PARAMETRIC ANALYSIS OF MODELS OF INTRACRANIAL SACCULAR ANEURYSMS

Krzysztof Szafrański*

* Department of Electricity, Szczecin University of Technology, Szczecin, Poland

kris790@wp.pl

Abstract: The aim of this paper is to present analysis of the mechanics of intracranial aneurysms and description of factors leading to their rupture. The general etiology of hemorrhages in the human body is presented. The mathematical models for intracranial saccular aneurysms based on Legendre functions have been prepared and the curvatures of aneurysms are compared to data received thanks to the magnetic resonance angiography (MRA). Moreover, minimisation of the difference between the actual radius calculated from the MRA data and the radius received from the Legendre-based function is presented. Basing on the achieved results the new approach to the possible reasons of the aneurysm's rupture is described. Therefore shape and curvatures are more significant parameters in the evaluation of aneurysm's rupture than dimension. Other results, including the comparison between Newtonian and non-Newtonian fluid used to simulate blood flow in the aneurysm and the relationship between dimensions of the aneurysm and wall shear stress are presented.

Introduction

Intracranial saccular aneurysm (ISA) is a localised distended sac affecting only a part of the circumference of the artery wall. About 2-5 % of the population can have ISA [1], but in case of a rupture 50-60 % of people with an aneurysm may die [2]. Treatments used for saccular aneurysms have a mortality rate reaching 18 %, hence it is critical to determine the risk of the aneurysm's rupture very quickly [2].

The main point of the investigations and tests is to determine the critical parameters for the risk of rupture. Mathematical models of aneurysms are typically based on Laplace's law which defines a linear relation between the circumferential tension and the radius. However, since the aneurysm wall is viscoelastic, a nonlinear model was developed to characterise the development and rupture of intracranial spherical aneurysms within an arterial bifurcation and the model describes the aneurysm in terms of biophysical and geometric variables at static equilibrium. A comparison is made between mathematical models of a spherical aneurysm based on linear and nonlinear forms of Laplace's law. The first form is the standard Laplace's law which states that a linear relation exists between the circumferential tension, T, and the radius, R, of the aneurysm given by $T = PR/2t$ where P is the systolic pressure. The second is a 'modified' Laplace's law which describes a nonlinear power relation between the tension and the radius defined by $T = ARP/2At$ where A is the elastic modulus for collagen and t is the wall thickness. Differential expressions of these two relations were used to describe the critical radius or the radius prior to aneurysm rupture. Using the standard Laplace's law, the critical radius was derived to be $Rc = 2Et/P$ where E is the elastic modulus of the aneurysm. The critical radius from the modified Laplace's law was $R = [2Et/P]2At/P$. Substituting typical values of E = 1.0 MPa, t = 40 microns, P = 150 mmHg, and A = 2.8 MPa, the critical radius is 4.0 mm for the standard Laplace's law and 4.8 mm for the modified Laplace's law [3].

Hence, the size is the primary parameter used for the prediction of rupture of an aneurysm. Reasearches prove that treatment is necessary for aneurysms larger than 10 mm, however, some small aneurysms less than 4 mm are not stable and can rupture more often than the bigger ones. Geometry of saccular aneurysms is not well explored, and as the main predictor the height-to width of an aneurysm (also called aspect ratio) is used [4]. According to tests performed by some scientists, the surface of the aneurysm has properties alike membrane, and it makes the curvature of a lesion a very important parameter leading to rupture. Following this assumption, it is necessary to calculate the curvatures in a precise and quick way. Calculations of curvatures have not been performed so far apart from some work done by Sacks et al. (1999) on abdominal aortic aneurysms, which included method for calculation of curvatures based on MRA [5]. The new method of calculation of curvatures by the means of Legendre functions is proposed in the following section, and it makes it possible to provide an almost real aneurysm geometry.

Another aspect of modelling of saccular aneurysms is determination of blood and lesion interactions. Therefore it is essential to compare the Newtonian fluid model, which mimics shear-thinning blood behaviour and the non-Newtonian fluid model. The choice of the model is necessary for determination of relationship between the aspect ratio and the wall shear stress.

