DYNAMICAL ANALYSIS AND CONTROL OF A SIMPLE NONLINEAR LIMB MODEL

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Abstract: Dynamic limb models possess strongly nonlinear behaviour that calls for the application of nonlinear system analysis and control methods. The aim of this study is to design, investigate and compare different controllers: a linear LQR for the locally linearized model, a nonlinear controller based on input-output linearization with pole-placement design and a fuzzy controller by using a simple nonlinear limb model. A nonlinear input-affine state-space model has been developed for a simple one-joint system with a flexor and an extensor muscle taking into account the nonlinear muscle and limb dynamics. Model analysis and verification were performed before controller design. The nonlinear controller based on input-output linearization outperformed the linear LQR and the fuzzy ones in both the regulation and the trajectory following control tasks while it required acceptable computing time. Results may be applied in the fields of designing and controlling artificial limbs, muscle prosthesis and in neuro-physiological investigations.

Introduction

Even the simplest limb model exhibits strongly nonlinear dynamic behavior that calls for applying the results of nonlinear systems and control theory. The analysis and control of limb models are important in the fields of designing and controlling artificial limbs, muscle prosthesis and in neuro-physiological investigations. The aim of this work is to investigate the difficulties related to the control of such strongly nonlinear system, as a human limb, and to design and to compare controllers for a simple nonlinear limb model. For this purpose one needs a nonlinear state space model suitable for the mathematical tools of linear and nonlinear control theory and system analysis.

There exist a number of papers in the literature that deal with the desing and investigation of various controllers for limb models. In the papers found in the area of biomechanics and movement control generally static (e.g. [15]) or dynamic (e.g. [6, 12]) optimization is applied in a feed-forward manner for controller design. In these studies the movement is controlled based on the minimization of some key performance variables, such as minimization of net force, net activation, fatigue etc. When the controller design is based on optimization, the nonlinear behavior is usually generally taken into account but the computing cost of the design is very high and these methods are generally not robust to the disturbances.

Thelen et al. [14] proposed the so called computed muscle control method to make faster optimization by applying feedback. Applying this method they computed the state of the muscles and then the value of excitation therefrom.

In the area of posture control the controller is generally designed by using engineering methods (feedback controllers) based on locally linearized system models. Therefore, these controllers are not able to take into account the nonlinear dynamics of the system but their design and operation are much faster than the ones based on dynamic optimization. The feedback nature of these controllers can provide stable response to disturbances and external interactions. Khang and Zajac [9], for example, designed an LQ-like (Linear Quadratic) controller for maintaining the standing posture with FES applying the dynamic equations linearized around the standing posture. Kooij et al. [16] also developed an LQ-like controller to the linearized system equations for maintaining the standing posture by integrating all available sensory information.

The application of nonlinear control theory has the potential to provide a fast and efficient controller that is able to take into account the nonlinear dynamics of the limb at the same time. However, so far, there are only a few such kind of controllers in the literature. A research on acceptable controllers (including input-output linearization) for the cycling problem was reported by Abbot in his degree thesis [1]. A study proposed by Sim et al. [13] has shown, that in the case of the pedaling problem, the control to achieve maximal acceleration for a simple skeletal system is of bang-bang type.

The short description of the model together with the results of the preliminary dynamic analysis and the applied control design methods are found in the next section. For more details we refer to a recent diploma thesis [3]. Thereafter the control results and the comparison of the controller performance are described. Finally the conclusions are given and possible future directions are shortly summarized.

Materials and Methods

Our first aim was to create a musculoskeletal model of a simple 1-degree-of-freedom one-joint system with a

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flexor and an extensor muscle (see in figure 1) suitable for nonlinear systems analysis and control.

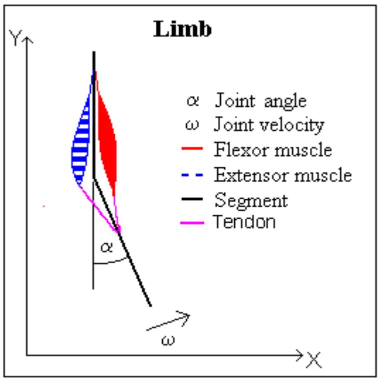

Figure 1: Simple limb model

The dynamic model of a one-joint system with two muscles contained the nonlinear dynamics of the limb [19] and the nonlinear dynamics of the muscle contraction [4, 7, 17, 18]. The nonlinearities of limb dynamics originate from the gravitational effects and the geometry of the model, while nonlinearities of muscle dynamics originate from the nonlinear properties of the muscles, such as force-length-velocity relation, activation dynamics, passive force and tendon nonlinear dynamics. Inputs of the system were the normalized activation signal of each muscle while its output was the joint angle.

