

## ON MODELING BOW ARM MOVEMENT IN CELLO PLAYING BY WHIP MOTION

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**Abstract:** In this paper, we introduce a whip dynamics to build a model of the bow arm movement in playing string instruments. Recently the physics of the whip motion was studied and its equation of motion was obtained by McMillen and Goriely. We give detailed consideration of the bow arm movement based on the equation. In particular we take up the problem of the speed control at bow direction change (“bow change” in short). Bow change is one of the hardest tasks in the bow movement, and we need to control the bow change speed depending on the music phrases. According to the whip equation of motion, the propagation speed of the maximum speed point is proportional to the square root of the whip’s stiffness. In case of the bow change, we can adjust the bow change speed by adjusting the impedance of the body trunk which corresponds to the main part of the whip. We investigate the validity of the whip model by measuring the deviation of the center of the gravity and muscles activation patterns during a simple bowing task.

### Introduction

Several studies have been made on the bow arm movement in playing string instruments [1], [2], [3], [4]. Since the bow arm movement is a very hard task in playing string instruments, it is expected that such studies would significantly help not only beginners but also professional players to improve their skills, to avoid fatigue from painful exercise and/or to get rid of performance slump.

Most of the previous studies attempt to identify the differences between novices and professionals by comparing their biophysical data during performance. Although these approaches provide possibly useful information to elucidate tacit knowledge of bowing skill, they are not always successful in extracting general laws of bow arm movement. This is because there are large variations amongst players on how to perform given music pieces. We refer to this difficulty as the problem of individual differences. There are two possible approaches to get around the problem: 1) to increase the number of the subjects to guarantee the validity of the induced laws by statistical test, or 2) to build a dynamical model of bow arm movement, and verify the experimental results by referring the model. In this paper, we adopt the second approach.

In our previous studies, we proposed several dynamical models of bow arm movements, e.g., a pendulum model, an inertia model and a stretch movement model. We then provided a physical interpretation of some aspects of bow arm movements [5], [6]. In the case of a pendulum model, for example, we regard the repetitive bow movement as a pendulum motion, and the model provides a numerical target value of the pendulum length to achieve the required bow movement cycle. More specifically, the model says that when the cycle is two seconds, then the pendulum length is around 1m. On the other hand, when we reduce the cycle by half, we then need to shorten the pendulum length to 1/4, namely 0.25m. This relationship continues further; if the cycle becomes 0.5 second, then the required pendulum length is around 0.06m. The last pendulum length is achieved by making the wrist as the supporting point of the pendulum, which in turn is realized by increasing the impedances of both the upper arm and the forearm. We actually conducted an experiment, the task of which included repeated bowing with gradually shortening the cycle of the bow change. We observed the predicted impedance adjustment according to the cycle of the bow change.

In the case of the stretch model, we wanted to verify the hypothesis that the human beings generate biggest power when they pull toward the center of the body. To do so we measured the relationship of the power and the angle between the bow direction and the frontal plane. We observed around 30% power reduction with the bow pulled parallel to the frontal plane, compared to the direction toward the center of the body.

In this paper, we introduce a whip model as our fourth physical model. Several researchers have reported the use of a whip model, in order to explain such movements as javelin throwing and badminton backhand stroke, in addition to our modeling the bowing arm movement in playing string instruments. The importance of the whip motion was also indicated for kendo, the Japanese art of fencing. However they only referred whip-like motion as a phase shift phenomenon of maximum speed occurrence from proximal extremity (e.g., the hip) to the distal end (e.g., hand or finger) without detailed whip dynamics analysis.

Recently the physics of the whip motion was studied and its equation of motion was obtained by McMillen and Goriely [7]. In this paper, we introduce the whip’s equation of motion and give detailed consideration of the bow arm movement based on the equation. In particular we take up the problem of speed control at bow direction

change (“bow change” in short). Bow change is one of the hardest tasks in bow movement, and we need to control the bow change speed according to the music phrases we play: rapid notes require fast bow change, and slow notes require slow bow change. The problem here is how to control such bow change speed. The whip motion model provides the answer. According to the whip equation of motion, the propagation speed of the maximum speed point is proportional to the square root of the whip’s stiffness. In case of the bow change, we can adjust the bow change speed by adjusting the impedance of the body trunk which corresponds to the main part of the whip.

