EMIT – A COMPARISON OF SENSITIVITY FOR 2D AND 3D MODELS

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Abstract: Electromagnetic impedance tomography (EMIT) is a new imaging technique. An image is reconstructed on the basis of measurements of electrical potential at the boundary of an object by means of electrodes and a magnetic field using coils around or over the object. A sensitivity function was evaluated for EMIT. It was compared for 2D and 3D objects. Analysis was performed on the basis of singular value decomposition (SVD) of the so-called forward operator. It was found that a 3D case is considerably more ill-conditioned that a 2D one.

Introduction

EMIT was proposed by Levy [6]. It is an extension of traditional electrical impedance tomography (EIT) on additional magnetic measurements. A reconstructed image is created on the basis of data obtained from measurements of electrical potential distribution at the boundary by means of electrodes and an exterior magnetic field using coils. EMIT is an example of the *joint inversion technique*, the integration of various groups of data sets into a single inverse algorithm, which is an efficient way to overcome the drawback of the sparseness of data in ETI that results when a limited number of electrodes is used. It is anticipated that additional measurements allow an underdetermined EIT problem to be turned into a well-posed one with a reasonable condition [6].

Sensitivity illustrates the relation between measured data and conductivity distribution inside the object under study. Analysis of sensitivity is an important means of testing the quality of a given model and an evaluated solution. It enables the accuracy and instability of the reconstruction solution to be indicated and consequently also ensures the correct interpretation of given results.

More information can be gained using the singular value decomposition (SVD) method. In this process a significant role is played by eigenvalues and singular values, the main products of SVD decomposition.

It is known that the mesh model is of considerable importance in the reconstruction process. It has an important influence on the quality of the reconstruction image. Of great significance also is the geometry of the elements of the mesh model and their number and design in space. These properties determine how real the current density will be inside the object studied. The analysis performed on paper is an attempt to ascertain the difference between solutions evaluated on the basis on various mesh models.

Methods

The essence of the forward problem in EMIT is to determine the potentials and magnetic field resulting from the injected current. The forward operator describing the dependence of the measured data on the conductivity of the imaging object can be defined by the following equation:

$$\mathbf{d} = F\{\sigma(\mathbf{x})\} + \delta, \quad (1)$$

where: $\mathbf{d}$ is a vector of data, $F\{\cdot\}$ is the forward operator, $\sigma(\mathbf{x})$ is the conductivity of the object and δ is the noise. For the discrete and linear case considered here the forward operator can be treated as a sensitivity matrix $F(\sigma)$:

$$d = F(\sigma)\boldsymbol{\sigma}. \quad (2)$$

When two different measurement data sets are included, potential v_e, and magnetic field v_m, the forward operator for EMIT can be given by the following equation:

$$d = \begin{bmatrix} \mathbf{v}_e \\ \mathbf{v}_m \end{bmatrix} = \begin{bmatrix} \mathbf{S}_e(\sigma) \\ \mathbf{S}_m(\sigma) \end{bmatrix} \boldsymbol{\sigma}, \quad (3)$$

where S_e is the sensitivity function evaluated using the relationship published by Geselowitz [2,3]:

$$S_e = \nabla\phi \cdot \nabla\psi = \frac{\partial\phi}{\partial x}\frac{\partial\psi}{\partial x} + \frac{\partial\phi}{\partial y}\frac{\partial\psi}{\partial y} + \frac{\partial\phi}{\partial z}\frac{\partial\psi}{\partial z}, \quad (4)$$

where ϕ and ψ are potentials calculated for different distributions of conductivity. S_m is the sensitivity function for the magnetic signal.

Assuming that the current is quasi-static, the divergence of the current density vanishes:

$$\nabla \cdot \mathbf{J} = 0 .$$

In this case the Biot-Savart law can be used to calculate the magnetic field:

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$$d\mathbf{B} = \frac{\mu_0}{4\pi} i d\mathbf{l} \times \frac{\mathbf{R}}{R^3}, \quad (5)$$

where: μ_0 - the magnetic permeability of the free space, i – current, **R** – the distance between current element and the measurement coil. **R** is equal to:

$$\left(x_0 - \frac{x_i - x_j}{2}\right)\mathbf{i}_x + \left(y_0 - \frac{y_i + y_j}{2}\right)\mathbf{i}_y + \left(z_0 - \frac{z_i + z_j}{2}\right)\mathbf{i}_z,$$

where: (x_0,y_0,z_0) is the location of the coil, (x_i,y_i,z_i) and (x_j,y_j,z_j) are co-ordinates of the current element nodes, $\mathbf{i}_x$, $\mathbf{i}_y$ and $\mathbf{i}_z$ are unit vectors. Thus it is assumed that the distance is calculated as the distance between the centre of the element dl and the coil centre. The length of **R** is assigned as |R|. The length of element $d\mathbf{l}$ is given as:

$$d\mathbf{l} = (x_i - x_j)\mathbf{i}_x + (y_i - y_j)\mathbf{i}_y + (z_i - z_j)\mathbf{i}_z.$$

The magnetic field is, therefore, calculated as the sum of all components associated with all currents flowing in each FEM element. Thus

$$\mathbf{B} = \frac{\mu_0}{4\pi} \sum_k i_k d\mathbf{l}_k \times \frac{\mathbf{R}_k}{R_k^3}, \quad (6)$$

where index k indicates the certain branch of each FEM element. Taking the approach proposed by Mura and Kagawa [7], current flowing between two nodes, or branches, can be evaluated from knowledge of the geometry and conductivity of the FEM element:

$$\mathbf{B} = \frac{\mu_0}{4\pi} \sum_{ij} \sigma_k C_k (\Delta V_k) d\mathbf{l}_k \times \frac{\mathbf{R}_k}{R_k^3}. \quad (7)$$

Further, it is assumed that only one component of the magnetic field, such as B_z., is measured. As a result, a sensitivity can be calculated:

$$S_m = \frac{C_{ij}^k [V_i - V_j]}{R^3}\left[R_x dl_y - R_y dl_x\right]. \quad (8)$$

where: C_{ij}^k - geometric co-efficient for the branch between nodes i and j of the k^{th} element, V_i, V_j - potential at the i^{th} and j^{th} node, σ_k conductivity of the k^{th} element.

Models

Both 2D and 3D models were used in the simulations. These were discretised on triangular and tetrahedric elements, respectively (fig. 1).

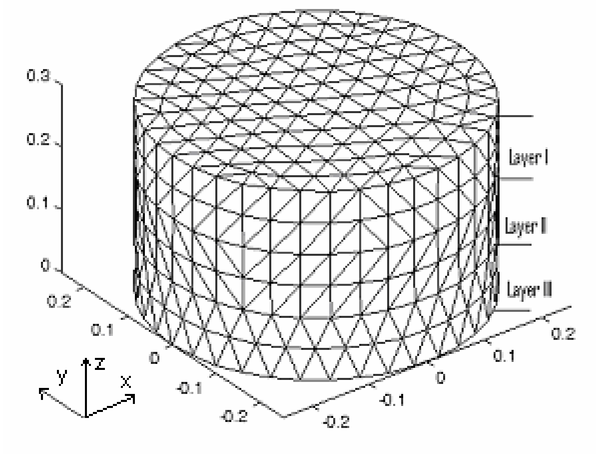


Figure 1: 3D mesh model divided into layers.

3D mesh models were examined for a different geometry. Detailed information about the geometry of 3D models may be found in the following table:

Table 1: Detailed information on the 3D models.

Number of layers	Number of mesh elements	Number of mesh nodes
I	1416	406
II	2696	659
III	4104	934

The 2D mesh model is presented in fig. 2. It is constructed in such a way that the arrangement of triangular elements and point co-ordinates are the same as on the top cover of the 3D mesh model presented in fig. 1.

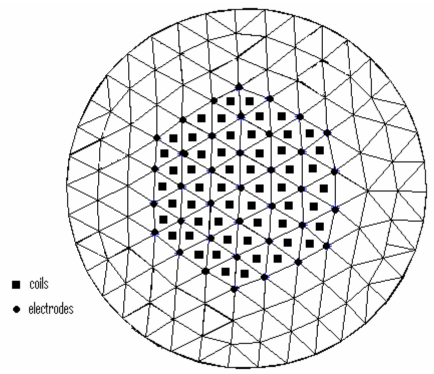


Figure 2: Top cover of the 3D mesh model with localisation of the electrodes and coils.