Finally, the impact of stent on the inflow in the aneurysm should be taken into account. This impact is described by the blockade ratio (C_α) and it facilitates determination of rupture of an aneurysm for any stent location [6].

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Materials and Methods

The computation of boundaries for aneurysms, which data are received in the form of MRA films, by the usage of splines and some modelling packages is pretty time-consuming and it is rather impractical for application during patient's test. The aneurysms are not ideal spheres, therefore I decided to use spheroidal functions (Legendre functions), which guarantees calculation accuracy and the possibility of alteration of shape.

Legendre functions can be described by:

$$P_n^m(x) = (-1)^m (1-x^2)^{m/2} \frac{d}{dx^m} P_n(x) \qquad (1)$$

where m and n are integer values, $P_n(x)$ is a Legendre polynomial and x is any argument. The boundary of the lesion can be described by:

$$r = \beta(1 + c \cdot P_n^m(\cos\phi) \cdot \cos(m\theta)) \qquad (2)$$

where r is the radius, ϕ is from $[0,\pi]$ and θ is from $[0,2\pi]$. ϕ and θ are angles in spherical coordinates, P_n^m is the Legendre function of the first kind, and β, c, n and m are shape parameters. The shape can be defined for any value of ϕ and θ, if the center point and the equation for radius are determined.

The variation of β changes the overall size of the figure and the variation of c changes the shape of the figure. Therefore the higher value of c, the bigger distortion of an aneurysm from a spherical geometry.

The exemplary shapes of aneurysms were presented below:

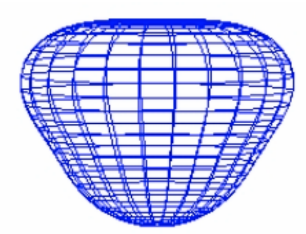

Figure 1: Shape of an aneurysm based on Legendre functions with n=3 and m=0.

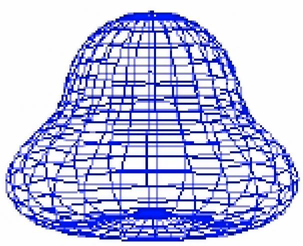

Figure 2: Shape of an aneurysm based on Legendre functions with n=3 and m=0 and c=0.4.

Firstly, the Legendre polynomials are transformed from spherical (r, ϕ, θ) into Cartesian (x, y, z) coordinates,

$$R = xi + yj + zk \qquad (3)$$

and

$$\begin{aligned} x &= r\cos\theta\sin\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\phi \end{aligned} \qquad (4)$$

and then every component is multiplied by a scalar (αx, γy, δz),

$$R = \alpha xi + \gamma yj + \delta zk \qquad (5)$$

This operation stretches the spheroid in three directions.

The difference between the actual radius and radius of an aneurysm calculated from the Legendre functions shall be minimised. The Marquardt-Levenberg regression code was used in order to determine parameters that minimise the sum of squares error (SSE), defined as:

$$SSE = \sum (r - r_{th})^2 \qquad (6)$$

These parameters *m, n, c, α, γ, δ, β, x0, y0, z0, rot 1, rot2, rot3*. *α, γ, δ* are the stretches in the *x, y* and *z* directions, *x0, y0* and *z0* define the center of the spheroid, and rot1, rot2 and rot3 define the Euler angles for the rigid rotation of an aneurysm shape.

The root mean squared error (RMS) is used as the error criterion and is given in milimeters (mm),

$$RMS = \sqrt{MSE} \qquad (7)$$

where

$$MSE = \frac{SSE}{N} \qquad (8)$$

The main point of the tests is the calculation of curvatures. The calculations of curvatures for Legendre spheroids were performed in MATLAB, while curvature maps were presented in Maple. According to the previously defined conditions the following can be determined:

$$R = r\sin\phi\cos\theta \cdot i + r\sin\phi\sin\theta \cdot j + r\cos\phi \cdot k \qquad (9)$$

