# LDL TRANSPORT FROM BLOOD FLOW TO THE ARTERIAL WALL IN A THREE DIMENSIONAL HUMAN CAROTID BIFURCATION

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**Abstract: The carotid arteries are a common site of atherosclerotic plaque formation, which has been linked to the blood flow patterns and the mass transport phenomenon. The purpose of this research was to study the lipid transport in a human carotid artery model, focusing on the effects of local geometric and hemodynamic factors on mass transfer from blood flow to vessel wall and its concentration at the luminal surface of the artery. The Reynolds number, 250, and the Schmidt number, $6.66 \times 10^5$, were selected to model the mass transfer of LDL macro molecules, and in order to see the effect of Reynolds and Schmidt numbers to mass transport, the model was analyzed with different conditions. The steady state flow was used for three dimensional carotid geometry. At the inlet, the blood flow was assumed a steady fully developed laminar velocity profile with a uniform LDL concentration. The vessel wall was assumed permeable to water and semi-permeable to LDL macro molecules. The problem was analyzed with the finite element method. The results show 29% increase of LDL concentration from inlet value at the luminal surface of the artery located in the separated flow region. The maximum value of LDL concentration occurred at the separation point.**

## Introduction

Atherosclerosis is one of the most important cardiovascular diseases. It is associated with macro molecules deposition in the subendothelial layer, intimal thickening, SMC proliferation and plaque formation [13]. Because the endothelial cells of the artery displays low permeability to macro molecules, the infiltration flow across the artery wall may cause a concentration polarization of lipids at the luminal surface of artery (blood-wall interface). This phenomenon causes increasing the lipid concentration from bulk value towards the interface. Hemodynamic factors are strongly correlated with the localization of atherosclerotic plaques. In areas with complex flow where the separation and recirculation flow occurs and the wall shear rate is low, the luminal surface lipid concentration may be elevated relative to the luminal concentration in regions without complex flows. It has been widely observed and noted clinically relevant plaque deposits are most common in complex flow regions such as curvature sites, branching and bifurcations. Aorta, Carotid, Coronary, Abdominal, and femoral arteries are examples of these complex flow locations [3, 4, 6, 7, 8, 9, 11]. In order to prevent, diagnose and treat this vascular disease, detailed knowledge of blood flow and mass transport is essential. The aim of present numerical study is to test this hypothesis.

Pervious studies were done in different geometries such as; straight tube [1,9], back step tube [4], stenosis geometry [11] and coronary artery [3]. In our previous works [10,12], first we studied the lipid transport from flowing blood to the arterial wall in a 45 degree bifurcation using finite difference approach, and then we studied the LDL transport in a two dimensional carotid bifurcation model.

Since the two dimensional model does not predict the flow pattern correctly and secondary flows will not be seen in the 2D model, in present study we extended our work to a more realistic human common carotid bifurcation geometry using the finite element approach. Our aim was to understand how this geometry influences flow patterns and how flow patterns affect the lipid transport and its luminal concentration. The blood flow was assumed steady state and the vessel wall was assumed permeable to water and semi-permeable to LDL molecules. The pressure at the artery outlets were selected values which lead to a physiologic flow rate division at the bifurcation of artery. Results independency of meshing and element size was obtained.

## Modeling and Analyses

Arterial mass transport study can be done in different ways, including *in vivo*, *ex vivo,* and *in vitro* experimentations, numerical methods, and analytical solutions. Each way has its strengths and drawbacks. In this study we were chosen the numerical method to simulate the mass transport of LDL molecules in blood

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flow in a three dimensional carotid artery with a steady state regime.

*Assumptions*

To investigate the effects of various physical and hemodynamic factors on LDL concentration at the luminal surface these simplifying assumptions were made:

- The concentration field of LDL species does not affect the velocity field in the artery;
- Blood is homogeneous, incompressible, Newtonian fluid with a viscosity, $\mu$=0.0035 Pa.s and a density, $\rho$=1050 kg/m$^3$ [1,3];
- The blood flow is a steady laminar Poiseuille with a parabolic velocity profile at the inlet;
- The vessel wall is permeable to water, and the water infiltration velocity is a constant value at different flow rates , $V_w$=4x10$^{-8}$ m/s [1,3];
- Permeability coefficient of the arterial wall to LDL molecules was selected, $K$=2x10$^{-10}$ m/s [1,3];
- Diffusivity of LDL molecules in blood is a same value in all direction, and is a constant value at the body temperate, $D$=5x10$^{-12}$ m$^2$/s [1,3];
- LDL flux at the vessel wall is a function of LDL concentration at the luminal surface of artery;
- The arterial wall is rigid.