### Materials and Methods

In this section, we introduce the whip’s equation of motion derived by McMillen and Goriely [7]. Whip motion can be simply viewed as the movement of a maximum speed point of the whip, under consideration of the properties of the physical whip object. Here we consider the whip motion in a two dimensional x–y plane as shown in Figure 1.

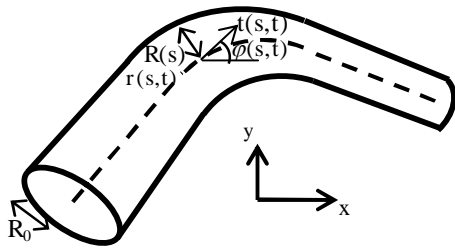


Figure 1: Whip Coordinate System

Let  $\mathbf{r}(s, t) \equiv (x(s, t), y(s, t))$  be the centerline of the whip rod in the x–y plane, where s is the arc-length and t is time, and  $\varphi$  be an angle between the tangent of the whip at (x,y) and the x axis as shown in Figure 1. Then the unit tangent vector  $\mathbf{t}$  is given by

$$\mathbf{t} = \frac{\partial \mathbf{r}}{\partial s} = \left( \frac{\partial x}{\partial s}, \frac{\partial y}{\partial s} \right) = (\cos \varphi, \sin \varphi). \quad (1)$$

Let  $\rho$  be the density of the whip, A be the cross section of the whip at (x,y), (F,G) be force vector at (x,y), E be Young’s modulus and I be geometric moment of inertia at (x,y). Then the following equations of motion hold:

$$\rho A \ddot{x} = F' \quad (2)$$

$$\rho A \ddot{y} = G' \quad (3)$$

$$\rho I \ddot{\varphi} = (EI\varphi')' + G \cos \varphi - F \sin \varphi \quad (4)$$

where  $(\bullet)'$  denotes differentiation with respect to the arc-length s, and  $(\bullet)\dot{\phantom{\bullet}}$  differentiation with respect to time t. Then we apply the following normalizing transformations:

$$t = \frac{R_0}{2} \sqrt{\frac{\rho}{E}} \tilde{t}, \quad s = \frac{R_0}{2} \tilde{s}, \quad \varphi = \frac{R_0}{2} \tilde{\varphi}, \quad \mathbf{r} = \frac{R_0}{2} \tilde{\mathbf{r}},$$

$$\mathbf{F} = E\pi R_0^2 \tilde{\mathbf{F}}, \quad c = \sqrt{\frac{E}{\rho}}, \quad \frac{\Delta s}{\Delta t} = c \frac{\Delta \tilde{s}}{\Delta \tilde{t}}, \quad \delta(s) = \frac{R^2(s)}{R_0^2}$$

where R(s) is the radius of the whip at s,  $R_0$  is the radius of the whip at a given reference point as shown in Figure 1 and  $\delta$  is the ratio of the cross-section area at s to the one at the reference point. It is known that c is the speed of the sound in the whip. Then we obtain the following equations (after the transformation, we drop ~ symbol for readability):

$$\delta \ddot{x} = F', \quad (5)$$

$$\delta \ddot{y} = G', \quad (6)$$

$$\delta^2 \ddot{\varphi} = (\delta^2 \varphi')' + G \cos \varphi - F \sin \varphi \quad (7)$$

