A MODEL OF THE THERMODYNAMICAL PROCESSES IN THE HUMAN BODY DURING A CARDIOPULMONARY BYPASS

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Abstract: A mathematical model is introduced which allows to estimate temperature fields in the body of an adult patient under extracorporeal circulation. To obtain a system which can be computed in real-time the geometry of the body is approximated by six cylinders: the torso, the head, two arms and two legs. Each cylinder contains an artery-vein-loop in the center and is constructed by concentric layers of tissue with different heat conduction coefficients. The model is designed to be integrated into a more complex system which describes the haemodynamical processes and the metabolism of the human body.

Introduction

During cardiac surgery the patient's body is put into hypothermia in order to reduce oxygen consumption and thus to prevent insufficient perfusion of vital organs and to ensure cerebral protection. For the supervision of this process a model of the thermodynamical processes is desirable which allows to estimate temperature fields in the human body in real-time.

For the design of the model, the special circumstances of the extracorporeal circulation have to be taken into account:

- The heat production of the body is equivalent to the basal metabolic rate which is reduced due to hypothermia.
- Amyostasia is suppressed.
- The operating room has a constant temperature of about 23 °C. Therefore evaporation can be neglected.
- As the lung is excluded from the circulatory system, no heat can be exchanged via respiration.
- Due to the applied anaesthetics, auto-regulation by the hypothalamus is constricted [1].

Methods

In order to estimate a temperature field in the human body during a cardiac surgery, the mathematical model has to be computable in real-time. This requires that the shape of the human body is approximated by simple geometric shapes. A six-cylinders-model, suggested by Werner [2], is outlined in figure 1.

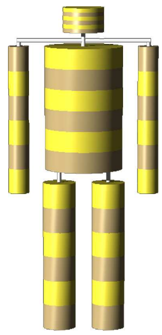

Figure 1: Six-cylinders-model of the human body

If the height h and mass m of the patient are known, the volume V of the body can be calculated [3]:

$$V = \frac{1000 \cdot m/\text{kg}}{0.0277(m/\text{kg})^{-0.3} \cdot (h/\text{cm})^{0.725} + 0.75} \cdot \text{m}^3 \quad (1)$$

The data given in table 1 leads to the dimensions of the cylinders.

Table 1: Characteristics of the cylinders [2, 4, 3, 5]

	head	torso	arm	leg
rel. length	12 %	38 %	40 %	50 %
rel. volume	5 %	45 %	7.5 %	17.5 %
rel. blood volume	20 %	55 %	5 %	7.5 %
rel. blood flow	21 %	42 %	9.5 %	11.5 %

Each cylinder consists of an artery-vein-loop and concentric layers of tissue as shown in figure 2. By this means a core-shell-model is obtained: The core represents muscles, organs and connective tissue, the shell represents fat tissue and skin. For simplicity matters, the artery and the vein are overlaid in the centre. The distribution of the blood volume is given in table 1. With this information the radii of the vessels can be calculated in such a way that the volume of the vessels is equivalent to the blood volume in the cylinder.

Analogously to the anatomy of the human body the blood flows from the aorta ascendens to the head, the arms and the torso. The output blood flow of these four cylinders forms the central venous return. Therefore the temperature in the vena cava can be calculated as a

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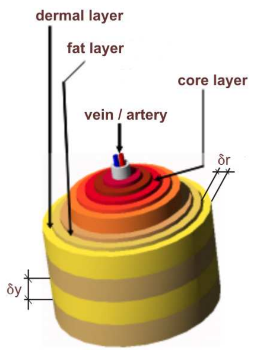

Figure 2: Schematics of a body cylinder

weighted average of the corresponding output temperatures. The weighting factors are the blood flow rates in the particular cylinders. Finally the leg cylinders are provided by a part of the arterial blood in the torso and contribute to the venous return of the torso.

Assuming that there is neither ingestion nor excretion during the cardiac surgery and neglecting the compliance of the vessels, the human body can be regarded as a thermodynamically closed system. Therefore the inner energy U can only change due to a heat flow Q over the boundaries of the system or the release of heat W within the system caused by irreversible energy conversions:

$$dU = dQ + dW\,. \tag{2}$$

Generally the change of the inner energy in a volume V can be expressed by

$$\frac{dU}{dt} = \rho \cdot c \iiint\limits_V \frac{\partial T}{\partial t}\, dV \tag{3}$$

if a constant density ρ and a constant heat capacity c are assumed. With the heat flow density $\underline{\dot{q}}$ the heat exchange with the environment is

$$\frac{dQ}{dt} = -\oiint\limits_{\partial V} \underline{\dot{q}}\, d\underline{A} = -\iiint\limits_V \operatorname{div} \underline{\dot{q}}\, dV\,. \tag{4}$$