*Governing Equations*

The steady and incompressible flow equations describe with continuity and Navier-Stockes equations as follows:

$$\nabla.\mathrm{v} = 0 \tag{1}$$

$$\rho(\mathrm{v}.\nabla\mathrm{v}) = -\nabla p + \mu\nabla^2\mathrm{v} \tag{2}$$

where **v** is a velocity vector considering of three dimensional components $u$, $v$, $w$ for $x$, $y$, z directions, and $p$ is blood pressure. In Equation 2 body force was negligible and was eliminated from the equation.

Steady state mass transport is another governing equation. The advection and diffusion of LDL molecules in blood flow describe as the following equation:

$$\mathrm{v}.\nabla C - D\nabla^2 C = 0 \tag{3}$$

where $C$ is the concentration of LDL molecules, and $D$ is the diffusivity of LDL molecules in blood. Using the coordinate and variable transformation:

$$\begin{aligned} x^* &= \frac{x}{R_0}, y^* = \frac{y}{R_0}, z^* = \frac{z}{R_0}, u^* = \frac{u}{U_0} \\ v^* &= \frac{v}{U_0}, w^* = \frac{w}{U_0}, p^* = \frac{p}{\rho U_0{}^2}, C^* = \frac{C}{C_0} \end{aligned} \tag{4}$$

Applying the characteristic length of the flow domain, $R_0$ (radius at the inlet of artery), the characteristic velocity, $U_0$ (mean inflow velocity), and the reference concentration, $C_0$ (LDL concentration at the inlet) the Navier-Stockes and mass transport equations can be transformed into non-dimensional forms:

$$\mathrm{v}^*.\nabla\mathrm{v}^* = -\nabla p^* + \frac{\nabla^2\mathrm{v}^*}{\mathrm{Re}} \tag{5}$$

$$\mathrm{v}^*.\nabla C^* - \frac{\nabla^2 C^*}{\mathrm{Re}\,\mathrm{Sc}} = 0 \tag{6}$$

In Equation 5 and Equation 6 coordinate components are $x^*$, $y^*$, $z^*$. The non-dimensional parameter which characterises the flow is the Reynolds number, Re=2$\rho$U$_0$R$_0$/$\mu$. Schmidt number is another non-dimensional parameter which characterises the transport, Sc= $\mu$/$\rho$D. The product of Reynolds and Schmidt numbers which relates the convective transport velocities to the diffusion velocities is Peclet number, Pe=ReSc=2U$_0$R$_0$/D. Mass transport processes in medium sized and large arteries are generally highly convection dominated, which is reflected in very large Peclet numbers.

*Boundary Conditions*

The blood flow equations were solved with the following boundary conditions (BC):

$$u = 2U_0 - 2U_0\left(\frac{r}{R_0}\right)^2, v = 0, w = 0 \quad at\ x=0 \tag{7}$$

This equation implies the Poiseuille velocity profile at the inlet of the artery, where $r$ is the radius distance from the central axis of vessel wall, $R_0$ is the inlet radius, and $U_0$ is mean inflow velocity.

It was assumed that the wall is water permeable uniformly. This boundary condition was applied as a constant water infiltration velocity at vessel walls:

$$\mathrm{v} = V_w\mathrm{n} \quad at\ r=R \tag{8}$$

where **n** is a unit vector normal to the vessel wall. Thus the velocity vector has no tangent component at vessel wall and no-slip condition is satisfied. $V_w$ is water infiltration velocity at vessel walls.

Pressure at the outlets prescribed as these constants:

$$p_e = 0.0\,\mathrm{Pa}\ at\ l=L_e,\ p_i = 28\,\mathrm{Pa}\ at\ l=L_i \tag{9}$$

This condition makes the ratio of outlet flow rate from internal carotid artery (ICA) to flow rate at inlet of common carotid artery (CCA), $Q_{ICA}/Q_{CCA}$=0.59 and for


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external carotid artery, $Q_{ECA}/Q_{CCA}$=0.41 [5]. $L_i$ and $L_e$ are outer wall lengths of internal and external carotid arteries.

The following boundary conditions were applied for mass transport equation:

$$C = C_0 \quad at \ x=0 \tag{10}$$

This condition implies a uniform LDL concentration at the inlet of artery.

