T \mathbf{F} - \mathbf{I} \right) \tag{2}$$

The aim is to find a deformed state, such that the principle laws of continuum mechanics are satisfied.

Principle Laws of Continuum Mechanics:

The governing equations of finite elasticity can be formulated by defining the stresses in a body based on the undeformed state (reference configuration) or the deformed state (current configuration) of the body. The modelling framework has been implemented with respect to the reference configuration and thus the equations below are written in terms of the undeformed state. There are three laws that govern the mechanics of tissue deformation.

The first law states that mass must be conserved during deformation:

$$\int_{V} \rho_0 dV = \int_{v} \rho dv \tag{3}$$

where ρ_0 and ρ represent the density of the body in the undeformed and deformed configurations, respectively and *V* and *v* represent the volumes in the undeformed and deformed configurations, respectively.

When the body is in equilibrium, all of the forces (body and traction) are in balance. This is achieved by satisfying the principle of conservation of linear momentum:

$$\frac{\partial}{\partial X_M} \left(T^{MN} \frac{\partial x_j}{\partial X_N} \right) + \rho_0 b^j = \rho_0 f^j \tag{4}$$

where T^{MN} are components of the second Piola Kirchhoff stress tensor (force per unit area of the undeformed body), b^j are the body forces (such as gravity), and f^j are components of the surface tractions acting on the body.

The final conservation principle is that of angular momentum, which states that the rate of change of total angular momentum must equal the vector sum of moments of the external forces acting on the system. This restriction ensures that the Cauchy (force per unit area in the deformed configuration) and second Piola Kirchhoff stress tensors are symmetric.

The stress equilibrium equations (4) can be expressed in an alternative form using the principle of virtual work as follows ([6]):

$$\int_{S_2} \mathbf{s} \cdot \delta \mathbf{v} dS + \int_{V} \rho \left(b^j - f^j \right) dV - \int_{V} \frac{1}{J} T^{MN} \frac{\partial x_j}{\partial X_M} \frac{\partial \delta v_j}{\partial X_N} dV = 0$$
(5)

where δv_j are the virtual displacements expressed in terms of the reference coordinate system, S_2 is the free boundary surface on which virtual displacements are applied, and *J* is the determinant of the deformation gradient tensor.

These nonlinear governing equations have been formulated in such a way that they can be solved using an iterative finite element modelling technique. A description of the finite element modelling method can be obtained from [12].

Membrane Theory

A majority of the research in biomechanical modelling of skin has been conducted to simulate incisions and wound closures ([4], [2]). Larrabee *et al* [4] modelled the skin as a two-dimensional elastic membrane with subcutaneous attachments to take the coupling of skin to subcutaneous tissue into account. Kirby *et al* [2] modelled skin deformation in three-dimensional space during wound closing.

In terms of characterising mechanical behaviour of skin with constitutie equations, we performed multiaxial extension experiments on the skin of the forearm in-vivo and fitted material parameters of available constitutive equations in literature to the data [3]. To date, no study has been published to the authors' knowledge to characterise the deformations through the thickness of the skin during multiaxial testing. The deformations in our study were modelled using a specialised form of the finite elasticity theory known as membrane theory [7]. This theory allows us to model the skin as a two dimensional surface and assumes that the deformation through the skin layer can be determined through incompressibility (conservation of volume) and the deformations in the in-plane directions. The stresses through the skin are assumed to be zero.

We propose to couple two dimensional surface elements of skin to three dimensional volume elements of internal breast tissue. Samani *et al* [10] stated that they used four noded membrane elements to model skin. However, the underlying theory used to compute the deformations (using ABAQUS) was not published. Ruiter *et al* [9] also attempted to model skin in a similar fashion, but only performed numerical experiments and concluded that the skin had no effect on the deformations. Therefore, formulation of the proposed method and its validation are critical steps in developing an accurate model of the breast.

The proposed method involves the addition of the nonlinear residuals (force and moment balances) for the associated degrees of freedom between the two problems (membrane theory for 2D skin and finite deformation theory for 3D breast tissue) to solve one global system which tightly couples the skin to the breast tissue. This first attempt is also justified by Ruiter's conclusion that a model simulating tight coupling between skin and breast tissue gave better results over those that assumed a loose coupling [9].

