COMPARISON OF MULTIPOLE EXPANSION AND LEVENBERG-MARQUARDT METHOD FOR THE LOCALISATION OF MAGNETICALLY MARKED CAPSULES IN THE INTESTINAL TRACT

B. Hilgenfeld*, H. Nowak** and J. Haueisen*

* Biomagnetic Centre, Department of Neurology, Friedrich Schiller University Jena, Germany
** Jena SQUID GmbH, Germany

bhi@biomag.uni-jena.de

Abstract: The localisation of pills and capsules in the intestinal tract can be accomplished by means of the Magnetic Marker Monitoring method. Here, the field of magnetically marked drug forms is measured continuously with sensor arrays outside the body. Different approaches exist to compute position and strength of the marker from the measurement data. We compare two such localisation algorithms, the multipole expansion method and the Levenberg-Marquardt method, in terms of their speed, accuracy and stability. Simulations are done with respect to the 195-channel vector magnetometer Argos 200. When tracking time series data, the multipole expansion method proves to be seven times faster than the Levenberg-Marquardt method. The localisation accuracy is equal for both methods, and depends mainly on the sensor geometry.

Introduction

The localisation of magnetically marked capsules and pills inside the human body is used for examinations of the intestinal tract and of the transport and dissolution of drug forms [1,2]. Markers act as permanent magnets, and can for their small dimensions be described as dipoles. Since no appropriate closed-form solutions for the determination of the dipole location from a noisy measurement are known, linearisation techniques and iterative search algorithms are used to accomplish this task.

The dipole localisation from noisy measurements can be formulated as a non-linear least squares problem and solved with the Levenberg-Marquardt method [3]. We compare the localisation characteristics of the Levenberg-Marquardt method and a new multipole expansion method [4].

Materials and Methods

Levenberg-Marquardt method: The Levenberg-Marquardt method is used to find an optimal solution in the space of **x** in order to minimise the quadratic norm of the error function vector **F**(**x**), see the left side of equation (1). The Levenberg-Marquardt method is provided with this error function **F** and the Jacobian matrix **J**, which contains the partial derivations of **F** with respect to the coordinates of the search space **x** as columns. We use the Levenberg-Marquardt implementation of the Matlab Optimisation Toolbox with cubic polynomial line search. The error function of our problem is the difference of the measured magnetic field and the dipole field of marker **m** at position $\mathbf{r}_\mathrm{q}$. As both have three dimensions, the dimensionality of **x** is six.

$$\min_{x\in\Re^n} \frac{1}{2}\left\|\mathbf{F}(\mathbf{x})\right\|_2^2 = \min_{\vec{m}(\vec{r}_\mathrm{q})} \sum_{i=1}^{N} \left(B_{\mathrm{meas}\,i} - B_{\mathrm{d},i}(\vec{m},\vec{r}_\mathrm{q})\right)^2 \quad (1)$$

The dipole field $B_{\mathrm{d},i}$ at sensor i with sensor normal direction d_i can be written as a dot product of function $\mathbf{F}_{\mathrm{d},i}$, which depends only on the dipole position $\mathbf{r}_q$, with marker strength **m**.

$$B_{\mathrm{d},i}(\vec{m},\vec{r}_\mathrm{q}) = \mathbf{F}_{\mathrm{d},i} \cdot \vec{m} \quad (2)$$

with

$$\mathbf{F}_{\mathrm{d},i} = \frac{\mu_0}{4\pi} \frac{3\cdot(\vec{r}\vec{d})\cdot\vec{r} - r^2\vec{d}}{r^5}\Bigg|_{\mathrm{p}i} . \quad (3)$$

Here vector and length r mark the position of the i-th sensor (denoted with p) with respect to the marker position (denoted with q). The corresponding Jacobi matrix contains three columns with derivatives with respect to the coordinates of the dipole and three columns with derivatives with respect to the marker strength components.

$$\mathbf{J} = -\left[\frac{\partial}{\partial x_{\mathrm{q}1}}, \frac{\partial}{\partial x_{\mathrm{q}2}}, \frac{\partial}{\partial x_{\mathrm{q}3}}, \frac{\partial}{\partial m_1}, \frac{\partial}{\partial m_2}, \frac{\partial}{\partial m_3}\right]\mathbf{B}_\mathrm{d} \quad (4)$$