The 2D and 3D mesh models have identical sets of electrodes (37) located at the surface of the object under consideration. They also have the same array of coils (54), which is located over the surface of the object. The height of the magnetic sensors above the plane parallel to the x-y plane equalled 0.1 for all the simulations carried out.

Singular value decomposition technique

Singular value decomposition (SVD) is a particular case of orthogonal decomposition of every matrix $A \in R^{m\times n}$ such as:

$$A = U\Sigma V^T \tag{9}$$

where $U \in R^{m\times m}$ and $V \in R^{n\times n}$ are unitary matrices referred to as *left* and *right singular vectors* respectively, and the $\Sigma \in R^{m\times n}$ diagonal matrix is in the form:

$$\Sigma = \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix}, \tag{10}$$

where $D \in R^{r\times r}$ and r=*rank*(A). Diagonal elements of the Σ matrix are positive *singular values*:

$$\Sigma_{ii} = \lambda_i \qquad \lambda_1 \geq \lambda_2 \geq ... \geq \lambda_r \geq 0. \tag{11}$$

It follows that

$$Av_i = u_i / \lambda_i \quad \text{and} \quad A^T u_i = v_i \lambda_i. \tag{12}$$

where A^T is the transpose matrix of *A*. In addition, the pseudo-inverse of matrix *A* is defined as:

$$A^+ = v_i \frac{u^T}{\lambda_i} \quad . \tag{13}$$

Sensitivity Analysis

The solution of the inverse problem given from equation (2) equals:

$$\tilde{\sigma} = F(\sigma)^{-1} d \ , \tag{14}$$

The estimated conductivity distribution $\tilde{\sigma}(\mathbf{x})$ from the given data will be different from the true model. The dependency of the conductivity distribution $\tilde{\sigma}(\mathbf{x})$ on the measurement errors can be precise but it varies with respect to small changes in $\mathbf{d}$. By inserting equation (13) into equation (14) the estimated parameter can be obtained in terms of SVD products:

$$\tilde{\sigma} = \sum_{i\in\Omega} v_i \frac{u_i^T * d}{\lambda_i}, \qquad \Omega \in \{i : \sigma_i > \delta\}. \tag{15}$$

The instability of the reconstruction solution when the equations become almost linearly dependent is nicely illustrated by equation (15). For a large number of measurements many of the computed singular values are very small, in which case the components of the spectrum $\sigma(\mathbf{x})$ related to them are stifled by the measurement process. Singular values tending to zero make no contribution to the measured values and inflate the reconstruction solution when a set of data contains measurement errors δ.

Analysis of the spectrum of singular values allows round-off errors in the computed values to be made closer. It has been shown [8] that perturbations δ of any size in any matrix cause perturbations of roughly the same size in its singular values:

$$\|\delta\Sigma\| = \|\delta A\| \tag{16}$$

The perturbations are measured relative to the norm of the matrices, which is equal to the largest singular values, $\|A\|_2 = \sigma_1$. All random perturbations in matrix *A* caused by perturbations δ are smaller than $\varepsilon\|A\|$, where ε is the floating-point accuracy parameter (the difference between the largest and the smallest singular value). Round-off error in the computed values should are less than ε. Hence, for small perturbations their influence on them may not be visible in assuming exactness.

The largest changes in the estimated conductivity spectrum which results from a small change in the observed data can be estimated by the norm of $\|A^+\|$, which, owing to the properties of SVD, equals:

$$\|A^+\| = \lambda_s^{-1} \ , \tag{17}$$

where λ_s is the smallest singular value close to zero [1].

The relative changes in conductivity distribution can be estimated by the *condition number*, defined as the ratio of the largest singular value to the smallest:

$$cond(A) = \frac{\lambda_1}{\lambda_s} \cdot \tag{18}$$

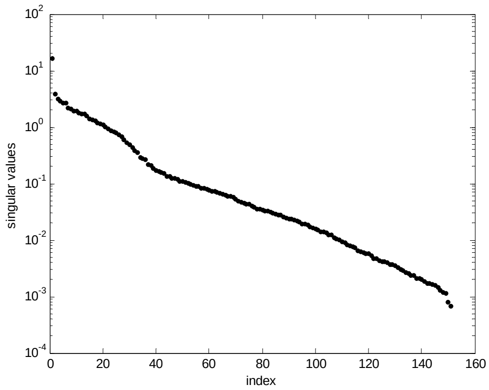
A condition number gives a simple indication of the accuracy of the reconstruction results. It is closely related to the size of *A* and is greater for a non-symmetric matrix. A high value of the condition number indicates extremely sensitive system equations to the values of system parameters. Generally, λ_i for *i*=1, ... *s* decays exponentially with *i*, so for a large *s* the condition number is extremely high and illustrates the ill-conditioned nature of the problem. If the condition number is greater than the relative error of the measurements, it may turn out the reconstruction result may be completely useless. A better reconstruction would be the sum of only those terms of equation (18) which have values of λ_i above a certain threshold δ, which should be smaller than the relative error of the measurements. The larger value of the minimum singular values makes the reconstruction problem more controllable and resilient. This is why the smallest singular value is sometimes known as the "Morari Resilience Index".

Sensitivity is determined by the optimal current pattern, which depends on object geometry and conductivity distribution [5]. The eigenfunction with the larger singular value indicates optimal current injection. In this case, the larger singular value informs us when two objects are distinguished from each other by measurements of precision ε. They are distinguished if $|\lambda_1| > \varepsilon$ and they are undistinguishable if $|\lambda_1| < \varepsilon$ [4].

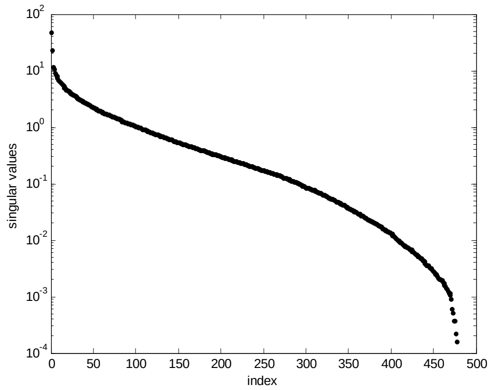
Results

The singular values of the EMIT forward operator were calculated for various models as presented in fig. 1.

Large singular values of the sensitivity matrices are attributed to the conductivity value adjacent to the surface of the model considered, whilst small ones are attributed to deep areas (located distant from the electrode and coil matrix) within the model. It can easily be noticed that changes in conductivity within this volume are almost undetectable.



a)



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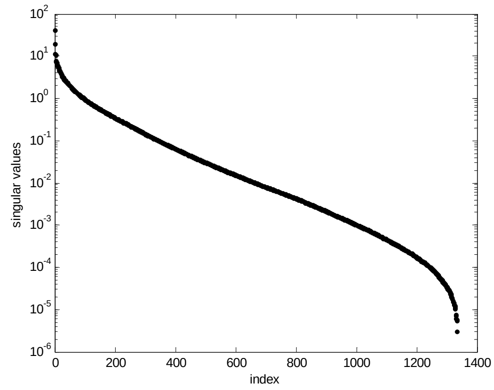

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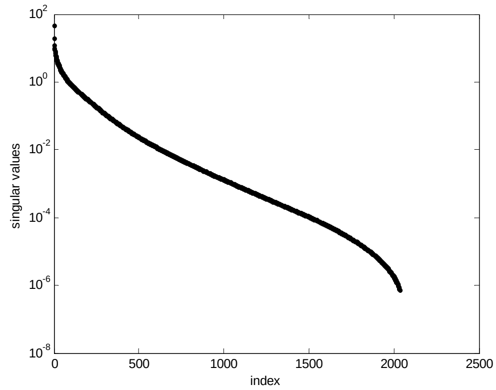

d)

Figure 3: Singular values of the forward operator matrix for a) a 2D model, and 3D models containing b) 1 layer, c) 2 layers, and d) 3 layers (log scale).

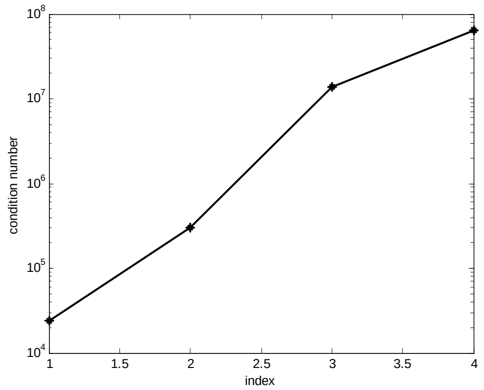

Figure 4: Condition number of the forward operator matrix for 1) a 2D model, and 3D models containing 2) 1 layer, 3) 2 layers, and 4) 3 layers (log scale).