By setting  $\xi = s - ct$  and converting the differentiations of both s and t to the differentiation of  $\xi$ , we obtain the following equations:

$$\delta c^2 (\cos \varphi)'' = F'', \quad (8)$$

$$\delta c^2 (\sin \varphi)'' = G'', \quad (9)$$

$$\delta^2 (c^2 - 1) \varphi'' = G \cos \varphi - F \sin \varphi \quad (10)$$

Now we consider a whip of infinite length which is horizontal at  $-\infty$  and the tension at  $-\infty$  is  $\alpha$ . Then, from (1), (8), (9) and (10), the following equations are obtained:

$$x(s, t) = s - 2\gamma \tanh\left(\frac{s - ct}{\gamma}\right), \quad (11)$$

$$y(s, t) = s - 2\gamma \operatorname{sech}\left(\frac{s - ct}{\gamma}\right), \quad (12)$$

where

$$\gamma^2 = \frac{\delta^2 (c^2 - 1)}{\delta c^2 - \alpha}.$$

Figure 2 is a graph of the locus of (x,y) defined by (11) and (12) computed by Mathematica. This graph shows that there is a loop in the middle of the whip which propagates fast from left to right as a wave. Note that both of the x and y axes of the figure represent the normalized coordinate of the whip plane.

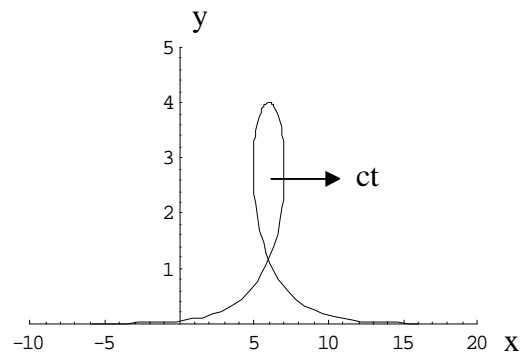


Figure 2: The locus of a whip rod defined by equations (11) and (12).

Equation (11) captures the behavior of the whip along x axis. By differentiating (11) twice, we obtain the condition where the maximum speed of the whip propagates by a function of time:  $s = ct$ . This equation states that the maximum speed point moves proportional to a constant  $c$ , the velocity of sound in the whip.

Furthermore, the speed of the sound  $c = \sqrt{\frac{E}{\rho}}$  is proportional to the square root of the Young's modulus  $E$ . That is, if you make the stiffness of the whip four times bigger, then the whip's propagation speed increases twice.

If we apply the whip model to the bow arm movement it is then possible to adjust the propagation speed of the maximum speed point by adjusting the stiffness of the body trunk. The problem is how to adjust the stiffness of the body trunk. It is well known that human beings adjust their impedance while performing various skill-based tasks. The impedance adjustment is directly related to a stiffness adjustment, and it is achieved by appropriately activating both of the agonist and the antagonist simultaneously. In our case, the impedance of the backbone is to be adjusted by activating both of the right and the left erector muscles of the spine. You can easily imagine this action: if you release the tension of the backbone, then you can swing your body easily and slowly, which corresponds to the low impedance situation. On the other hand, if you increase the tension of the backbone, then the swing width decreases and the move becomes faster, which corresponds to the high impedance situation.

It is very important to control the speed of body movement, and especially the speed of the bow change, by adjusting the impedance of the body trunk. The controllability makes it possible to select the right way to perform any given task.

## Results

We conducted two experiments to investigate the bowing mechanisms. In the first experiment, we measured the locus of the center of the gravity to find the relationship between the note length and the deviation of the center of the gravity. In the second experiment, we measured the electromyograms of various muscles to find the relationship between music note patterns and back muscles activities.

Figure 3 shows the performance task of the first experiment. In our case, we observed that the loci of the center of gravity during bow change tasks vary with note lengths: that is, whole notes, half notes, quarter notes, eighth notes and 16<sup>th</sup> notes. We selected this task because performing the bow changes for different note lengths require different timing for preparing the movements.

The equipment we used in our experiment is a force plate by Kistler, 9286AA. We measured the loci of the center of the gravity during performing the above task.

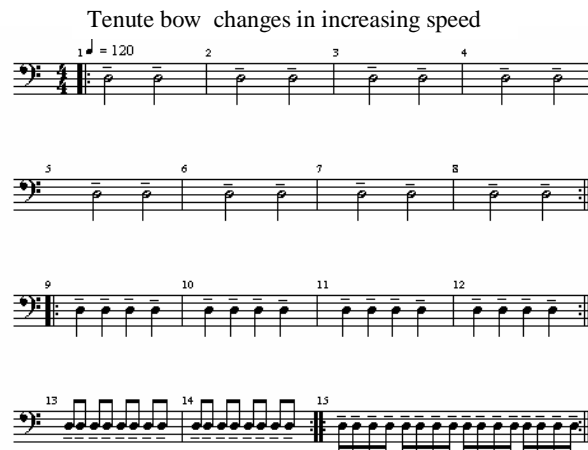


Figure 3: The first task for investigating the effects of the notes' length in preparing the bow change.