The derivative with respect to the j–th dipole coordinate is:

$$\frac{\partial B_{\mathrm{d},i}}{\partial x_{\mathrm{q},j}} = \frac{\mu_0}{4\pi\cdot r^7}\Big[15x_j\cdot(\vec{r}\vec{d})\cdot(\vec{r}\vec{m}) - 3r^2\cdot\Big((\vec{r}\vec{d})\cdot m_j + (\vec{r}\vec{m})\cdot d_j + (\vec{m}\vec{d})\cdot x_j\Big)\Big] \quad (5)$$

The derivation with respect to the j–th dipole strength component is:

ISSN: 1727-1983 © 2005 IFMBE

$$\frac{\partial B_{\mathrm{d},i}}{\partial m_j} = -\frac{\mu_0}{4\pi}\,\frac{3\cdot\left(\vec{r}\vec{d}\right)\cdot x_j - r^2\cdot d_j}{r^5} \tag{6}$$

Equations (1) to (6) facilitate the calculation of **F** and **J** for the six-dimensional Levenberg-Marquardt localisation.

Three-dimensional search space: In order to reduce **x** to the three dimensions of the marker position $\mathbf{r}_q$, the linear sub problem of finding the optimal marker strength **m** for a given position $\mathbf{r}_q$ is solved:

$$\min_{\vec{m}}\left\|\mathbf{B}_{\mathrm{meas}} - \mathbf{F}_{\mathrm{d}}\left(\vec{r}_p\right)\cdot\vec{m}\right\|_2^2 \longrightarrow$$
$$\mathbf{F}(\mathbf{x}) = \mathbf{F}\left(\vec{r}_q\right) = \left(\mathbf{I} - \mathbf{F}_{\mathrm{d}}\cdot\left(\mathbf{F}_{\mathrm{d}}^{T}\cdot\mathbf{F}_{\mathrm{d}}\right)^{-1}\cdot\mathbf{F}_{\mathrm{d}}^{T}\right)\cdot\mathbf{B}_{\mathrm{meas}} \tag{7}$$

The Levenberg-Marquardt method is provided with this error function **F** and the Jacobian matrix **J**, which is the partial derivation of **F** with respect to the Cartesian components of the dipole.

$$\mathbf{J} = -\left[\frac{\partial}{\partial x_{\mathrm{q}1}}, \frac{\partial}{\partial x_{\mathrm{q}2}}, \frac{\partial}{\partial x_{\mathrm{q}3}}\right]\left(\mathbf{F}_{\mathrm{d}}\cdot\left(\mathbf{F}_{\mathrm{d}}^{T}\cdot\mathbf{F}_{\mathrm{d}}\right)^{-1}\cdot\mathbf{F}_{\mathrm{d}}^{T}\cdot\mathbf{B}_{\mathrm{meas}}\right) \tag{8}$$

The *j*–th row of **J** can be expressed as

$$\begin{aligned}\frac{\partial\mathbf{F}}{\partial x_{\mathrm{q},\mathrm{j}}} = \mathbf{F}' = -\Big\{&\mathbf{F}_{\mathrm{d}}'\cdot\left(\mathbf{F}_{\mathrm{d}}^{T}\cdot\mathbf{F}_{\mathrm{d}}\right)^{-1}\cdot\mathbf{F}_{\mathrm{d}}^{T}\\ &-\mathbf{F}_{\mathrm{d}}\cdot\left(\mathbf{F}_{\mathrm{d}}^{T}\cdot\mathbf{F}_{\mathrm{d}}\right)^{-1}\cdot\left(\mathbf{F}_{\mathrm{d}}^{T\prime}\cdot\mathbf{F}_{\mathrm{d}} + \mathbf{F}_{\mathrm{d}}^{T}\cdot\mathbf{F}_{\mathrm{d}}'\right)\\ &\qquad\cdot\left(\mathbf{F}_{\mathrm{d}}^{T}\cdot\mathbf{F}_{\mathrm{d}}\right)^{-1}\cdot\mathbf{F}_{\mathrm{d}}^{T}\\ &+\mathbf{F}_{\mathrm{d}}\cdot\left(\mathbf{F}_{\mathrm{d}}^{T}\cdot\mathbf{F}_{\mathrm{d}}\right)^{-1}\cdot\mathbf{F}_{\mathrm{d}}^{T\prime}\Big\}\cdot\mathbf{B}_{\mathrm{d}}\end{aligned} \tag{9}$$