The dynamics segments of the limb were supposed to be rigid. The nonlinear equation (1) below describes the limb dynamics, i.e. how torques act on the moving limb part:

$$\frac{d\omega}{dt} = \frac{1}{\Theta + ml_{com}^2}\left(M + ml_{com}\cos\left(\alpha - \frac{\pi}{2}\right)g\right) \qquad (1)$$

where α [rad] is the joint angle, ω [rad/s] is the angle velocity, Θ [kgm^2] is the moment of inertia defined to the mass-center point of the bone, m [kg] is the mass of the moving limb part, l_{com} [m] is the distance between the moving limb part's center of mass point and the joint axis, M [Nm] is the resulting joint torque, and g [m/s^2] is the gravitational acceleration. The correction term $\frac{\pi}{2}$ means that the direction of the first, fix segment was vertical as it can be seen in figure 1.

The crucial component of the model was the part that generates the exerting muscle forces. A muscle model was converted into a state-space form where the following eight state variables were applied:

- Joint angle: α
- Joint angle velocity: ω
- Muscle activation states (2 pieces): q_χ
- Tendon lengths (2 pieces): l_χ^T
- Tendon extracting velocities (2 pieces): v_χ^T

where $_\chi =_{flexor},_{extensor}$ refers to the type of muscle.

Torque is computed by equation (2)

$$M = F_{flexor}d_{flexor} - F_{extensor}d_{extensor} \qquad (2)$$

where F_{flexor} [N] and $F_{extensor}$ [N] are the forces of flexor and extensor muscle respectively, acting on the joint, and d_{flexor}, $d_{extensor}$ [m] are the moment arms of the flexor and extensor muscle, respectively. Force of the flexor muscle is computed by equation (3):

$$F_{flexor} = F_{flexor}^{max} \, FL\left(l_{flexor}^{CE}\right) FV\left(v_{flexor}^{CE}\right) q_{flexor} + F_{flexor}^{PE} + F_{flexor}^{lig} \qquad (3)$$

where F_{flexor}^{max} [N] is the maximal force of flexor muscle, $FL(l_{flexor}^{CE})$ is a normalized nonlinear force-length relationship [17], which can be computed from the muscle's length, $FV(v_{flexor}^{CE})$ is the normalized, nonlinear force-velocity relation, q_{flexor} is the activation state of the flexor muscle (a first order activation-dynamics [18] was used), F_{flexor}^{PE} [N] is the passive force generated by the flexor muscle and F_{flexor}^{lig} [N] describes the passive forces generated by the ligaments and bones in the neighborhood of 0 [rad] and π [rad] is the joint angle. For example, the function of $FL(l_{flexor}^{CE})$ [17] can be seen in figure 2.

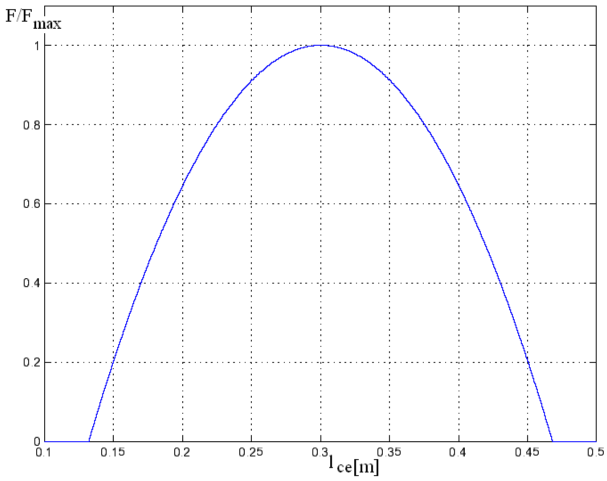

Figure 2: Force-length relationship: $FL\left(l^{CE}\right)$

The nonlinear properties of muscles were described based on [17, 18] but the static nonlinear functions were approximated to fit them better to the control purpose. For example, the figure 3 shows force-velocity relation $FV(v^{CE})$. The original function described by Hill [7] and extended by van Soest and Bobbert [17] was not continuously differentiable, so to avoid computational problems, we used an approximating smooth function to meet the requirements of nonlinear analysis (the parameters of the function were found by parameter fitting): Similar functions were used for the extensor muscle.