The infitesimal movement on the surface is defined by:

$$dR = R,_{\theta}\, d\theta + R,_{\phi}\, d\phi \qquad (10)$$

where $R,_{\theta}$ and $R,_{\phi}$ are the base vectors:

$$R,_{\theta} = \frac{\partial R}{\partial \theta} \qquad (11)$$

$$R,_{\phi} = \frac{\partial R}{\partial \phi} \qquad (12)$$

The curvatures are calculated as the maximal and minimal values of K_n, which is called the normal curvature and is strictly related to the curvature vector k:

$$k_n = -K_n n \qquad (13)$$

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where k_n is the normal component of the curvature vector, defined by:

$$k = \frac{dt}{dS} = k_n + k_t \tag{14}$$

The impact of k_t is not taken into account in this paper. n and t are perpendicular, and after taking the derivative with respect to S, the following is obtained:

$$\frac{dn}{dS} \cdot t + \frac{dt}{dS} \cdot n = 0 \tag{15}$$

K_n can be defined as:

$$K_n = \frac{dR \cdot dn}{dR \cdot dR} \tag{16}$$

where:

$$\begin{aligned} dn &= n,_\theta\, d\theta + n,_\phi\, d\phi \\ dR &= R,_\theta\, d\theta + R,_\phi\, d\phi \end{aligned} \tag{17}$$

Finally K_n can be expressed as [7]:

$$K_n = \frac{L(d\theta)^2 + 2Md\theta d\phi + N(d\phi)^2}{E(d\theta)^2 + 2Fd\theta d\phi + G(d\phi)^2} \tag{18}$$

where E, F and G are defined as:

$$\begin{aligned} E &= R,_\theta \cdot R,_\phi \\ F &= R,_\theta \cdot R,_\phi \\ G &= R,_\phi \cdot R,_\phi \end{aligned} \tag{19}$$

and L, M and N are second fundamental magnitudes:

$$\begin{aligned} L &= R,_\theta \cdot n,_\theta = -R,_{\theta\theta} \cdot n \\ M &= \frac{1}{2}(R,_\theta \cdot n,_\phi + R,_\phi \cdot n,_\theta) = -R,_{\theta\phi} \cdot n \\ N &= R,_\phi \cdot n,_\phi = -R,_{\phi\phi} \cdot n \end{aligned} \tag{20}$$

The formula for K_n can be modified by defining direction λ as $\lambda = d\phi/d\theta$:

$$K_n(\lambda) = \frac{L + 2M\lambda + N\lambda^2}{E + 2F\lambda + G\lambda^2} \tag{21}$$

The maximal (η_1) and minimal (η_2) values of the normal curvature were received upon calculation of $dK_n/d\lambda = 0$ as follows:

$$\begin{aligned} \eta_1 &= (K_n)_1 = \frac{L + 2M\lambda_1 + N\lambda_1^{\,2}}{E + 2F\lambda_1 + G\lambda_1^{\,2}} \\ \eta_2 &= (K_n)_2 = \frac{L + 2M\lambda_2 + N\lambda_2^{\,2}}{E + 2F\lambda_2 + G\lambda_2^{\,2}} \end{aligned} \tag{22}$$

where λ_1 and λ_2 are the directions of the maximal and minimal curvatures.

(23)

$$\{\lambda_1, \lambda_2\} = \frac{-(LG - NE)}{2(MG - NF)} \pm \frac{\sqrt{(LG - NE)^2 - 4(MG - NF)(LF - ME)}}{2(MG - NF)}$$

Results

Modelling of aneurysm by the usage of Legendre functions makes it possible to improve on RMS values. The average RMS values for five aneurysms, which data were taken for calculation from MRA films, were 0.45 mm for the standard Legendre function, and 0.77 mm for the sphere. Therefore the general improvement is about 42 %. The further improvement of 2 to 12 % was achieved thanks to the modification of the Legendre function by scalar multiplies. The results of optimisation are presented in the following table by the comparison of RMS values for unmodified Legendre spheroid (values were obtained by the Marquardt-Levenberg search algorithm) and modified (by scalar multiplies) Legendre spheroid.

Table 1: Comparison of RMS values for two kinds of Legendre spheroids.