The rows of **F**´ for each sensor *i* and derivation direction *j* compute as follows:

$$\begin{aligned}\mathbf{F}_{\mathrm{d},i}' = \frac{\partial\mathbf{F}_{\mathrm{d},i}}{\partial x_{\mathrm{q}j}} = \frac{\mu_0}{4\pi\cdot r^7}\Big[&15\cdot\left(\vec{r}\vec{d}\right)\cdot x_j\vec{r}\\ &-3r^2\cdot\left(d_j\vec{r} + \left(\vec{r}\vec{d}\right)\cdot\vec{e}_j + r_j\vec{d}\right)\Big]_{\mathrm{p}i}\end{aligned} \tag{10}$$

Equations (7) to (10) facilitate the calculation of **F** and **J** for the three-dimensional Levenberg-Marquardt localisation.

Homogeneous field suppression: Disturbing fields result from sources which are remotely located compared to the magnetic marker position and the sensor geometry. For this reason, their magnetic field measured is predominantly homogeneous. Contrary the magnetic field of the magnetic marker is strongly non-homogeneous. The suppression of homogeneous fields from measurement vectors and dipole fields results in a high suppression of disturbing fields, and only in a small degradation of localisation accuracy. The homogeneous field suppression is performed by left side matrix multiplication of all magnetic field vectors with

$$\mathbf{M}_g = \mathbf{I} - \mathbf{D}\cdot\left(\mathbf{D}^{T}\cdot\mathbf{D}\right)^{-1}\cdot\mathbf{D}^{T}\,. \tag{11}$$

Here Matrix **D** contains the normal directions of the sensors in its rows. Mathematically equation (11) is a solution to a linear least squares problem, comparable to equation (7). Since **D** does not change, $\mathbf{M}_{\mathrm{g}}$ must be computed only once. It can be used both for the six-dimensional and the three-dimensional implementation of the Levenberg-Marquardt localisation method.

Multipole expansion method: The multipole expansion of the magnetic field measurement (eq. 1) gives form functions $\mathbf{F}_{\mathrm{m}}$ and multipole moments $\mathbf{c}_{\mathrm{m}}$ for the field of the marker as well as form functions $\mathbf{F}_{\mathrm{ex}}$ and coefficients $\mathbf{c}_{\mathrm{ex}}$ for disturbing fields from remote sources. Their respective fields are depicted in Fig. 1.

$$\begin{aligned}\vec{B}_{\mathrm{m|ex}}\left(\vec{r}\right) = &\frac{\mu_0}{4\pi}\sum_{j=1}^{3}\left(F_{\mathrm{m|ex}}^{i,j}\left(\vec{r}\right)\right)_{i=1}^{3} c_j^{\mathrm{m|ex}}\\ &+\frac{\mu_0}{4\pi}\sum_{k=1}^{3}\sum_{j=1}^{3}\left(F_{\mathrm{m|ex}}^{i,j,k}\left(\vec{r}\right)\right)_{i=1}^{3} c_{j,k}^{\mathrm{m|ex}}\\ &+\frac{\mu_0}{4\pi}\sum_{l=1}^{3}\sum_{k=1}^{3}\sum_{j=1}^{3}\left(F_{\mathrm{m|ex}}^{i,j,k,l}\left(\vec{r}\right)\right)_{i=1}^{3} c_{j,k,l}^{\mathrm{m|ex}} + \ldots\end{aligned} \tag{12}$$

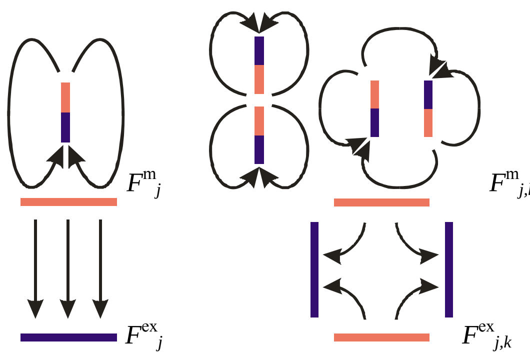

Figure 1: Dipole F^{m}_j and quadrupole fields $F^{\mathrm{m}}_{j,k}$, homogeneous fields F^{ex}_j and gradient fields $F^{\mathrm{ex}}_{j,k}$.