Phantom Validation Studies

We systematically validate modelling assumptions by measuring the accuracy of our model in predicting deformations of a silicon gel phantom model (Fig. 1). We have validated our modelling framework in predicting surface deformations of a homogeneous phantom and found the model predicted the experimental deformations with an RMS error of 1.8mm due to gravity loading under tilt conditions (the characteristic length of the gel is 120mm). Fig. 1 shows a predicted deformation matching the data cloud scanned when the gel was tilted at an angle of 15.6⁰ as in Fig.1(a).

We have extended these phantom experiments to include a thin layer of stiffer material (a rubber membrane) in order to validate our 2D/3D model coupling. We started by modelling a cuboid gel in a cantilever setup as shown in Fig.2.

Methods

In order to simulate the deformation in Fig. 2, we needed to estimate the constitutive relation that describes the relationship between the strain imposed on the material and resulting stress. There are a number of constitutive relations in the literature to describe different types of material mechanical behaviour. We found that the homogeneous gel could be accurately modelled using a neo-Hookean constitutive equation with a single material parameter c_1 [8].

$$W = c_1(I_1 - 3) \tag{6}$$

where I_1 the trace of the Cauchy Green deformation tensor and is known as the first principle invariant [7]. We have found that the rubber membrane can also be described using this constitutive equation [5].



(a)



Figure 2: Cantilever beam set up of gel set with rubber on the top surface.

Discussion

We have formulated and validated a coupling mechanism between two-dimensional membrane elements and three-dimensional volume elements in order to model the effects of skin on underlying tissues. The mechanism predicted deformations of a phantom with an RMS error of 1mm. We intend to use this mechanism to model the coupling of skin to breast tissue.

The phantom used for this validation was not intended to represent the breast. Validation studies with simple materials allow us to identify sources of error in the model in a systematic manner. This study has shown that it is possible to accurately capture the deformations of a soft tissue, membrane composite. Although the materials used in the validation study are simplistic, our modelling framework also has the ability to model the anisotropy of skin and the inhomogeneity of breast tissue. In addition to phantom validations, we are working on clinical validation of our modelling framework. We have developed a semiautomatic method of creating patient-specific finite element geometries (Fig. 4 for mechanics computations [8].

It is anticipated that a systematically validated, anatomical FE model of the breast will be a valuable tool to combine information from multiple images of the breast for clinicians to perform accurate diagnosis of breast cancer.

Figure 1: (a) Phantom gel used to validate our homogeneous model predictions. (b) Comparison of predicted deformation and scanned surface data. Wire frame indicates undeformed configuration. RMS error 1.8mm

The value of the neo-Hookean parameter was estimated for the gel using the cantilever beam experiment similar to that in Fig. 2. The value of the parameter for the membrane was estimated using the multiaxial rig developed for testing mechanical properties of skin in-vivo [5], [3]. A nonlinear parameter optimisation technique was used to estimate this parameter and is outlined for the two materials in [1] and [3]. The estimated values were then used in the 2D/3D coupling formulation to model the deformation in Fig. 2. The deformation was measured by scanning the deformed gel using a FASTSCANTM laser scanner.

Results

 c_1 was estimated for the silicon gel cube to be 0.65kPa. The rubber membrane parameter value was estimated to be 59 kPa.

Fig. 3(a) shows the deformation fitting the data cloud. The RMS error in the prediction was 1mm for displacements in the order of 21mm. Fig.3(b) shows the effect of the thin membrane on the z-coordinate value of a point on the free end of the cantilever beam.





Figure 3: (a): Predicted deformation matching data cloud (red markers) for a cantilever gel beam. (b) and (c) showing effect of 2D membrane on deformation of cuboid. (b): z-coordinate of a point in the undeformed (white wire-frame) compared to the z-coordinate of the same point in the 2D/3D problem (grey cuboid with green wireframe). (c): z-coordinate of a point in the homogeneous deformed model (grey cuboid with green wireframe) compared to z-coordinate of the same point in the undeformed state (white wireframe). The membrane reduced the vertical displacement by approximately 11mm

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Figure 4: Patient specific finite element geometry created from semi-automatic algorithm.

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