Finally the energy from sources of heat in the considered volume can by obtained from the power density $\dot{w}$:

$$\frac{dW}{dt} = \iiint\limits_V \dot{w}\, dV \tag{5}$$

With the equations (3)-(5) the energy conversion theorem (2) can be written as a partial differential equation:

$$\rho \cdot c \cdot \frac{\partial T}{\partial t} = -\operatorname{div} \underline{\dot{q}} + \dot{w} \tag{6}$$

In the sequel cylindric coordinates (r, φ, z) will be used. Due to the symmetry of the model it can be assumed that there is no dependence on the angle φ. The heat flow density therefore only consists of a radial $\dot{q}_r$ and a axial $\dot{q}_z$ component. This results in the following representation of equation (6):

$$\rho \cdot c \cdot \frac{\partial T}{\partial t} = -\frac{1}{r} \cdot \dot{q}_r - \frac{\partial \dot{q}_r}{\partial r} - \frac{\partial \dot{q}_z}{\partial z} + \dot{w} \tag{7}$$

In the following the sources of heat and the basic mechanisms of heat transfer in the cylinders as well as the heat exchange with the environment are considered in detail. A heat flow over the boundaries is caused by the blood which enters or leaves a cylinder as well as the convective/conductive heat flow from the skin into the environment and the emission of radiation. Inside the cylinder, the mass flow of the blood transports heat through the vessels (convection) and the temperature gradient between the core and the shell of the body causes a conductive heat flow in the tissue. Moreover, the release of energy in the core layers due to metabolism has to be considered.

In the vessels, the mass flow of the blood $\dot{m}_B$ causes an axial heat flow:

$$\dot{q}_z = c_B \cdot \dot{m}_B \cdot T_B\,. \tag{8}$$

Moreover there is a conductive heat flow due to the radial temperature gradient[1]

$$\dot{q}_r = -\lambda_B \cdot \frac{\partial T_B}{\partial r} \tag{9}$$

with the heat conduction coefficient of blood λ_B. From equation (7) follows with $\dot{w} = 0$:

$$\rho_B \cdot c_B \cdot \frac{\partial T_B}{\partial t} = \lambda_B \left(\frac{1}{r} \cdot \frac{\partial T_B}{\partial r} + \frac{\partial^2 T_B}{\partial r^2} \right) - c_B \cdot \dot{m}_B \cdot \frac{\partial T_B}{\partial z} \tag{10}$$

The boundary conditions are the temperature of the blood which enters the vessel $T_B(z=0)$ and the heat flow to the tissue $\dot{q}_r(r_B, z)$.[2] This heat flow is caused by the difference between the temperature of the blood $T_B(r_B, z)$ and the core $T_C(r_B, z)$ and can be modelled as a thermal transfer through a wall:

$$\dot{q}_r(r_B, z) = -\lambda_B \cdot \left. \frac{\partial T_B}{\partial r} \right|_{r=r_B} = k \cdot \left(T_B(r_B, z) - T_C(r_B, z) \right) \tag{11}$$

The constant k denotes the heat transfer coefficient.

In the core heat is transferred in the radial direction (cf. equation (9)). Again axial conduction is neglected. Moreover the power density caused by metabolism $\dot{w}_{\mathrm{BMR}}$ has to be considered:

$$\rho_C \cdot c_C \cdot \frac{\partial T_C}{\partial t} = \lambda_C \left(\frac{1}{r} \cdot \frac{\partial T_C}{\partial r} + \frac{\partial^2 T_C}{\partial r^2} \right) + \dot{w}_{\mathrm{BMR}} \tag{12}$$

The energy consumption of the patient equals the basal metabolic rate (BMR) under the conditions of a surgery. It can be obtained from the patient's height, mass, age and gender [3]. However, the consumption is reduced under hypothermia (see table 2). With the correct value for the

[1] Axial conduction is neglected.
[2] r_B denotes the radius of the vessel.

energy consumption at the current body core temperature and regarding the amount of consumption in the different parts of the body (torso: 60 %, head: 20 %, limbs: 20 % [6]) the power density in the core can be calculated for each cylinder.

