

# EXPERIMENTAL IDENTIFICATION OF THE ELECTROMECHANICAL TRANSFER FUNCTION OF ULTRASOUND TRANSDUCERS

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**Abstract:** Many advanced ultrasound imaging techniques rely onto knowledge of the electromechanical transfer characteristics of the transducer. State of the art models based on equivalent circuits, finite element modelling or measured impulse responses suffer from miscellaneous practical deficiencies. An experimental identification of a linear transducer model overcoming some of these problems is presented. The method is based on parametric system identification techniques and suitable broadband input signals. Appropriate measurement conditions are derived from numerical simulations of diffraction effects. The approach which does not require any advance information about the physical properties of the piezoelectric crystal is shown to result in an accurate model for a common medical transducer.

## Introduction

A piezoelectric ultrasound transducer in pulse echo imaging is used to transform an electrical signal produced by some driving circuit into an acoustical wave and vice versa to transform a pressure wave which impinges onto the transducer into an electrical signal. Since acoustical properties of the transduction medium are to be evaluated by an analysis of the received electrical signal, the connection between the acoustic and electric signals of a transducer is of great importance in medical ultrasound imaging. Coded excitation for example makes use of broadband signals which must be adapted to the transfer characteristic of the ultrasound probe in order to transmit sufficient energy. Range and resolution of ultrasound systems can potentially be enhanced by cancellation of the electroacoustic transfer characteristic by deconvolution techniques. More general all attempts to simulate ultrasonic pulse echo imaging systems depend on a realistic description of the relation between electric and acoustic signals.

Complicated measurement schemes have been set up to acquire those properties of piezoelectric crystals that determine their electroacoustic behaviour. A combination of such measurements with equivalent circuits, or finite element simulations provides analytic models of the transducer. In many practical cases it is however favourable to assume linear relations and express the

electroacoustic behaviour of a transducer in terms of a single transfer function. A transfer function provides intuitive insight into transducer characteristics, like centre frequency bandwidth and duration of postoscillations and is suited to be included into software for the simulation of ultrasound imaging systems. Moreover experimental methods to derive a transducer model do not comprise the necessity to know the properties of the piezoelectric crystal, its backing, and matching layers.

The most common method to measure the transfer function of a transducer is to position the transducer opposite to some reflector and excite it with an electrical pulse signal. The echo signal is measured and its Fourier transformation is taken as the transfer function of the transducer. It is well known that for non focussing transducers even a single point scatterer gives rise to a complex echo signal with overlapping contributions from plane and edge wave components due to diffraction. It is therefore problematic to consider the echo signal to be the electromechanical impulse response. Removing the diffraction effects has successfully been done by numerical deconvolution [1]. Since the diffraction depends on the geometry of the transducer and its oscillation modes, deconvolution is only applicable to situations in which detailed knowledge of the geometrical and mechanical properties of the transducer is available. It is advantageous to find a measurement set-up in which the diffraction proves to be negligible. By employing the work of Rhyne [2] some authors [1, 3, 4] pointed out that the diffraction effects between a circular non focussing transducer with pistonlike behaviour and an infinite perfectly reflecting plane can be neglected if the plane is placed in the nearfield of the transducer. In this case the assumption that the measured echo signal due to an impulse excitation resembles the electromechanical impulse response is thus true. There is some evidence that this holds also for circular transducers having a Gaussian velocity distribution [5]. We will show by numerical computations that the result is valid for rectangular transducers, finite sized targets of sufficient extends, and non uniform movement of the transducer surface as well. This gives strong support that the measurement principle is applicable even if the exact properties of the transducer are unknown.

Direct measurement of the impulse response belongs to the class of non parametric system identification meth-

ods [6]. It is afflicted with practical difficulties due to the inability to create pulses of sufficiently short duration, transient behaviour of the transducer and a poor signal to noise ratio of the measured impulse response. We therefore suggest the use of parametric system identification methods based on ARX- or alternatively ARMAX-system models and linear frequency modulated excitation signals. These methods, which have, to our knowledge, not yet been applied to ultrasound transducers, potentially overcome the mentioned drawbacks associated with needle shaped signals.