The model verification has been performed by simulation where the model response has been tested against engineering intuition. Gravitational effect and muscle force generation were verified by running the simulation with

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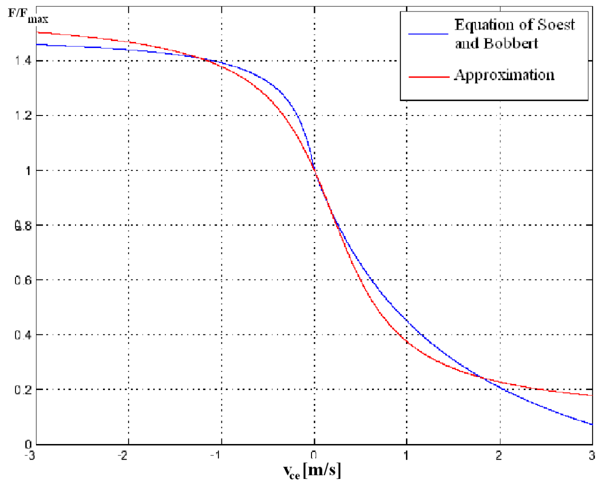

Figure 3: $FV(v^{CE})$ and its approximation

different initial conditions and different muscle activations.

The model analysis aimed at determining the dynamic properties that influence the controller design (stability, controllability, observability, zero dynamics etc. [8]). The steady-state point selected for local analysis purposes was at the $\frac{\pi}{2}$ [rad] joint angle, because of the definition of the control aim (see later) and the simplification of the computations. At this joint angle we can neglect the passive force, and the forces of the ligaments and bones.

The model analysis were performed on the locally linearized model by using standard linear analysis techniques. Methods for investigating structural controllability and observability were applied for the entire set of the locally linearized systems, for determine the global properties of the model [2].

The relative degree of the system was determined using graph-theoretic methods (structure graph) [2]. Moreover, the stability of the zero dynamics was investigated by using local linearization in several points [5].

The quadratic stability region of the steady-state point at $\frac{\pi}{2}$ [rad] joint angle was estimated with Ljapunov methods [11]. To estimate this region, the behavior of $\frac{dV}{dt}$ (the time-derivative of the quadratic Ljapunov function candidate V) was investigated in the neighborhood of the state-point, along the directions parallel to the axes $q_{flexor}, q_{extensor}, \omega, v^T_{flexor}, v^T_{extensor}$ by using cuts of the function $\frac{dV}{dt}$. Note that the function $\frac{dV}{dt}$ does not change along α, l^T_{flexor} and $l^T_{extensor}$ because of the Hamiltonian properties of the system [5]).

The control aim was to control the output of the system in the neighborhood of $\frac{\pi}{2}$ [rad] joint angle. Based on flexion-extension motion, three basic tasks of the control system were specified:

- Stabilization: the closed loop system should be stable in the region we apply the selected control method.
- Regulation and trajectory following: a sinusoidal trajectory should be followed within a tolerance limit.
- Disturbance-rejection: controller should not be seriously sensitive for disturbances.

The tolerance limit was defined as an acceptable difference between the reference signal and the output of the system, that was 0.05 [rad] in this case.

Three type of disturbances were applied: in the case of trajectory following an external load and muscle fatigue disturbances were used, and in the case of regulation spontaneous muscle activity was present. The external load was 1 [kg], and its distance from the joint was 0.3 [m]. The fatigue was modelled as a drift in the τ_{act} parameter [18]: $\tau_{act}(t) = \tau_{act0}(1+0.5t)$. The spontaneous muscle activity was modelled as an additive term in the dynamical equation of the muscle in the form:

$$\frac{dq}{dt} = ... + |1.5(sin(14t)sin(16t))| \quad (4)$$

Three different control methods were investigated:

- *Linear MISO (multiple input, single output) LQ regulator*
  A locally linearized system model obtained around the steady-state point at $\pi/2$ [rad] and centered variables were applied to design a standard LQR [11].
- *Input-output linearization and pole-placement (IOL-PP)*
  The flexor muscle's activation signal was used as the only input to the system (the extensor muscle's activation signal is identically zero) to get a SISO (single input single output) structure [8]. An internal constant nonlinear state feedback was applied for the input-output linearization, and an external feedback of the transformed coordinates was used for pole placement controller design (see figure 4).

