A 6 DEGREES OF FREEDOM KINEMATICAL MODEL OF THE KNEE FOR THE DESIGN OF A NEW REHABILITATION DEVICE

L. Seddiki, K. Guelton, L. Afilal, and J. Zaytoon

CReSTIC, University of Reims, Reims, France

Lynda.seddiki@univ-reims.fr; kevin.guelton@univ-reims.fr

Abstract: Isokinetics devices are widely used in the field of knee rehabilitation. This kind of apparatus often solicits the knee joint in the sagittal plane with one degree of freedom. With the development of exoskeletons and intelligent prostheses, more precise control applications to assistive robotics for rehabilitation are possible. Our aim is to design a new apparatus for the 3 dimensional control of knee joint movements for sports training and rehabilitation. The first step to design such apparatus is to specify its kinematics. This paper presents a kinematical model of the knee joint that will be employed to specify the future Device kinematics and a preliminary control scheme.

Introduction

Isokinetics devices are widely used in the field of knee rehabilitation. This kind of apparatus often solicits the knee joint in the sagittal plane with one degree of freedom [1]. With the development of exoskeletons and intelligent prostheses, more precise control applications to assistive robotics for rehabilitation are possible. Our aim is to design a new apparatus for the 3 dimensional control of knee joint movements for sports training and rehabilitation. This apparatus have to be used as an evaluator of the knee pathologies as well as a rehabilitation device.

Different kinds of models have been presented in the literature: Phenomenological mathematical knee models and anatomically based mathematical Knee models [2]. In a first step, we need a kinematical based model of the knee to specify the kinematics of the future rehabilitation device. The chosen model, which is presented in the next part, is a hybrid of phenomenological and anatomical consideration of the knee in the sense where it doesn't have to be very precise but respect the whole knee kinematics.

Afterward, to test the efficiency of the proposed model and to have preliminary results on the controllability of the future device, the model is stabilized with a proportional derivative controller.

Geometrical and kinematical model of the knee

Numerous planar kinematical knee joint models have been presented in previous literature. Some 3D models have been also presented but with some assumptions relating to the application for which they were destined to. The interests of many of these models are discussed in [3]. Knee models are based on 1, 2 or 3 degree of freedom and more recently on 6 degree of freedom [4]. In this study, six degrees of freedom have been considered for the knee modelization. This model remains as an open chain of 3 rotoïd joints (flexion-extension, adduction-abduction and intern-external rotations) and 3 translational joints (medio-lateral, the anterior-posterior and tibial axial translations) connecting the femur to the tibia, figure 1.

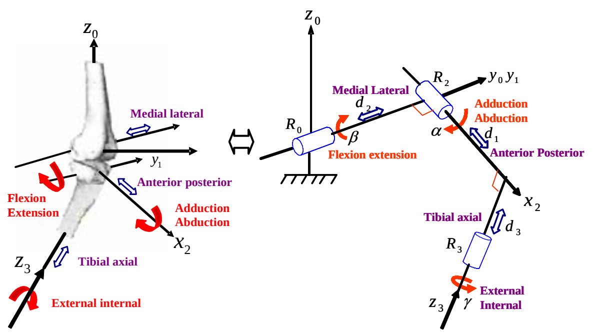

Figure 1: 3D kinematical model of the knee

The structure of the future rehabilitation device is then chosen with a similar kinematics as the above kinematical knee model. Then, in order to compute a simulation of this model, a mathematical description of the kinematics is needed [5][6][7]. That one is obtained from the derivative of the direct geometrical model described hereinafter.

The direct geometrical model describes the coordinates of the effectors (position and orientation from a point fixed to the tibia) in a reference frame R_0 (attached to the femur) related to the articular variables:

$$X = f(q) \qquad (1)$$

With:

- $q = [\alpha\ \beta\ \gamma\ d_1\ d_2\ d_3]^T$: articular variables vector including all the knee degrees of freedom (articular space),
- $X = [x\ y\ z\ \phi\ \theta\ \psi]^T$: tibia's position and orientation vector expressed in the reference frame fixed to the femur (operational space),
- L : length of the tibia,
- α : adduction-abduction rotation,

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- β : flexion-extension rotation,
- γ : external internal rotation,
- d_1 : anterior posterior translation,
- d_2 : medial lateral translation,
- d_3 : tibia axial translation.