The matrix notation of significant elements of the multipole expansion of marker and disturbing fields and residual fields $\mathbf{B}_{\mathrm{res}}$ is

$$\mathbf{B}_{\mathrm{meas}} = \mathbf{B}_{\mathrm{m}} + \mathbf{B}_{\mathrm{ex}} + \mathbf{B}_{\mathrm{res}} = \frac{\mu_0}{4\pi}\mathbf{F}\cdot\mathbf{c} + \mathbf{B}_{\mathrm{res}}\,. \tag{13}$$

The suppression of spatially correlated noise is achieved by neglecting the outer multipoles and the marker localisation works by combining dipole and quadrupole. In this manner the coefficients **c** found by solving eq. 13 may efficiently be used. A comparison of the multipole expansion (eq. 12) with a Taylor series of the marker dipole field, see [4], gives equality of marker strength

 ISSN: 1727-1983

and dipole moments (14a) and a linear relationship between quadrupoles $\mathbf{c}_q$, marker moment $\mathbf{m}$ and marker position $\mathbf{r}$` (14b).

$$\text{(a)}\ \vec{m} = \left(c_j^{\mathrm{m}}\right)_{j=1}^{3}, \quad \text{(b)}\ \mathbf{m}\cdot\vec{r}\,' = \mathbf{c}_q^{\mathrm{m}} \qquad (14)$$

Here, the quadrupole vector $\mathbf{c}_q$ is a linear independent representation of the quadrupole tensor (15a) and Matrix $\mathbf{m}$ contains the elements of the marker moment (15b).

$$\text{(a)}\,\mathbf{c}_q^{\mathrm{m}} = \left.\begin{pmatrix} c_{1,1}-c_{2,2} \\ c_{3,3}-c_{2,2} \\ c_{1,2}+c_{2,1} \\ c_{2,3}+c_{3,2} \\ c_{3,1}+c_{1,3} \end{pmatrix}\right|^{\mathrm{m}}, \ \text{(b)}\,\mathbf{m} = \begin{bmatrix} m_1 & -m_2 & 0 \\ 0 & -m_2 & m_3 \\ m_2 & m_1 & 0 \\ 0 & m_3 & m_2 \\ m_3 & 0 & m_1 \end{bmatrix} \qquad (15)$$

The principle behind eq. 14b is visualised in fig. 2 for shifts along and perpendicular to the marker direction.

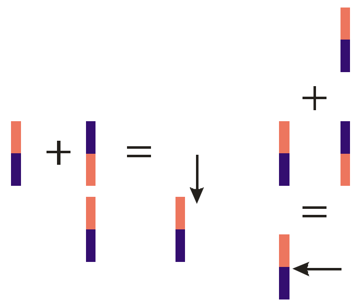

Figure 2: Localisation principle using dipoles and quadrupoles.

Sensor noise and localisation accuracy: The effect of low sensor noise (sufficiently high SNR) on the localisation result can be described with a linearised model. The corresponding localisation error results from

$$\min_{\vec{m}} \left\| \mathbf{B}_n - \mathbf{J}\cdot\mathbf{x} \right\|_2^2, \qquad (16)$$

which is again a linear least squares problem and can be solved with the pseudo inverse of the Jacobi matrix $\mathbf{J}$:

$$\mathbf{x} = \left(\mathbf{J}^T\cdot\mathbf{J}\right)^{-1}\cdot\mathbf{J}^T\cdot\mathbf{B}_n \qquad (17)$$