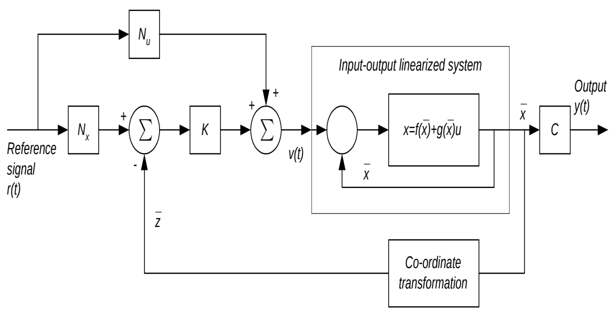

Figure 4: The method of input-output linearization and pole-placement (IOL-PP)

- *Simple Fuzzy-controller*
  The following variables were used as input for the fuzzy inference system: (1) difference between the reference signal and the output, (2) joint angle velocity and (3) activation states of the muscles [10]. These variables were chosen because there were the

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minimum suitable variables necessary to reach predefined accuracy. Gauss-like membership functions were used for both the input and output variables. Min-max implication algorithm and centroid defuzzification was used. The rules and memberships of the fuzzy-controller functions were designed in intuitive and experimental ways. The advantage of fuzzy controller the was that it required only 4 state-space variables for computing the feedback (the other methods required all 8 state-space variables for the feedback), and it did not require the mathematical equations of the model.

Results

The model analysis started with the model verification. Bearing in mind the aim of the model construction (control studies), and the simplifications used in the model building process, the model behaved as one expects, thus the model was verified and was accepted for controller design purposes.

Results of the *local linear analysis* in the steady-state point at $\frac{\pi}{2}$ [rad] joint angle were as follows:

- The linearized system was at the edge of stability (because of the Hamiltonian properties of the system).
- The linearized system was controllable and observable.

The local linear analysis was performed in other steady-state points also, e.g. $\alpha = 0\ rad$, were similar results were obtained.

The figure 5 depicts an example of *stability analysis* using Lyapunov method. It shows $\frac{dV}{dt}$ as a function of the state-space variable pair $q_{extensor}$ and ω, from where a cut of the stability region can be estimated. Similar figures of

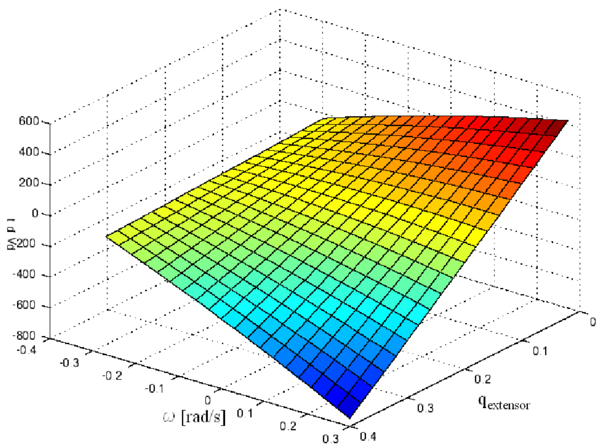

Figure 5: The change of $\frac{dV}{dt}$ along $q_{extensor}$ and ω

cuts can be made in the case of other state-space variable pairs. The points where the value of the function $\frac{dV}{dt}$ is negative, belong to the asymptotic stability region of the steady-state point.

Furthermore, the system was found to be both locally and structurally controllable and observable.

The relative degree of the system was 3. This implies that we had to use first, second and third order Lie-derivatives to determine the the static nonlinear feedback of the input-output linearization . The zero-dynamics of the system was found to be stable.

The regularization properties of the controllers were investigated both in the ideal case (i.e. without disturbance) and with disturbances.