## Methods and materials

*Electromechanical Transfer Function:* The electromechanical behaviour of a piezoelectric transducer is governed by the relation between the physical properties voltage  $u(t)$  and current  $i(t)$  on the electrical side and pressure  $p(\mathbf{r}_S, t)$  onto and normal velocity  $v_n(\mathbf{r}_S, t)$  of the transducer surface on the mechanical side [7]. Here  $\mathbf{r}_S$  denotes a position on the surface of the transducer and  $t$  denotes time. The relation between pressure  $p(\mathbf{r}, t)$  and particle velocity  $\mathbf{v}(\mathbf{r}, t)$  at some point  $\mathbf{r}$  inside a transduction medium with density  $\rho$  is given by

$$\rho \frac{\partial \mathbf{v}(\mathbf{r}, t)}{\partial t} = -\nabla p(\mathbf{r}, t), \quad (1)$$

while the relation between voltage and current depends on the impedance  $Z_i(t)$  of the driving circuit. In a given experimental set-up the behaviour of the transducer can thus be described by just one quantity on the electrical and one quantity on the acoustical side. Since a transmit-receive-switch is usually used to decouple the electrical circuit for transmission,  $Z_i(t)$  is time dependent and it is necessary to distinguish between the two conditions transmission and reception. The transducer in transmit mode can then be regarded as a device which reacts to some input voltage  $\hat{u}(t)$  with a displacement of its surface at position  $\mathbf{r}_S$  with a normal velocity  $v_n(\mathbf{r}_S, t)$ . In receive mode it is convenient to describe the transducer as being pressure sensitive i. e. to react to the total active pressure  $P(t)$  onto its surface  $S$  with an output voltage  $\hat{y}(t)$ . Under the assumption that time and space variables are separable at the surface of the transducer one can write [8]

$$v_n(\mathbf{r}_S, t) = \Gamma(\mathbf{r}_S)v(t) \quad (2)$$

and accordingly

$$P(t) = \iint_S \Gamma(\mathbf{r}_S)p(\mathbf{r}_S, t)dS. \quad (3)$$

The weighting function  $\Gamma(\mathbf{r}_S)$  is called velocity distribution and accounts for position dependent sensitivities of the transducer surface.

If nonlinear behaviour is neglected, the transducer can be described as a scalar causal linear time invariant system (LTI-system) with the impulse response  $h_{\text{Trm}}(t)$

$$v(t) = \int_{-\infty}^{\infty} h_{\text{Trm}}(t - \tau)\hat{u}(\tau)d\tau \quad (4)$$

in transmit, impulse response  $h_{\text{Rec}}(t)$

$$\hat{y}(t) = \int_{-\infty}^{\infty} h_{\text{Rec}}(t - \tau)P(\tau)d\tau \quad (5)$$

in receive mode and the surface velocity distribution  $\Gamma(\mathbf{r}_S)$ . The pulse-echo electromechanical impulse response is defined to be the convolution of the impulse responses in transmit and receive mode:

$$h_{\text{PE}}(t) = \int_{-\infty}^{\infty} h_{\text{Trm}}(t - \tau)h_{\text{Rec}}(\tau)d\tau. \quad (6)$$

Its Fourier transformation

$$H_{\text{PE}}(f) = \int_{-\infty}^{\infty} h_{\text{PE}}(t)e^{j2\pi ft}dt. \quad (7)$$

is the electromechanical transfer function of the transducer.

*Experimental Set-up:* We derived the transfer function  $H_{\text{PE}}(f)$  of a piezoelectric ultrasound transducer for medical applications. The exciting frequency modulated binary signals (pseudochirps)  $\hat{y}(t)$  were created by the one channel ultrasound device PCM 100 (Inson GmbH, St. Ingbert). The same device was used to amplify and digitize the echo signal trace  $\hat{u}(t)$  at a sample frequency  $f_S = 25$  MHz. As a medium for sound propagation we used distilled water, which has previously been degassed by heating it to 80°C for one hour. The reflection from a circular planar reflector with a diameter of 80 mm in a distance of 8 mm to the transducer surface was measured. The reflector was built of an air cavity separated from the surrounding water by a thin plastic membrane. It was placed inside a quadratic basin with 300 mm side length. The electromechanical transfer function  $H_{\text{PE}}(f)$  of the transducer has been determined by means of a parametrized model which relates the echo signal  $\hat{y}(t)$  to the input signal  $\hat{u}(t)$  in the time domain.