The relation (1) is often written by means of a homogeneous transformation matrix ${}^0T_3$ which depends on the angular and the translational variables of the knee and anthropometric values. ${}^0T_3$ is deduced from the combinations of all the translations and rotations with respect to the above defined kinematics (figure 1). It yields:

$$\begin{aligned}{}^0T_3 &= rot(y,\beta).tr(y,d_2).rot(x,\alpha)\\ &\quad .tr(x,d_1).tr(z,d_3).rot(z,\gamma).tr(z,L)\end{aligned} \tag{2}$$

Where a matrix $rot(j,u)$ denotes the rotation of u (rad) around the j axis and $tr(j,v)$ the translation of v (mm) along the j axis.
Equation (2) can also be written as:

$$ {}^0T_3 = \begin{bmatrix} & {}^0A_3 & & {}^0P_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Where ${}^0A_3$ is the orientation matrix of the tibia's frame R_3 explained in the reference frame R_0 (fixed to the femur).

$$ {}^0A_3 = \begin{bmatrix} s_x & n_x & a_x \\ s_y & n_y & a_y \\ s_z & n_z & a_z \end{bmatrix} \text{ with :}$$

$$\begin{aligned}
s_x &= \cos(\beta)\cos(\gamma)+\sin(\beta)\sin(\alpha)\sin(\gamma)\\
s_y &= \cos(\alpha)\sin(\gamma)\\
s_z &= -\sin(\beta)\cos(\gamma)+\cos(\beta)\sin(\alpha)\sin(\gamma)\\
n_x &= -\cos(\beta)\sin(\gamma)+\sin(\beta)\sin(\alpha)\cos(\gamma)\\
n_y &= \cos(\alpha)\cos(\gamma)\\
n_z &= \sin(\beta)\sin(\gamma)+\cos(\beta)\sin(\alpha)\cos(\gamma)\\
a_x &= \sin(\beta)\cos(\alpha)\\
a_y &= -\sin(\alpha)\\
a_z &= \cos(\beta)\cos(\alpha)
\end{aligned} \tag{4}$$

The tibia's position is done by the vector ${}^0P_3$. It depends on the articular variables written in the reference frame R_0:

$$ {}^0P_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (d_3-L)\sin\beta\cos\alpha+d_1\cos\beta \\ (L-d_3)\sin\alpha+d_2 \\ (d_3-L)\cos\alpha\cos\beta-d_1\sin\beta \end{bmatrix} \tag{5}$$

As the position of the tibia is defined by ${}^0P_3$, we need now to describe its orientations. These orientations are obtained using the Euler angle method [8] with the sequence (zyz) and where ϕ, θ and ψ are respectively the rotation around z_0, the rotation around y_n and the rotation around z_n, figure 2.

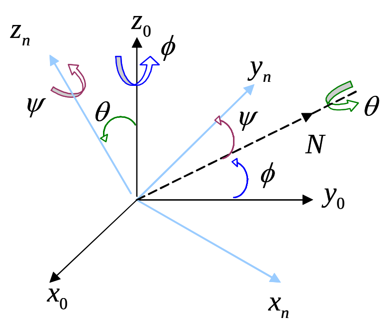

Figure 2. Euler Angles, sequence (zyz) [8]