Parameters and criteria	Aneurysm	
	unmodified	modified
c	0.29	0.0099
beta	4.01	2.1185
m	1	2
n	2	4
alpha	1	1.71
gamma	1	2.63
delta	1	1.17
x_0	0.08	0.03
y_0	0.09	-0.1044
z_0	0.06	0.0476
rot1	-1.3	0.7055
rot2	-1.47	0.025
rot3	-1.15	-1.75
RMS (mm)	0.4015	0.3667
improvement over unmodified		8.67 %

The modified Legendre spheroids are shown in the figures below, and the minimal and maximal values of curvatures are bordered by different colours by the means of Maple.

Another important field of my researches is estimation of blood flow in the aneurysm by using aspect ratio

$$AR = \frac{H}{D_n} \tag{24}$$

where H is the height of the aneurysm and D is the width of the aneurysm.

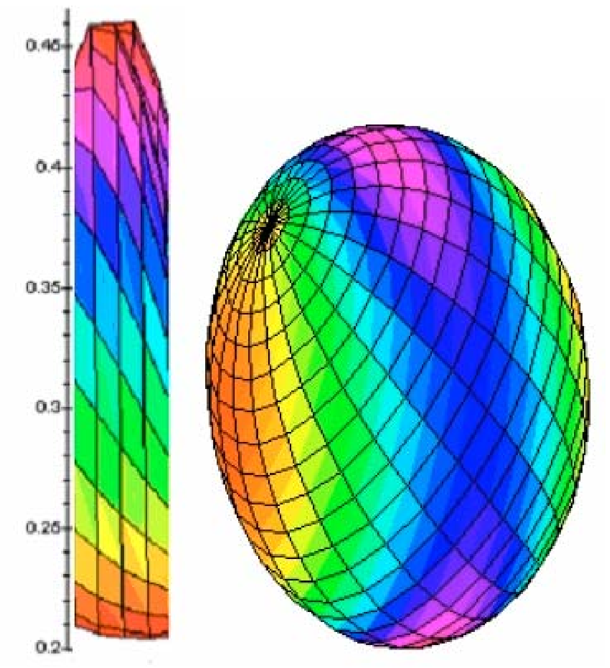

Figure 3: Maximal curvature values η_1, ranging from 0.2 to 0.46 for the exemplary aneurysm.

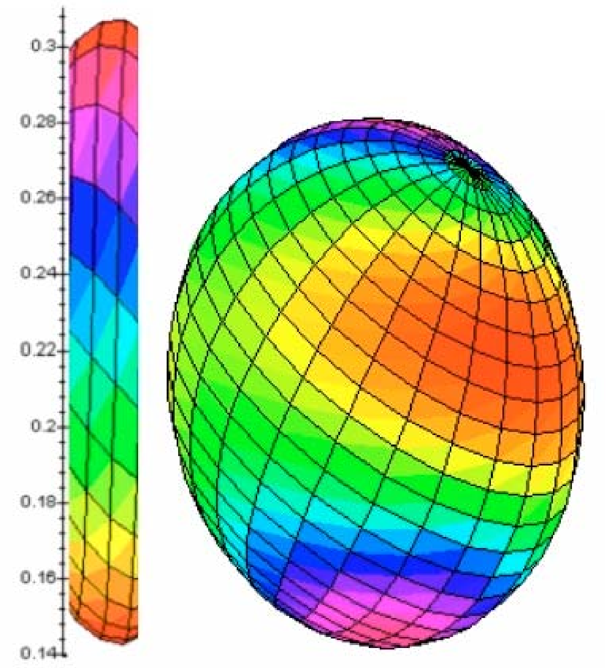

Figure 4: Minimal curvature values η_2, ranging from 0.14 to 0.32 for the exemplary aneurysm.