Computations and simulations were performed in Matlab® on standard PCs. The integrals in (15) and (17) have been solved by an iterative Simpson's algorithm, while the spatial impulse responses have been calculated by employing the ultrasound simulation software Field II [9]. For minimization of (25) the Nelder-Mead simplex algorithm with the outcome of the least-squares method as initial values has been used.

*Diffraction Effects:* The incident pressure field  $p_0(\mathbf{r}, t)$  from a finite transducer placed inside an infinite rigid baffle at a point  $\mathbf{r}$  in a homogeneous medium with density  $\rho$  and sound speed  $c$  is [8]

$$p_0(\mathbf{r}, t) = \rho \iint_S \frac{\partial}{\partial t} \frac{v_n(\mathbf{r}_S, t - |\mathbf{r} - \mathbf{r}_S|/c)}{2\pi|\mathbf{r} - \mathbf{r}_S|} d\mathbf{r}_S. \quad (8)$$

Substituting (2) this can be expressed as the convolution

$$p_0(\mathbf{r}, t) = \rho \int_{-\infty}^{\infty} \frac{\partial}{\partial \tau} h_{\text{Sp}}(\mathbf{r}, t - \tau)v(\tau)d\tau \quad (9)$$

with the spatial impulse response

$$h_{\text{Sp}}(\mathbf{r}, t) = \iint_S \Gamma(\mathbf{r}_S) \frac{\delta(t - |\mathbf{r} - \mathbf{r}_S|/c)}{2\pi|\mathbf{r} - \mathbf{r}_S|} d\mathbf{r}_S. \quad (10)$$

Every point  $\mathbf{r}_R$  on a reflector surface can approximately be assumed to react to the incident pressure like a monopole with a particle velocity  $v_R(\mathbf{r}_R, t) = K_R p_0(\mathbf{r}_R, t)$ , thus to emit a spherical pressure wave

$$p_s(\mathbf{r}, t) = \rho K_R \frac{\partial}{\partial t} \frac{p_0(\mathbf{r}_R, t - |\mathbf{r} - \mathbf{r}_R|/c)}{2\pi|\mathbf{r} - \mathbf{r}_R|}, \quad (11)$$

where  $K_R$  is a constant which accounts for the scattering strength. This approximation is not strictly valid due to the fact that the particles of the reflector surface can not freely move [10, p. 300], but has proven to provide results for plane surface reflectors normally aligned to the transducer which are in good agreement with experimentally measured echoes [11, 12]. Using the reciprocity theorem [10, pp. 642–643] which states, that a small source and a receiver can be interchanged, it is evident, that the integrated pressure  $P(\mathbf{r}_R, t)$  onto the transducer surface  $S$  due to the spherical wave  $p_s(\mathbf{r}, t)$  can be calculated from the spatial impulse response that determines the pressure at the point reflector:

$$P(\mathbf{r}_R, t) = \rho \int_{-\infty}^{\infty} \frac{\partial}{\partial t} h_{Sp}(\mathbf{r}_R, t - \tau) v_R(\mathbf{r}_R, \tau) d\tau, \quad (12)$$

where  $h_{Sp}(\mathbf{r}, t)$  has been defined in (10). In a linear model, the echo wave from a finite sized target can be calculated by summing up the contributions of all point sources i. e. by integrating over the reflector surface  $R$ . The total pressure onto the transducer is thus

$$P(t) = \iint_R P(\mathbf{r}_R, t) d\mathbf{r}_R. \quad (13)$$

The Combination of (6), (9) and (13) provides a complete system model for the experimental setup described above:

$$\hat{y}(t) = \hat{u}(t) * h_{PE}(t) * \frac{\partial}{\partial t} h_{RC}(t), \quad (14)$$

where the convolution integrals have been denoted by the shorthand symbol  $*$  and  $h_{RC}(t)$  is the so called radiation coupling function

$$h_{RC}(t) = \iint_R \frac{\partial}{\partial t} h_{Sp}(\mathbf{r}_R, t) * h_{Sp}(\mathbf{r}_R, t) d\mathbf{r}_R, \quad (15)$$

which accounts for diffraction effects between the transducer and the reflector. For our purposes it is more illustrative to examine diffraction effects in the frequency domain, therefore the pressure coupling transfer function

$$H_{RCp}(f) = \int_{-\infty}^{\infty} \frac{\partial}{\partial t} h_{RC}(t) e^{j2\pi ft} dt \quad (16)$$

is introduced.

If the reflecting plane is assumed to be infinite in space, calculation of the radiation coupling function can be simplified by using the concept of mirror sources [10, pp. 36–37] as it has been done for a circular piston transducer opposite to a rigid plane [2]. The computation of  $h_{RC}(t)$  reduces to the computational less demanding equation

$$h_{RC}(t) = \iint_M h_{Sp}(\mathbf{r}_M) d\mathbf{r}_M. \quad (17)$$

Here  $h_{Sp}(\mathbf{r}_M)$  is the spatial impulse response of the transmitter at a point  $\mathbf{r}_M$  on the surface of the mirror transducer, which is thought to be placed opposite to the real transducer at twice the distance plane–transducer. We will show that for a reflector of 80 mm diameter, contributions from the edge of the reflector to the output signal  $\hat{y}(t)$  are negligible and (17) is therefore applicable to our measurements.

In the system model (14) the influences of the radiation coupling function  $h_{RC}(t)$  and the electromechanical impulse response  $h_{PE}(t)$  are separated from each other. As will be shown later our experimental set-up ensures negligible influence of diffraction onto the output voltage  $\hat{y}(t)$ . It is thereby possible to obtain the electromechanical behaviour of the transducer from measurements of  $\hat{y}(t)$  to some known input  $\hat{u}(t)$  by applying a parametrized model which describes solely the electromechanical transducer behaviour. Since the input and output signals that are processed by the ultrasound device PCM 100 are digitalized we will now consider the sampled counterparts of  $\hat{u}(t)$ ,  $\hat{y}(t)$  as system in- and output and denote them as  $\hat{u}(k)$  and  $\hat{y}(k)$ , where  $k \in \mathbb{N}$  and  $t = k/f_S$ .

*System Identification:* A standard system model is the autoregressive model with an exogenous variable (ARX-model) which can be denoted as a linear difference equation:

$$\tilde{y}(k) = \sum_{\mu=0}^m b_{\mu} \hat{u}(k - \mu) - \sum_{\nu=1}^n a_{\nu} \tilde{y}(k - \nu) + n(k). \quad (18)$$

The first two terms describe a lti-system with an impulse response

$$\tilde{h}_{PE}(k) = \sum_{\mu=0}^m b_{\mu} \delta(k - \mu) - \sum_{\nu=1}^n a_{\nu} \tilde{h}_{PE}(k - \nu) \quad (19)$$

while the last addend adds a disturbance which accounts for those parts of  $\tilde{y}(k)$  that can not be expressed by an impulse response due to time-variant, stochastic, or nonlinear system behaviour. If the disturbance can be assumed to be the moving average of some white noise signal  $\varepsilon(k)$  with Gaussian amplitude distribution of zero mean, the ARX-model can be extended to the autoregressive moving average model with an exogenous variable (ARMAX-model):

$$\tilde{y}(k) = \sum_{\mu=0}^m b_{\mu} \hat{u}(k - \mu) - \sum_{\nu=1}^n a_{\nu} \tilde{y}(k - \nu) + \sum_{\omega=1}^o c_{\omega} \varepsilon(k - \omega), \quad (20)$$

a structure commonly used in control engineering [6]. The coefficients  $b_{\mu}$ ,  $a_{\nu}$  and possibly  $c_{\omega}$  are adjustable parameters which are to be estimated for an appropriate model order  $m$ ,  $n$  and possibly  $o$ .