Then the orientation matrix ${}^0A_3$ can be written as:

$$ {}^0A_3 = rot(z_0,\phi).rot(y_n,\theta).rot(z_n,\psi) \tag{6}$$

Left multiplying (6) by $rot(z_0,-\phi)$ we obtains:

$$ rot(z_0,-\phi).{}^0A_3 = rot(y_n,\theta).rot(z_n,\psi) \tag{7}$$

Expanding (7), it yields:

$$\begin{bmatrix} \cos\phi\, s_x+\sin\phi\, s_y & \cos\phi\, n_x+\sin\phi\, n_y & \cos\phi\, a_x+\sin\phi\, a_y \\ -\sin\phi\, s_x+\cos\phi\, s_y & -\sin\phi\, n_x+\cos\phi\, n_y & -\sin\phi\, a_x+\cos\phi\, a_y \\ s_z & n_z & a_z \end{bmatrix} = \begin{bmatrix} \cos\theta\cos(\psi) & -\cos\theta\sin\psi & \sin\theta \\ \sin(\psi) & \cos\psi & 0 \\ -\sin\theta\cos(\psi) & \sin\theta\sin\psi & \cos\theta \end{bmatrix} \tag{8}$$

Identifying the first and the second member of (8), we find:

$$\begin{aligned}
\phi &= \arctan(a_y / a_x)\\
\theta &= \arctan((\cos\phi.a_x+\sin\phi.a_y)/a_z)\\
\psi &= \arctan\left(\frac{-\sin\phi.s_x+\cos\phi.s_y}{-\sin\phi.n_x+\cos\phi.n_y}\right)
\end{aligned} \tag{9}$$

Then, the direct kinematical model of the knee can be defined from the direct geometrical model as:

$$ \dot{X} = J(q).\dot{q} \tag{10}$$

Where $J(q)$ is the jacobian matrix depending on the articular configuration q. Owing to $X = f(q) \in \mathbb{R}^6$, $J(q)$ is obtained by the following equation:

$$J_{ij} = \frac{\partial f_i(q)}{\partial q_j} \quad i = 1,\ldots 6;\ j = 1,\ldots,6 \tag{11}$$

Where J_{ij} is the i^{th} line and the j^{th} column element of the jacobian matrix $J(q)$.

Inverse kinematical control

In this section, our aim is to compute the inverse kinematical model in a way to evaluate its behavior in the operational space. That is to say the controlled variables are the translational and rotational coordinates of the tibia included in the vector X. The inverse kinematical model is deduced from (10) and is written as:

$$\dot{q} = J^{-1}(q).\dot{X} \tag{12}$$

The inverse kinematical model (12) is known instable in open loop. Owing to that, we apply a proportional derivative corrector (PD) in the control scheme detailed figure 3. Moreover, the use of this kind of corrector within the closed loop system allows the implementation of the inverse kinematics algorithm and avoids the drift due to numerical integration [9][10].

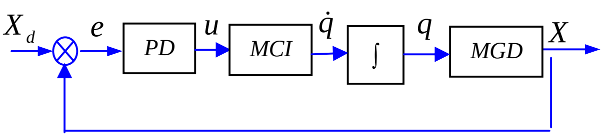

Figure 3: Control scheme design.

The PD control law is written as:

$$u = k_p.e + k_v\dot{e} \tag{13}$$

Where u is the input vector that is homogeneous to the tibia's velocities, X_d is the desired trajectory vector, e is the input output error vector, k_p is the proportional gain matrix and k_v the derivative gain matrix, k_p and $k_v \in \mathbb{R}^{6\times 6}$.

$$\begin{aligned} e &= X_d - X \\ \dot{e} &= \dot{X}_d - \dot{X} \end{aligned} \tag{14}$$

The gain matrix k_p and k_v are tuned with several trials in a way such a good step response is obtained.

Results and discussion

Figure 4 show the step response, using the PD controller, where the simulated movement is an extension from $\theta = -\pi/3$ to $\theta = 0$ at time $t = 0.2s$. The static error tends toward zero and steady state time is quite slow, about 0,4 s, regarding to physiological capabilities of the human knee. Of course the same results have been observed about all the operational coordinates.