Basing on the Ujie [8] conclusion it was important to determine the relation between the aspect ratio (AR) and the wall shear stress. Ujie confirmed that an aneurysm with AR > 1.6 has a recirculation region inside the aneurysm, while the aneurysm with AR < 1.6 has no recirculation region and blood can pass through the aneurysm. The tests prove that aneurysm of large AR, but smaller than 1,6 have smaller wall shear stress than aneurysms of AR close to 0.5. The wall shear stress of the aneurysm normalised against that of the rest of the region decreases, if the aspect ratio increases. The relationship between AR and wall shear stress is presented below:

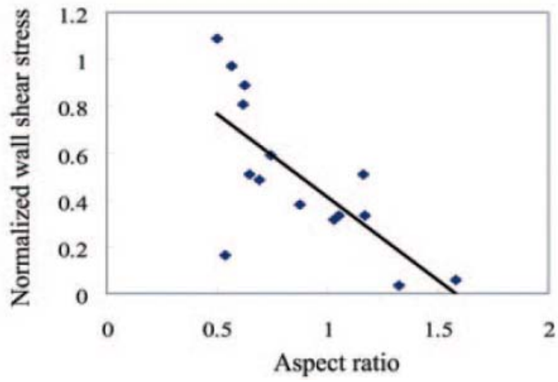

Figure 5: Relationship between aspect ratio (AR) and normalised wall shear stress.

The above described research did not require the walls of the aneurysm to be elastic, as the aim of the research was to estimate the morphological impact on the hemodynamics and therefore rigid walls and Newtonian flow of blood were assumed.

Acceptance of initial assumptions is essential, because some tests prove that the Newtonian model and the non-Newtonian model do not differ too much. Numerical tests in a two-dimensional symmetric bifurcation with a non-symmetric aneurysm show that difference in velocity field between non-Newtonian and Newtonian model is small. Experimental comparison of 56 % aqueous solution of glycerol solution and 0.07 % Separan solution with shear-thinning properties similar to blood gave comparable results [9], [10].

Another important problem related to modelling of saccular aneurysms is effect of treatment on the changes of aneurysmal flow. Nowadays, the rupture of saccular aneurysms is prevented by intravascular stenting. Therefore the right placement of stents is crucial for the eventual formation of thrombus inside the aneurysm and finally for eliminating of aneurysm from cardiac circulation [11]. The impact of the stent on the flow inside the aneurysm can be effectively described by the blockade coefficient:

$$c_\alpha = \frac{Nd}{L} \tag{25}$$

where N is the number of stent loops, d is the diameter of the wire and L is the length of the stent. The most important aspect of design of stent is analysis of the blockade coefficient in relation to wall shear stress and total strength inside the aneurysm [12]. My tests of the

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blockade coefficient proved that the risk of aneurysmal rupture is the smallest, if stent is placed like helix and and C_α amounts to 75 %. Moreover, the effect of the blockade coefficient on the flow inside aneurysm is worth checking. The average value of the wall shear stress after putting the stent of the blockade coefficient of 75 % decreases to 51 % of the value for aneurysm without inserted stent.

Discussion

Modelling of saccular aneurysms is an important basis for the future treatment and rupture prevention of aneurysms. Therefore the proper shape shall be assumed, as the sphere is the too simplified curvature.

According to international researches on intracranial saccular aneurysms the size cannot be used for prediction of risk of rupture in aneurysms of diameter less than 10 mm [13]. Thanks to the usage of Legendre spheroids it is possible to evaluate curvatures at any location of an aneurysm. As it was stated in the section of results, the general improvement of RMS values for the standard Legendre function is about 42 % in relation to the sphere. The average improvement rate of RMS value in case of modification of Legendre spheroid was 2 to 12 %, however the maximal improvement of η was 37 %. The model of an aneurysm based on Legendre function can be still modified, as RMS values for the modified Legendre spheroid are as big as 0.85 mm comparing to 0.2 mm resolution of MR and 0.4 mm of SOMATOM Sensation 64 CT scanner.

The main fault of the Legendre spheroid is the smoothing effect of the surface of a lesion, hence the curvature values of the Legendre spheroid do not differ too much. Although splines can be used for evaluation of curvatures, they increase computation time to 9-10 hours, which makes it an almost impractical method. The computation time for the Legendre spheroid is about 3 minutes, while for the modified Legendre spheroid amounts to about 40 minutes. More complex functions than Legendre functions will be tested in the future in order to eliminate smoothing effect and decrease RMS.