The results of the experiment are shown in Figure 4 where the horizontal axis is time (second) and the vertical axis is the deviation of the player's center of gravity projected to a frontal plane. This figure shows center of gravity loci of four different subtasks corresponding half notes, quarter notes, eighth notes and the 16<sup>th</sup> notes altogether by folding the entire graph at those points where each new subtask starts. Since the projection of the center of gravity to a frontal plane reflects the body swing along frontal plane, it is interpreted as showing pre-shaping for the bow change. In this figure, we observe two important facts. First, the amplitudes of the center of the gravity deviation are bigger when the note length is longer. This means that the player swings wider when playing the longer notes. This tendency is natural, because the player can produce the longer notes more softly. On the other hand, when the player plays shorter notes, he or she tends to make sharper sound by restricting their body movement smaller. There is one exception: the case of playing 16<sup>th</sup> notes (purple thin line in Figure 4). The reason why they are played with more amplitude than the eighth notes is due to the difficulty of the task.

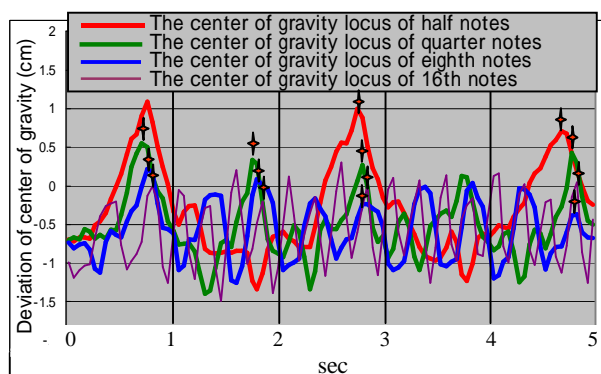


Figure 4: The deviation of the center of gravity in performing whole notes, half notes, quarter notes, eighth notes and 16<sup>th</sup> notes.

Secondly, the lag time for preparing the bow change becomes shorter when the note length is shorter. This fact gives a support for the whip model which claims players adjust impedance of back muscles in order to control the propagation speed of the phase shift depending on the music notes.

The task of the second experiment is the same as that of the first one. In this case, we measured both the right and the left erector muscle of spine during the performance. Surface EMG signals were measured with disposable Ag-Cl bipolar surface electrodes (EL503, BIOPAC Systems, Inc. CA, USA) using a telemetry system (Syna Act MT11, NEC Medical Systems, Tokyo, Japan), whose time constant value was set to 0.01, filtered with a Hamming windowed 6Hz FIR low pass filter. After rectifying and filtering we normalized the EMG data by the maximum voluntary contraction (MVC), which was measured at the same time. Figure 5 shows activity patterns of these two muscles during performing a sequence of half notes and Figure 6 those for 16th notes. The vertical axis is activity ratio of each muscle relative to the corresponding MVC.

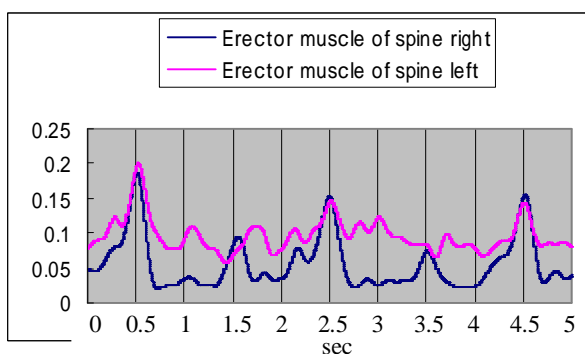


Figure 5: The activation patterns of the right and the left erector muscles in performing a sequence of half notes.