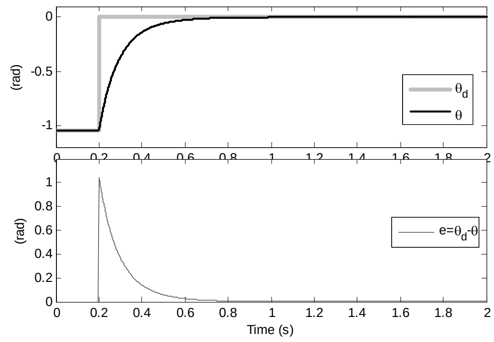

Figure 4: step response on θ.

To test the kinematical behavior of the closed loop system, sinusoidal desired trajectories are employed. Figure 5 shows, on θ, that the error between the desired trajectories and the outputs are weak (error max about 0.05 rad). After that, we assume that the use of the PD controller is an adequate solution to simulate the knee kinematics without disturbance.

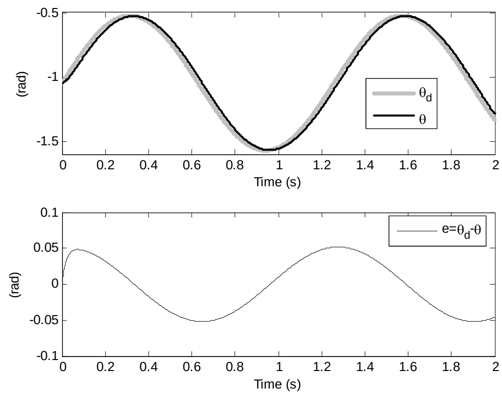

Figure 5: Simulation with sinusoidal inputs.

In order to implement that controller in a real time application, a disturbance d has been introduced to the above control scheme to simulate a measurement error on the flexion angle as describe in figure 6. In that simulation, d corresponds to pulses starting at 0.2s and 1.2s and with a bandwidth of 0.02s.

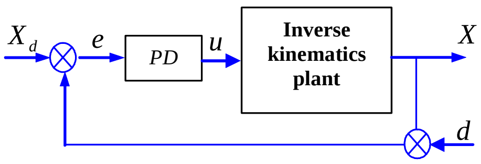

Figure 6: Control scheme with measurement error simulation.

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Figure 7 shows that the closed loop system with the PD controller is not able to reject this perturbation. Consequently, it is necessary to synthesize a robust controller to improve the system performances which is the aim of our future works.

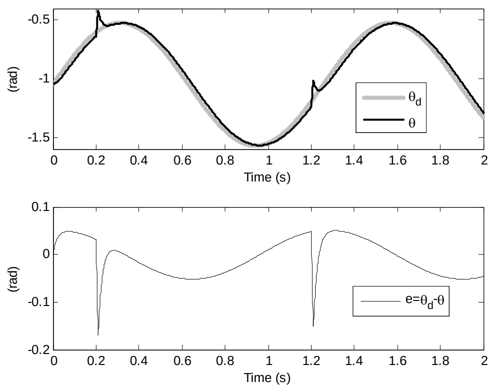

Figure 7: behavior of PD controller in the presence of disturbances.

When a disturbance on the medio-lateral translation is applied, it doesn't affect a lot the other variables because of the small range of this degree of freedom. Then, in order to simplify the future apparatus, this degree of freedom should be neglected and only the adduction-abduction will be considered in the frontal plane.

Conclusions

This paper presents a 3D kinematical model of the knee to be used to design a new rehabilitation device. A preliminary control scheme is then proposed. Simulation results show that the model is able to follow the set points. Then, the future device can use this controller to achieve simple tasks like rehabilitation in flexion extension without disturbance. Nevertheless, the difficulty to control such device is highlighted especially when measurement error in the close loop are observed. Consequently, in future studies, the control scheme will be extended with a non-linear controller to compensate for the non-linear behavior of the kinematical model if complex tasks are required. A robust non linear sliding mode controller [11] associated with a dynamical uncertain model of the future apparatus will show how the whole system (including the apparatus as well as the knee) behave on the presence of reference trajectories obtained from motion capture.

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