In order to approximate the true system behaviour as close as possible by the impulse response  $\tilde{h}_{PE}(k)$  the contribution of  $n(k)$  to the model (18) must be minimized. A natural criterion would be the difference between the output signal of the model  $\tilde{y}(k)$  and the measured output  $\hat{y}(k)$  under the assumption of zero disturbance:

$$e(k) = \tilde{y}(k) - \hat{y}(k)|_{n(k)=0}. \quad (21)$$

The series  $e(k)$  is the so called output error. Substituting (18) into this equation leads to

$$e(k) = \sum_{\mu=0}^m b_{\mu} \hat{u}(k-\mu) - \sum_{v=1}^n a_v \hat{y}(k-v) - \hat{y}(k). \quad (22)$$

The output signal  $\hat{y}(k)$  of the system model is however not known prior to the estimation of the unknown parameters  $b_{\mu}$  and  $a_v$ . To employ a linear estimation the output signal of the model  $\hat{y}(k)$  is therefore replaced by the measured signal  $\hat{y}(k)$  in (22):

$$e'(k) = \sum_{\mu=0}^m b_{\mu} \hat{u}(k-\mu) - \sum_{v=1}^n a_v \hat{y}(k-v) - \hat{y}(k). \quad (23)$$

The variable  $e'(k)$  is called equation error. The unknown parameters  $b_{\mu}$ ,  $a_v$  and possibly  $c_{\omega}$  can be estimated by minimizing an appropriate functional of the equation error  $e'(k)$ . The sum of squared errors over a defined time horizon

$$E(m, n, b_{\mu}, a_v) = \sum_{k=1}^K e'^2(k) \quad (24)$$

for the ARX-model or

$$E(m, n, o, b_{\mu}, a_v, c_{\omega}) = \sum_{k=1}^K e'^2(k) \quad (25)$$

for the ARMAX-model respectively is the most commonly applied error functional. Minimization of  $E(m, n, b_{\mu}, a_v)$  corresponds to linear regression and is usually named least-squares method, while minimization of  $E(m, n, o, b_{\mu}, a_v, c_{\omega})$  provides the maximum-likelihood estimator for the Gaussian white noise  $\varepsilon(k)$  in (20) and is therefore called maximum-likelihood method.

Parameters  $\tilde{a}_v^s$  and  $\tilde{b}_{\mu}^s$  from the least-squares method will be unbiased estimates for  $a_v$  and  $b_{\mu}$  respectively only if the equation error  $e'(k)$  is white noise [13]. That is generally not true because of its correlation with past system outputs as analysis of (22) reveals. However, the bias is small in case of a high signal-to-noise ratio. The maximum-likelihood method requires that  $\varepsilon(k)$  is Gaussian white noise to make the estimated parameters  $\tilde{a}_v^{ml}$  and  $\tilde{b}_{\mu}^{ml}$  unbiased estimates for  $a_v$  and  $b_{\mu}$  respectively, which can be assumed in many practical cases. Minimization of (25) is a nonlinear task and can therefore only be performed iteratively, while finding the minimum with respect to (24) is a linear problem.

## Results

The radiation coupling function  $h_{RC}(t)$  between a circular piston transducer with a radius of 3.2 mm and a circular reflecting plane of radius 10 mm placed opposite to each other in a distance  $d = 8$  mm is shown in Figure 1. The initial high peak corresponds to the reflected plane wave component, while the slight curvature which starts at 14  $\mu$ s is present due to reflections from the edge of the reflector. The edge contribution is much smaller than

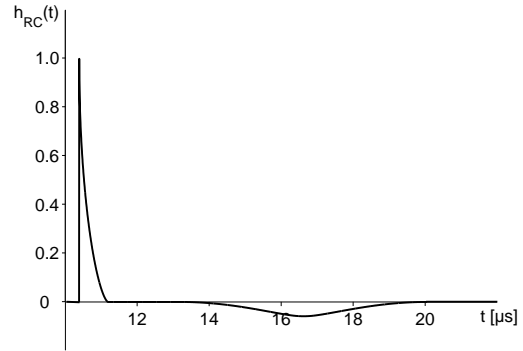


Figure 1: Radiation coupling function of a circular piston transducer (radius 3.2 mm) and a circular reflecting plane (radius 10 mm) situated in a distance of 8 mm to each other.