Conclusions

Since vascular changes are sensitive to minor changes in the environment, these changes can be controlled by modelling of saccular aneurysms. Aneurysms resemble membranes and their stresses can be managed by local curvatures. The shapes are not ideal spheroids, therefore the application of Legendre function is the correct solution. Thanks to the usage of Legendre spheroids the RMS values can be reduced of up to 40 % and even more. The Marquardt-Levenberg search algorithm was also used. The computation time is reasonable for the Legendre spheroids, which predispose them to modelling of lesions. Nevertheless the size of aneurysm shall be taken into account as well, as there is relationship between the aspect ratio (AR) and normalised wall shear stress. Such a relationship makes it possible to classify aneurysms which are more prone to ruptures.

The choice of the initial conditions is essential for simplification of further calculations, especially if blood flow is analysed.

Finally, the impact of stents used to harbour aneurysms on blood flow and wall shear stress inside the aneurysm is worth testing. The critical value for which the probability of aneurysmal rupture is slight was found and there is a need to look for critical relationships between stent, vortical flow and wall shear stress for different shapes of stent.

References

[1] The International Study of Unruptured Intracranial Aneurysms Inverstigation (1998): 'Unruptured intracranial aneurysms - Risk of rupture and risks of surgical intervention'. *New England Journal of Medicine*; 339: 1725-1733.

[2] VAN GIJN, J. and RINKEL, G. (2001): 'Subarachnoid hemorrhage: diagnosis, causes and management'. *Brain* ; 124:249-278.

[3] HADEMENOS G. J., MASSOUD T., VALENTINO D.J., DUCKWILER G., VINUELA F. (October 1994): 'A nonlinear mathematical model for the development and rupture of intracranial saccular aneurysms'. *Neurol. Res.*; 16(5):376-84.

[4] PARLEA L., FAHRIG R., HOLDSWORTH D.W., and LOWNIE S.P. (1999): 'An analysis of the geometry of saccular intracranial aneurysm'. *Am J. Neuroradiol*, 1079-1089.

[5] SACKS M.S., VORP D.A., RAGHAVAN M.L., FEDERLE M.P., WEBSTER M.W. (1999): 'In vivo three-dimensional surface geometry of abdominal aortic aneurysm'. *Annals of Biomedical Engineering 27*, 469-479.

[6] BURLESON A.C., STROTHER C.M., and TURITTO V.T. (1995), 'Computer Modelling of Intracranial Saccular and Lateral Aneurysms for the Study of their Hemodynamics". *Journal of Neurosurgery,* **37**, 774-784.

[7] KRAUS H. (1967): 'Thin elastic shells an introduction to the theoretical foundations and the analysis of their static and dynamic behavior'. John Wiley & Sons, New York.

[8] UJIE H., TACHIBANA H., HIRAMATSU O., et al. (1999): 'Effects of size and shape (aspect ratio) on the hemodynamics of saccular aneurysms: a possible index for surgical treatment of intracranial aneurysms'. *Neurosurgery 45, 119-129, 1999.*

[9] PERKTOLD K., PETER R. and RESCH M. (1989): 'Pulsatile non-Newtonian blood flow simulation through a bifurcation with an aneurysm'. *Biorheology*, **26**, 1011-1030.

[10] THIRIET M. et al. (1998): 'Computational models of flow in cerebral aneurysms'. *J. Biomechanics*, **31**, spt. 1, 20.

[11] DOTTER C.T. and JUDKINS M.P. (1964): 'Transluminal Treatment of Arteriosclerotic Obstruction: Description of a New Technique and a Preliminary Report of its Application'. *Circulation*, **30**, 654-670.

[12] AENIS M., STANCAMPIANO A.P., WAKHLOO A.K. and LIEBER B.B. (1997): 'Modelling of Flow in a Straight Stented and Non-Stented Side Wall Aneurysm Model'. *ASME Journal of Biomechanical Engineering*, **119**, 206-212.

[13] WIEBERS D.O. (1998): 'Unruptured intracranial aneurysms – risk of rupture and risks of surgical intervention'. *The New England Journal of Medicine 339*, 1725-1733.

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