The results of the regulation task in the case of *ideal circumstances* can be seen in figure 6. The LQ controller

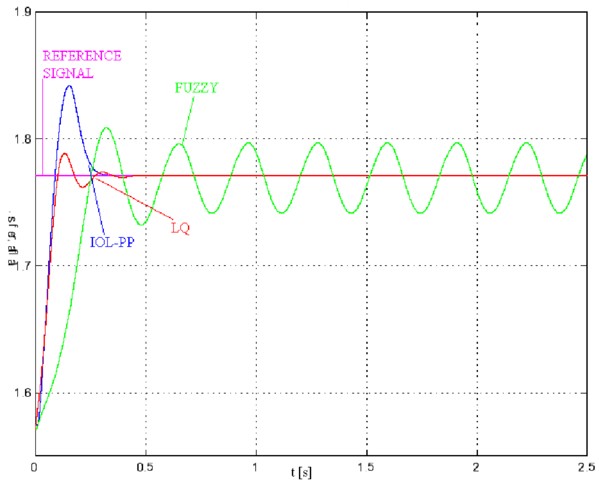

Figure 6: Regulation without disturbance

provides the best performance (the less overshot and short settling time) in this case, and the fuzzy controller is far the worst producing oscillation around the reference value. The IOL-PP controller has a greater overshot than the LQR but its settling time is similar to the LQR's settling time. Both the LQ and IOL-PP controller produced an output that was within the tolerance limit.

The results of the regulation task *with disturbances* described above can be seen in figure 7. Both the LQ and IOL-PP controllers are found to be sensitive to the disturbances. The overshot of the IOL-PP controller is greater than that of the LQR, but the their settling times are very similar. These values are not very sensitive to the spontaneous muscle activity as disturbance. In the case of the same disturbances and starting conditions, the fuzzy controller fails to prove stable control. The waves appear in the output signal at those times, where the disturbance amplitude reaches it's maximum (disturbance is not shown in the figure 7).

The trajectory following properties were also investigated in both the ideal case and with disturbances.

The results of the trajectory following task in the case of *ideal circumstances* (i.e. without disturbance) can be

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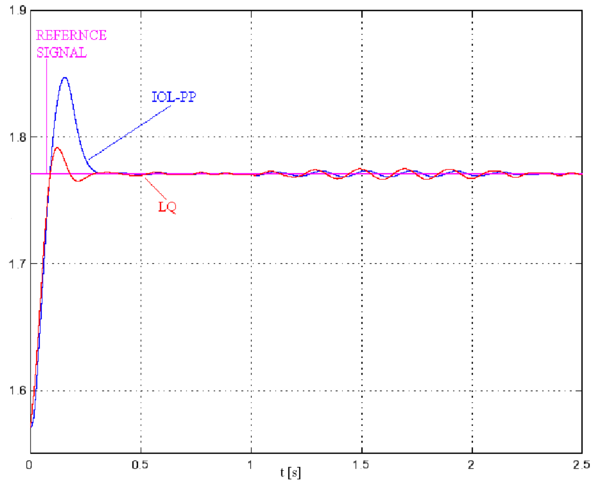

Figure 7: Regulation with spontaneous muscle activity

seen in figure 8, where the applied reference signal was sinusoidal. Both the LQ and the IOL-PP controllers can follow the trajectory and their outputs are smooth but they has got a delay. The delay of the LQ controller is greater than the delay of the IOL-PP. In the case of fuzzy controllers waves can be seen around the reference trajectory that is the consequence of the oversimplified nature of the fuzzy controller. All controllers produced an output within the tolerance limit.

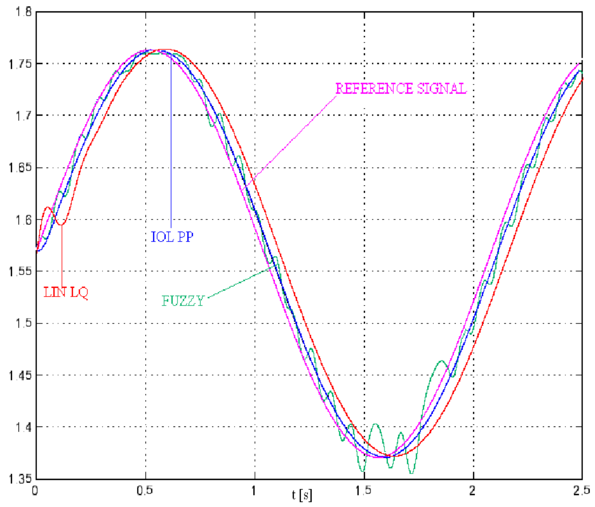

Figure 8: Trajectory following without disturbance

The results of the trajectory following task *with disturbances* described above can be seen in figure 9. The output of the LQ controller is not smooth anymore, waves around the required trajectory can be found. Disturbances cause constant shift and higher delay in the case of the IOL-PP controller. In the case of the same disturbances and starting conditions, the fuzzy controller fails to prove stable control.