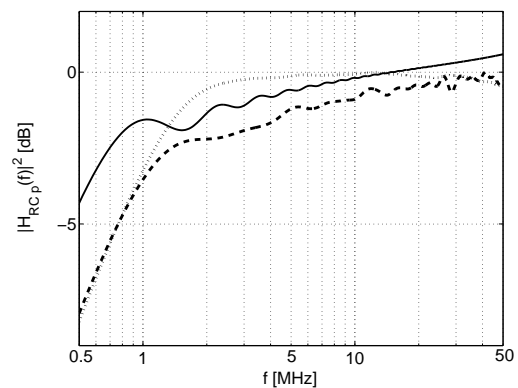


Figure 2: Pressure coupling amplitude response for transducers opposite to a plane reflector in 8 mm distance. Circular piston transducer: solid line; Rectangular piston transducer: dashed line; Rectangular transducer with non uniform velocity distribution: dotted line.

the plane wave contribution, so that its influence onto the echo signal  $\hat{y}(t)$  might well be neglected. The influence is even lower for larger reflectors as the one we used in our experimental set-up. The reflecting plane can thus be regarded infinite and the concept of mirror sources (17) might be applied for calculation of radiation coupling functions.

The pressure coupling amplitude response  $|H_{RCp}(f)|^2$  corresponding to the radiation coupling function in Figure 1 is depicted in Figure 2 (solid line). Additionally the pressure coupling amplitude responses of a rectangular piston transducer with side lengths 5 mm x 4 mm (dashed line) and a rectangular transducer with the same dimensions but velocity distribution  $\Gamma(x, y) = \sin(x\pi/5 \text{ mm} + y\pi/4 \text{ mm})$  (dotted line) are plotted for the same transducer – reflector distance. Diffraction effects are negligible, i.e. the pressure coupling amplitude response is relatively flat, for frequencies above 2 MHz for all transducers. Our investigations indicate, that the edge frequency above which diffraction effects can be neglected depends

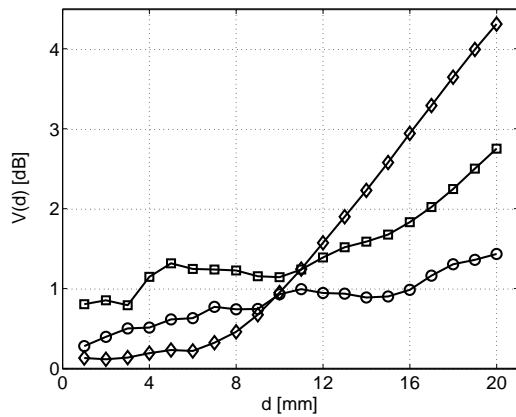


Figure 3: Variability of the pressure coupling amplitude response in the frequency range 2 MHz upto 8 MHz as a function of distance between transducer and reflector. Circular piston transducer: circles; Rectangular piston transducer: squares; Rectangular transducer with non uniform velocity distribution: diamonds.

mainly on the transducer size and the distance to the reflector but less on the actual transducer geometry or mode of oscillation i.e. velocity distribution. The variability of the pressure coupling amplitude response  $|H_{RCp}(f)|^2$  between 2 MHz and 8 MHz

$$V(d) = \max |H_{RCp}(f)|^2 - \min |H_{RCp}(f)|^2 \quad 2 \text{ MHz} < f < 8 \text{ MHz}$$

is plotted as a function of distance  $d$  between transducer and reflector in Figure 3 for the three transducer types described above.