The vector $\mathbf{x}$ contains the localisation offset corres-ponding to the noise field vector $\mathbf{B}_n$. If we use the six-dimensional Jacobi matrix, $\mathbf{x}$ contains also the corres-ponding offset for the marker strength at position 4 to 6. If the sensor noise vector $\mathbf{B}_n$ contains the standard deviation of N_{sen} independent Gaussian noise processes, the corresponding localisation error $\mathbf{x}$ contains also Gaussian distributed variables, which compute to

$$x_j = \sqrt{\sum_{i=1}^{N_{sen}} \left(\left[\left(\mathbf{J}^T\cdot\mathbf{J}\right)^{-1}\cdot\mathbf{J}^T\right]_{j,i}\cdot B_{n,i}\right)^2}\ . \qquad (18)$$

SQUID system Argos 200: The sensor geometry of this multichannel system from AtB (Advanced technologies Biomagnetics, Pescara, Italy) is depicted in figure 3. The triplet design with orthogonally oriented sensors enables full vector field measurements. The sensor triplets are placed on a ground plane (z=0) and three reference planes above.

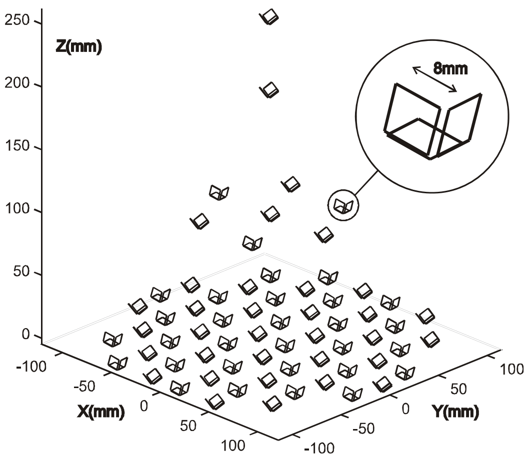

Figure 3: Sensor geometry of the Argos 200 Squid magnetometer array with orthogonal sensor triplets.

Results

The localisation speed depends on the number of iteration steps needed to find the marker dipole with the desired spatial accuracy. The improvement in the marker position after one iteration step depends highly on the starting distance between search point and marker position. For the multipole method this relation is given in figure 4. If this distance is not bigger than 20 mm, 2-3 steps of the multipole expansion method suffice to reach an accuracy of less than 0.1 mm.

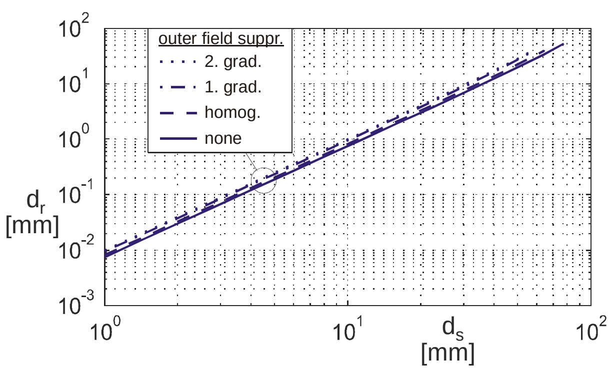

Figure 4: Remaining multipole localisation error depen-ding on search distance for one iteration step, marker position 300 mm beneath the measurement plane

The Levenberg-Marquardt algorithm facilitates the same desired accuracy within 2 iteration steps. For each step the Levenberg-Marquardt search direction and the optimal step length (via cubic polynomial line search) must be computed consecutively. For this reason several function calls including the computation of dipole field and Jacobi matrix are necessary per iteration step.

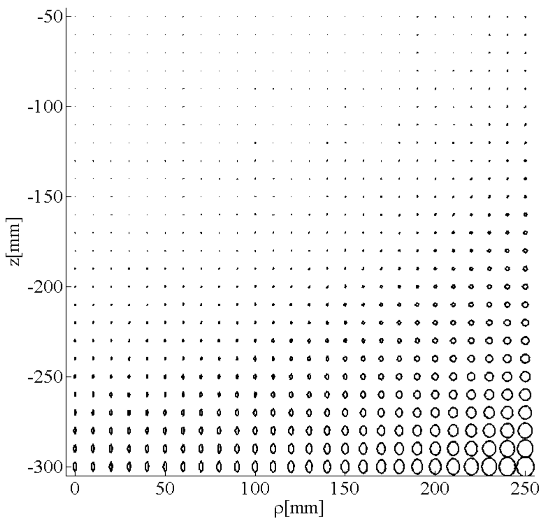

Figure 5: Localisation standard deviation, 2 Amm^2 horizontal marker, 100 fT sensor noise