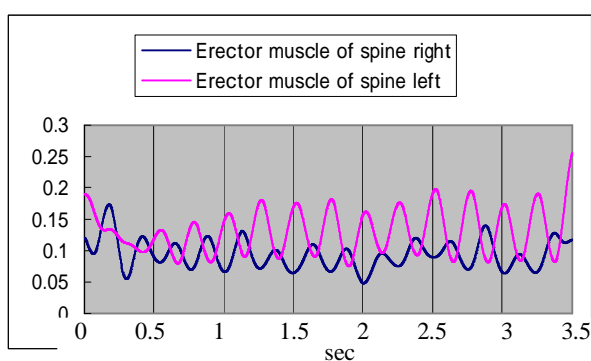


Figure 6: The activation patterns of the right and the left erector muscles in performing a sequence of 16th notes.

We notice a big difference between them: in case of the half notes playing, the right and the left erector muscle of spine are almost synchronizing, but in case of the 16th notes playing, they are completely in opposite phase. This suggests that the player is employing

different modes in playing these two tasks. This fact suggests the importance of the back muscles usage in performing the bowing tasks.

## Discussion

We can now show the feasibility of our hypothesis about controlling the bow speed by the back muscles impedance adjustment, by looking at two different kinds of evidence.

First, we consider the remarks typical of music teachers' advice on students' posture. They first insist on the importance of posture with a straight backbone. One of the authors had an experience that a violin teacher pushed his student from the back to make sure that he stands up steadily to resist such sudden force from the back. We have also observed a sitting violinist playing with an almost standing posture. He claimed that this kind of posture produced his best performance.

Second, consider the results of experiments stated above. Both of the two experiments showed the importance of the back muscles in performing the bow change. The first result is related to pre-shaping in performing the bow change. We tend to lean our body slightly to prepare the change of the bowing direction. This slight leaning produces a force to pulling the bow toward the center of the body after changing the bowing direction. Since it takes time for leaning the body, there may be a case that such leaning action cannot be finished in time when the music note requires fast action. In such a case, we need to make the impedance of the back muscles higher and to make the propagation speed of the backbone whip faster.

The result of the second experiment is rather surprising. We did not expect such a big mode change in performing such simple tasks. One possible reason why there appeared anti-synchronization pattern is to realize the fast movement of the 16<sup>th</sup> notes by activating the right and the left erector muscles alternatively.

## Conclusions

In this paper, we introduced the whip's equation of motion and tried to explain the bow arm movement based on the model. First, we observed the phase shift phenomenon in the equation  $s = ct$  describing the maximum speed propagation along the whip. It is interesting that the phase shift is actually proportional to the time whose coefficient  $c$  is the speed of sound in the whip. Since  $c$  is proportional to the square root of the Young's modulus, there is the possibility of controlling the propagation time by adjusting the stiffness of the whip. In our case, what we need to do in controlling the bow change speed is to adjust the stiffness of the back muscles. It is not always necessary to propagate the whip wave very fast. There may be a case that slower propagation is preferred. In general, a music piece with slow tempo does not require a fast preparation and furthermore, slow preparation often yields a better and richer sound. The reason why we can obtain better

sound by slow preparation is to be clarified in our future work. One possible interpretation is the principle of noise avoidance in slow preparation. In addition, in practice, it is hard to keep the back muscles strained to achieve the high impedance and therefore we need to rest time after time. If players can choose the adjustment level of impedance depending on the speed of the notes, the problem will be solved. The controllability during performance is essential in increasing skill as well as in avoiding fatigue from heavy performance.

In this research, we tried to validate the whip model for the bow change. We succeeded in providing several observations in supporting our hypothesis. However we did not provide a direct evidence to prove the whip model hypothesis. We need to show that a phase shift phenomenon of maximum speed occurrence from proximal extremity (e.g., the hip) to the distal end (e.g., hand or finger) really happens in conjunction with the high impedance of the back muscles. To show this fact, we can employ a method developed by Ueno and Furukawa [4] which finds peak timing points of various joints efficiently.

## References

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