Table 2: Relationship between body temperature and oxygen consumption [7]

temperature	oxygen consumption
37°C	100 %
29°C	50 %
22°C	25 %
16°C	12 %
10°C	6 %
6°C	3 %

Analogously to equation (11), the thermal transfer between the vessels and the core is a boundary condition. However it has to be taken into account that there is a heat flow to the artery *and* to the vein:[3]

$$\begin{aligned} \dot{q}_r(r_B,z) &= \lambda_B \cdot \left.\frac{\partial T_B}{\partial r}\right|_{r=r_B} \\ &= -k \cdot \left(T_{B,\text{art.}}(r_B,z) - T_C(r_B,z)\right) \\ &\qquad - k \cdot \left(T_{B,\text{ven.}}(r_B,z) - T_C(r_B,z)\right) \end{aligned} \tag{13}$$

Furthermore the temperature at the outer radius of the core $T_C(r_C,z)$ equals the temperature of the shell $T_S(r_C,z)$. This is a boundary condition for both the core and the shell.

Finally the radial conduction in the shell has to be modelled. It is assumed that there is no metabolism in the shell layer. The following partial differential equation results:

$$\rho_S \cdot c_S \cdot \frac{\partial T_S}{\partial t} = \lambda_S \left(\frac{1}{r} \cdot \frac{\partial T_S}{\partial r} + \frac{\partial^2 T_S}{\partial r^2}\right) \tag{14}$$

At the outer radius of the shell r_S (which equals the radius of the cylinder) the boundary condition is defined by the heat flow from the skin into the environment:

$$\dot{q}_{\text{skin}} = -\lambda_S \cdot \left.\frac{\partial T_S}{\partial r}\right|_{r=r_S} \tag{15}$$

Under the conditions of a cardiopulmonary bypass the heat flow $\dot{q}_{\text{skin}}$ results from convection/conduction and emission of radiation:

$$\begin{aligned} \dot{q}_{\text{skin}} = \zeta \cdot \Big(& h_{\text{conv/cond}}\big(T_S(r_S) - T_{\text{room}}\big) \\ &+ \sigma\big(\varepsilon T_S^4(r_S) - \alpha T_{\text{room}}^4\big)\Big) \end{aligned} \tag{16}$$

In this equation $h_{\text{conv/cond}}$ is the convectional/conductional heat transfer coefficient, σ is the Stefan Boltzmann constant, ε is the emission coefficient and α is the absorption coefficient. The shell temperature T_S and the room temperature T_{room} are absolute temperature values. The expression enclosed in brackets is the value for the heat flow from an unclothed human. However, the patient is covered by a blanket which reduces the heat flow by a factor 5 to 10 [8]. This is modelled by the parameter ζ which should be in the range 0.1 to 0.2.

[3] The heat flow is defined to be positive if it leads from the considered volume to its environment.

In the previous paragraphs the required equations to calculate the temperature fields in the cylinders have been deduced. To solve the partial differential equations on a computer, the *Finite Differences Method* (FDM) is applied. This means that the differential quotients are approximated by difference quotients. In this case the first order differential quotients are approximated by right hand difference quotients

$$\frac{\partial f}{\partial x} \approx \frac{f(x+\Delta x) - f(x)}{\Delta x} \tag{17}$$

or central difference quotients

$$\frac{\partial f}{\partial x} \approx \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}\,. \tag{18}$$

The second order difference quotients are

$$\frac{\partial^2 f}{\partial x^2} \approx \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{\Delta x^2}\,. \tag{19}$$

To apply the FDM the time t as well as the coordinates r and z have to be discretized:

$$t_k := t_0 + k \cdot \Delta t \qquad k = 0,\ldots,K-1 \tag{20}$$
$$r_m := r_0 + m \cdot \Delta r \qquad m = 0,\ldots,M-1 \tag{21}$$
$$z_n := z_0 + n \cdot \Delta z \qquad n = 0,\ldots,N-1 \tag{22}$$

For example equation (14) is implemented in the following way:

$$\begin{aligned} &T_S(t_{k+1},r_m,z_m) = \\ &\quad \frac{\lambda_s \cdot \Delta t}{\rho_s \cdot c_S \cdot \Delta r} \cdot \frac{1}{r_m}\Big(T_S(t_k,r_{m+1},z_n) - T_S(t_k,r_m,z_n)\Big) \\ &\quad + \frac{\lambda_s \cdot \Delta t}{\rho_s \cdot c_S \cdot \Delta r^2} \cdot \Big(T_S(t_k,r_{m+1},z_n) - 2T_S(t_k,r_m,z_n) \\ &\qquad + T_S(t_k,r_{m-1},z_n)\Big) + T_S(t_k,r_m,z_n) \end{aligned} \tag{23}$$

In the outer layer $m = M-1$ the boundary condition (15) has to be used to calculate the discrete derivatives with respect to r:

$$\dot{q}_{\text{skin}} = -\lambda_S \cdot \frac{T(t_k,r_M,z_n) - T(t_k,r_{M-2},z_n)}{2\Delta r} \tag{24}$$

Results

To evaluate the model the following data has been recorded during a cardiac surgery:

- arterial blood flow
- arterial temperature
- venous temperature
- nasal temperature
- rectal temperature

The sample time of the measured data is 60 s, the accuracy is ±0.1 °C. The temperatures of the arterial and venous blood were measured between the oxygenator and the patient's body. A homogeneous body temperature of 34 °C was assumed as an initial condition. The simulation was executed with a sample time of 1 ms. The figures 3-5 compare the measured venous, nasal and rectal temperature with the simulated venous temperature and core temperature curves of the head and the body cylinder.