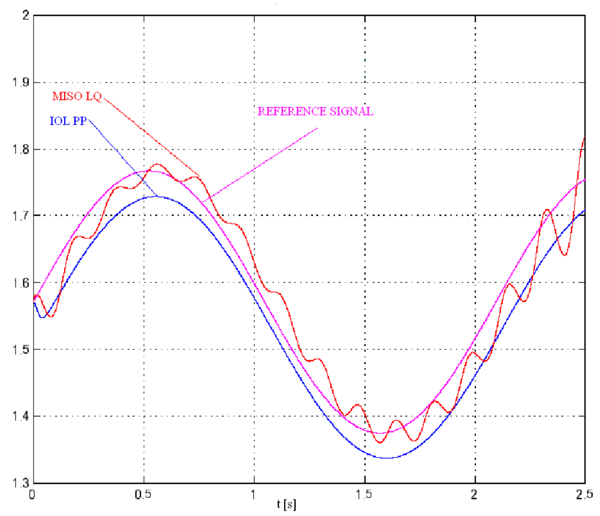

Figure 9: Trajectory following with external load and fatigue disturbances

Discussion

In the case of ideal circumstances (i.e. without disturbances) all of the LQ regulator, the IOL-PP conroller and the fuzzy controller provided acceptable output, but the fuzzy controller produced a highly oscillatory output signal. The output was within the tolerance limit for both the LQR and the IOL-PP controller. The LQ controller provided the best regulation type control with the smallest overshot and settling time, and the IOL-PP controller was just a little bit worse because it had got a greater overshot. In the case of regulation with disturbances the LQ and IOL-PP controllers generated similar output but IOL-PP produced greater overshot.

The controllers succeed to achieve trajectory following in the ideal case. The LQ and the IOP-PP controllers provided more smooth output than the fuzzy controller. Probably this phenomenon could have been eliminated with more tuning of the rules and membership functions in fuzzy controller. In the case of the trajectory following task with disturbances the LQ controller provided less smooth output than the IOL-PP, and both became less accurate. Both the LQ and the IOL-PP controllers provided stable control in the case of disturbances.

The fuzzy controller was very sensitive for disturbances, and failed to achieve stability in the case of appearing disturbances. We have to note, that, as far, the rules and membership functions of the fuzzy controller were designed and tuned only in the case of ideal circumstances and mainly for the trajectory following task.

The linear LQ controller required the less computer capacity (30 *sec* in Pentium II, 1.2 GHz, 128 MB RAM), while the nonlinear input-output linearization required more time (2 *min*) and fuzzy controller required much more time (25 *min* with 13 rules).

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Conclusions

The methods of nonlinear systems and control theory were applied to a simple limb model with one joint and two muscles. The main problem to be solved for designing a nonlinear controller based on input-output linearization is the complexity of the functions needed for the linearizing feedback. Therefore, we approximated the nonlinear static functions in the model with smooth functions of a relatively simple form.

Standard linear model analysis performed on the locally linearized version of the model as well as nonlinear model analysis were used to prepare the controller designs. Three types of controllers were compared: a linear LQR designed for the locally linearized model, a nonlinear controller based on input-output linearization and pole-placement and a fuzzy controller.

Two types of control tasks, trajectory following and regulation were investigated both with and without disturbances. It has been found by simulation that both these tasks can be better solved with nonlinear control based on input-output linearization as compared to both the LQR and the fuzzy controler. The required computing capacity was the highest in the case of the fuzzy controller and the less in the case of the LQ controller. If the expectations are not too high or the computing capacity should be low, an LQR can also be successfully applied. If the model equations or some important parameters are not available, a fuzzy control can also be applied but its proper design and tuning requires substantial efforts and skill.

Directions for further research includes the extension of nonlinear control studies to the case when the gamma-loop effect is also taken into account. For this purpose, our model has already been extended with the simplified model of the gamma-loop [3]. In addition, it would be possible to design a nonlinear loop-shaping PD controller [5] by utilizing the Hamiltonian properties of the model, that is also a subject of our future work.

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