The transducer under test was modelled by a discrete linear system of order  $n = m = 6$  using a sampling frequency  $f_s = 25$  MHz. Increasing the system order further did not lead to significant reduction of the output error  $e(k)$  as is illustrated in Figure 4, where the error is plotted as a function of the system order. Only subtle differences between the ARX-model derived by the least-squares method and the ARMAX-model derived by the maximum-likelihood method could be noticed. The electromechanical transfer function  $H_{PE}(f)$  of the transducer has been derived from the ARX-model. The corresponding Bode diagram is depicted in Figure 5. The diagram shows the bandpass characteristic of the transducer. It has got a centre frequency of 4.9 MHz and a 3 dB-bandwidth of 1.6 MHz.

A good agreement between the modelled and measured behaviour of the transducer could be recognized in the time domain. In Figure 6 the modelled and measured system response to a linear frequency modulated signal (3 MHz - 6 MHz) not previously used for model derivation is shown. The sum of squared output errors was below 2% of the total signal energy.

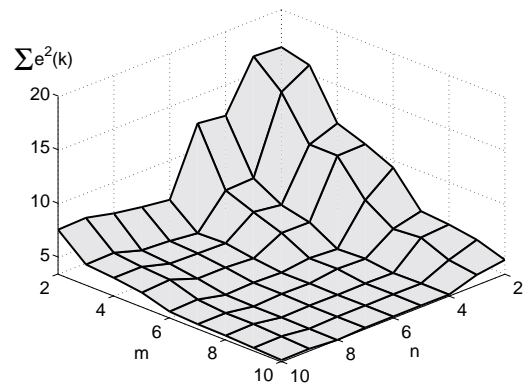


Figure 4: Dependency of the output error on the order of the system model.

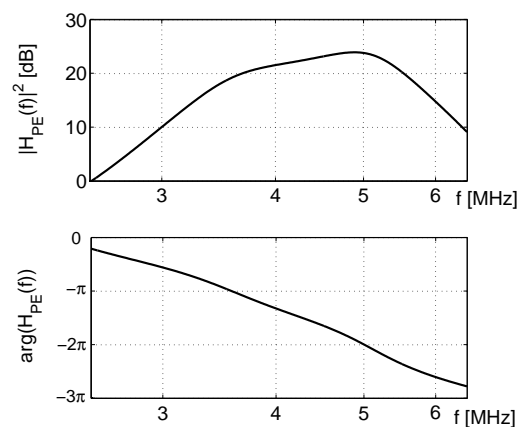


Figure 5: Bode diagram of the system model.

## Discussion

We described a method to derive the electromechanical transfer function of ultrasonic transducers from measurements of echo signals due to excitation with frequency modulated signals. This approach proved to be useful in deriving a realistic transducer model. The influence of diffraction effects have been outlined and a measurement set-up which minimizes those effects has been suggested.

It turned out, that the least-squares method and the maximum-likelihood method gave very similar results. We already stated that both methods lead to unbiased estimates for the unknown parameters  $a_v$  and  $b_\mu$  in absence of noise and provide very similar results in low noise situations. Since in our measurements a very high signal to noise ratio was attained, similarity of least squares results and maximum likelihood results is in good agreement with expectations. Because of its computational advantages the linear least squares method appears to be preferable for the derivation of the transfer functions of ultrasound transducers in practise.

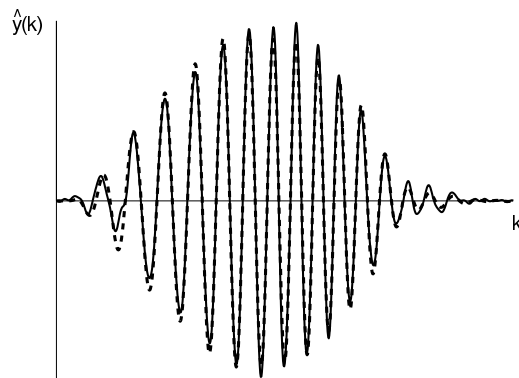


Figure 6: Comparison of the measured (solid line) and modelled (dashed line) system output.

### Acknowledgement

The work is funded by the Federal Ministry of Education and Research, Germany and Inoson GmbH, St. Ingbert, Germany in the aFuE program. We thank both institutions for their support.

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