The condition number for different reconstruction models is presented in fig. 4. It can be observed that the best value of the condition number was obtained for the 2D model. However, it increases rapidly with the complexity of the reconstruction model. The highest values of the condition number were obtained for the 3D reconstruction models with 2 and 3 layers. For these models small errors in measurement can produce large errors in the conductivity distribution of the reconstructed image.

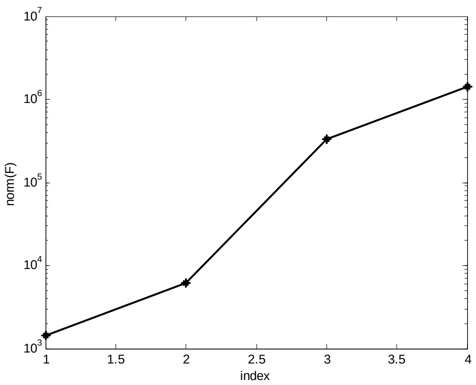

Figure 5: The norm of the forward operator matrix for 1) a 2D model, and 3D models containing 2) 1 layer, 3) 2 layers, and 4) 3 layers (log scale).

In fig. 5 it may clearly be seen that the largest changes in the estimated conductivity spectrum, which result from small changes in the observed data, can be estimated for more complex mesh models. This means that for them the reconstruction problem is extremely ill-posed.

Discussion

Sensitivity analysis is an important step in the reconstruction process. It can provide significant information about the properties of the forward operator matrix. It is a useful tool for finding the relationship between the design of the mesh and the distribution of a given set of data. Analysis of singular values enables sensitive areas of the mesh models to be identified and allows for sensitivity to perturbation of the data. The distribution of singular values can also inform us of the conditioning of the problem considered and provide an indication of the accuracy of the reconstruction image and the instability of the inverse solution.

Information obtained from the sensitivity analysis is important for the modification of the mesh model. The sensitivity analysis allows image resolution to be increased and noise sensitivity to be reduced by reducing the condition number of the inverse problem. Analysis of the singular value spectrum is also significant for correct interpretation of the reconstruction result.

Conclusions

The exactness of the reconstruction image should be greater but less real for the 2D model. The difference between 2D and 3D in the sensitivity of the system solution ranges from 10^2 to 10^4, the highest value being for the 3D reconstruction model with 3 layers. The ill-conditionig of the reconstruction problem increases with the complexity of model mesh. In this case a given solution is also more sensitive to perturbations in the set of data.

References

[1] AESELOWITZ .B., HARNILTON R., KUHEL A. (2004): ‘Spectrum reconstruction from dose measurements as a linear inverse problem’, *Phys. Med. Biol.*, vol. **49,** pp. **5087-5099**.

[2] FRIEDEL S. (2003): Resolution, stability and efficiency of resistivity tomography estimated from a generalized inverse approach *Geophys. J. Int.* vol. **153**, pp. 305–316.

[3] GESELOWITZ D.B. (1971): ‘An application of electrocardiographic lead theory to impedance pletysmography’, *IEEE Trans. Biomed. Eng*, vol. **18,** pp. **138-41**.

[4] ISAACSON D. (1986): ‘Distinguishability of conductivities by electric current computed tomography’, *IEEE Trans. Med. Imag*, vol. **MI-5,** pp. **91-95**.

[5] JANCZULEWICZ A., WTOREK J (2003): ‘EIT reconstruction based on noisy data’, *Proceed. of I Ntional Conf. On Information Technolog.*, vol. **1,** pp. **17-25**.

[6] LEVY S., ADAM D. and BRESLER Y. (2002): ‘Electromagnetic impedance tomography (EMIT): A new method for impedance imaging’, *IEEE Trans. On Medical Imaging*, vol. **21** no. **6,** pp. **676-87**.

[7] MURAI T. AND KAGAWA Y. (1985): Electrical impedance computed tomography based on a finite element model, *IEEE Trans. on Biomedical Engineering*, vol. **32** no. 3, pp. **177-184**.

[8] MathWorks, Internet site address: http://www.mathworks.com/moler/eigs.pdf.