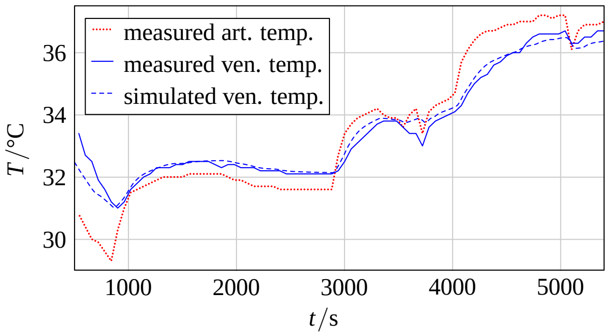

Figure 3: Measured and simulated venous temperature

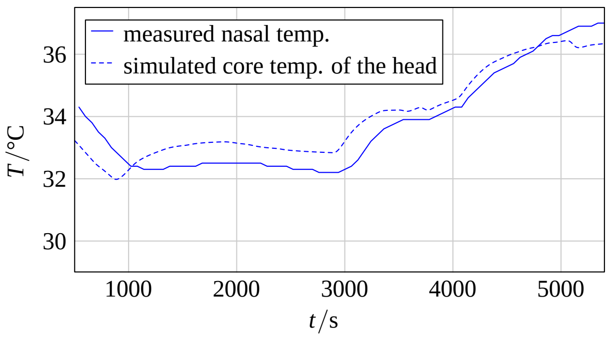

Figure 4: Measured nasal temperature and simulated core temperature of the head

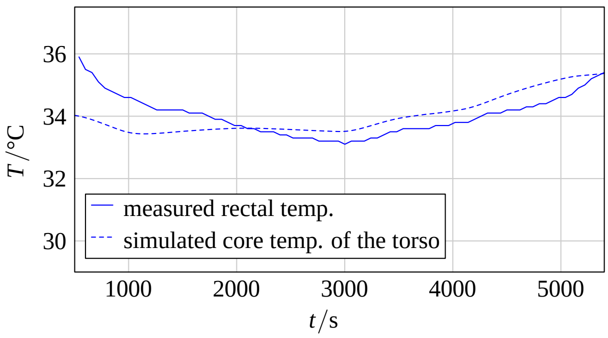

Figure 5: Measured rectal temperature and simulated core temperature of the torso

Discussion

Figure 3 shows that after an initial transient period the deviation between measured and estimated venous temperature is less than 0.5 °C. Obviously the cylinder model is suitable to estimate the cycle time of the blood through the body and the energy exchange with the tissue. The latter is influenced by the parameter k in equation (11). As this parameter cannot be measured and is not given in any literature it has been determined empirically from several data records.

Due to the positioning of the probes which are not placed in the immediate core, the measured nasal and rectal temperatures do not exactly represent the core temperature of the head or the torso. Therefore, in the figures 4 and 5, these temperatures are not compared to the estimated temperature of the innermost radial core layer but to the temperature of a layer closer to the shell.

In contrast to the venous temperature, the deviations are higher in case of the nasal and rectal temperature. Nevertheless the run of the curve is represented well in both cases apart from an initial transient phenomenon of about 2,000 s in case of the rectal temperature. The long duration of this phenomenon shows that it is necessary to choose a suitable initial temperature of the body. In the example the real initial temperature values are different in the individual cylinders. Therefore it might be convenient to choose individual values for each cylinder rather than one temperature value which is valid for the whole body.

Another source of error ist the fact that the model does not consider the site of surgery, where size and depth of the wound, evaporation, suction of blood and irrigation affect the temperature.

Conclusions

The temperature model presented in this paper is computable in real-time and enables the estimation of the temperature of the blood, the core and the shell in different parts of the body of an adult patient. The model is going to be integrated into an observer system which is being developed at the Institute of Industrial Information Technology, Universität Karlsruhe. It provides information about the haemodynamics and metabolism of the patient during a cardiac surgery. Furthermore, the temperature model will be used to design a temperature controller for the heater-cooler-device of a heart-lung-machine.

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