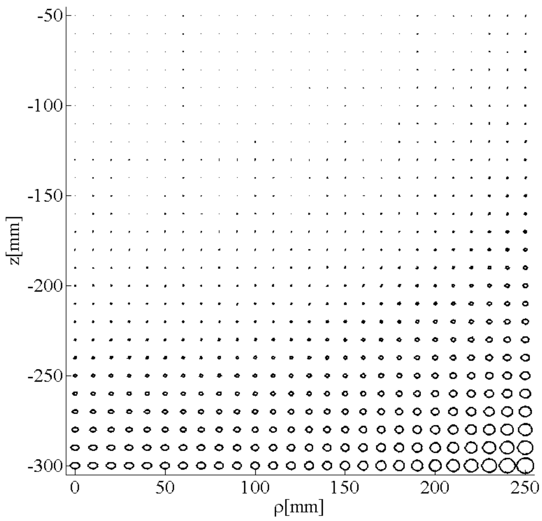

Figure 6: Localisation standard deviation, 2 Amm^2 vertical marker, 100 fT sensor noise

Numerical tests for the behaviour of both localisation techniques have been done with the tracking of actual patient data recorded with the ARGOS system. With a 2.40 GHz Pentium 4 CPU a localisation speed of 225 data points per second could be reached for the multipole method compared to 32 data points per second when using the Levenberg-Marquardt method. With the multipole expansion method, the number of function calls needed for one time step equals the number of iterations, and is normally not higher than 3. The Levenberg-Marquardt method with cubic polynomial line search requires 11 function calls for 2 iteration steps for most data points of the same measurement.

The dependence of the localisation accuracy on the level of spatially uncorrelated noise is equal for both methods. Figures 5 and 6 show the standard deviation of the localisation result for different marker positions below the measurement plane (z-value) and offset to the symmetry axis (ρ-value). The signal to noise ratio for the chosen marker strength and noise level is 77 at the nearest sensor. The influence of the marker direction on the localisation deviation is small due to the vector properties of the sensor array. The localisation deviation is predominantly circular and depends mainly on the distance between sensor array and marker.

The localisation radius around the marker position gives the range of stable localisation with the multipole method. This radius has been found to be above 70 mm for the Argos 200 system and a marker depth of 300 mm [4]. This is sufficient for the successive localisation of marker positions from time series data.

Conclusions

The multipole expansion is an advantageous method especially for the online tracking of magnetically marked capsules. The tracking of marker movement data implicates short search distances, since the preceding localisation result may always be used as a search point for the next marker position. This ensures stability and time efficiency of the multipole expansion method. Furthermore the multipole expansion method combines numerically efficiently the suppression of spatially correlated disturbing fields and the localization of the magnetic dipole, since both use the same set of parameters, the multipole coefficients.

References

[1] KOSCH O., OSMANOGLOU E., WEITSCHIES W., WIEDENMANN B., MÖNNIKES H., TRAHMS L. (2003): 'Online localisation of a magnetized capsule for investigation of the gastrointestinal passage', *Biomed. Tech.*, **48**, Suppl 1, pp. 244-245

[2] KOSCH, O., WEITSCHIES, W., TRAHMS, L. (2004): 'On-line localization magnetic markers for clinical applications and drug delivery studies', Proc. of the 14th BIOMAG 2004, Boston, USA, pp. 261-262

[3] MORÉ, J. J. (1977): 'The Levenberg-Marquardt Algorithm: Implementation and Theory' in Watson G. A. (Ed): 'Numerical Analysis, Lecture Notes in Mathematics 630', (Springer Verlag), pp. 105-116

[4] HILGENFELD B., HAUEISEN J. (2004): 'Simultaneous suppression of disturbing fields and localization of magnetic markers by means of multipole expansion', *Biomagn. Res. Technol.* 2004 Sep 1, 2:6, Internet site address: http://www.biomagres.com