In addition to Equation 10, another important condition was used is:

$$V_w C_w - (\frac{\partial C}{\partial n})_{r=R} = KC_w \quad at \ r=R \tag{11}$$

where $C_w$ is the luminal LDL concentration at the endothelial surface, and $K$ is a net uptake mass transfer coefficient for the vessel wall. This boundary condition is the mass conservation law at the blood-wall boundary, and it states that the amount of LDL passing through the vessel wall is determined as a difference between the amount carried to vessel wall by a filtration flow and the amount which diffuses back to the main stream. The overall mass transfer coefficient of LDL at the artery wall was assumed, $K$=2x10$^{-10}$ m/s which generally reported in literatures under physiologic conditions [1,3,6].

*Numerical Simulation*

The common carotid artery originates at the aortic arch and bifurcates into the external and internal carotid arteries at the neck level. The external carotid supplies blood to the facial area and internal carotid to the brain region. The diameter of the common carotid is about 6-8 mm and the flow rate is 270-480 mL/min. The geometry parameters were used for this model is based upon the data described by Ku *et al.* [2]. The finite element method (FEM) was implemented using ANSYS-FLOTRAN (ANSYS, Inc., Southpointe, Canonsburg, PA, USA) to simulate the flow and transport problem. The model was created in the software and was meshed in its mesh generator with FLUID 142 element. This element is defined by maximum eight nodes. The degrees of freedom for the element are velocities, pressure, temperature and concentration.

Since the LDL molecules are large in dimensions, the diffusivity of LDL is very small, so the Peclet number becomes extremely large. This makes difficult numerical solving of mass transport equation particularly at vessel walls, and in separation areas. In order to have a correct solve of equations, the geometry was meshed with non-uniform grid which the most elements of grid were limited to these regions where the fluid velocity was relatively low. The finest grid had 308000 elements. The model was solved with different grid configurations and it was confirmed that satisfactory results were independent of meshing and element size. Figure 1 shows the geometry meshing and the finer mesh near the boundary wall. The solution was obtained by a finite element method, and SIMPLEF was chosen for solution algorithm which desecrates the momentum and transport equations with a StreamLine Upwind/Petrov-Galerkin (SUPG) approach.

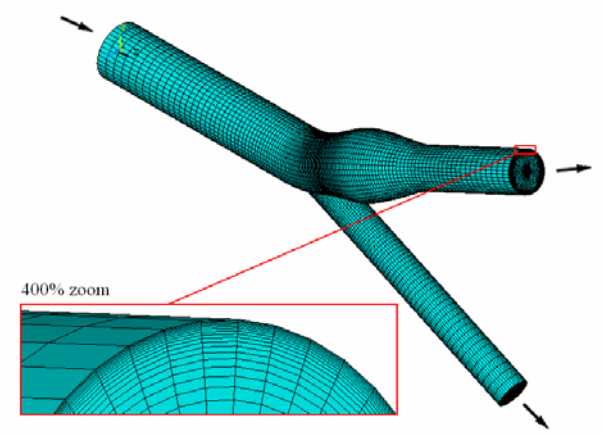


Figure 1: Carotid bifurcation model and the finite element mesh, arrows indicate the direction of the inflow and outflows.

## Results

*Flow patterns*

Simulations were carried out at various Reynolds and Schmidt numbers (Reynolds was evaluated at the inlet of the vessel). At first calculation flow rate at the inlet of common carotid artery was selected 275 mL/min [11], and the Reynolds and Schmidt numbers were, Re=250 and Sc=6.66x10$^5$. Figure 2 shows the velocity in the middle section of the artery to depict the flow pattern in the vessel.

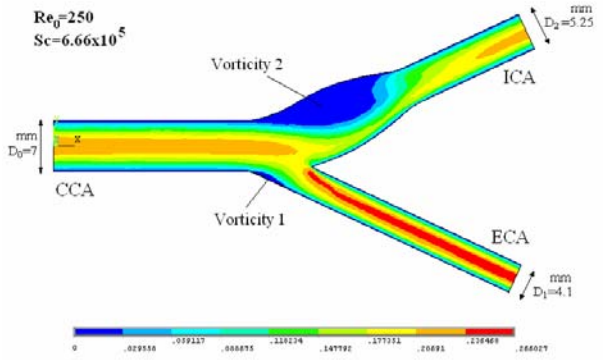


Figure 2: Flow patterns in carotid model at the inflow Reynolds number, Re=250 and Schmidt number, Sc=6.66x10$^5$. Velocity contour at the middle cross sections of the vessel.


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As shown in Figure 2, flow separations occurred at locations $S_1$, and $S_2$ near the artery bifurcation at the outer walls. The separated flows reattached to the walls at locations $R_1$ and $R_2$, distal to the point of flow separations. In these regions flow recirculation occurred and the wall shear rate was low. At the inlet of the artery the Poiseuille velocity profile was applied, and other velocity profiles were obtained from calculation. After the bifurcation velocity profile changed to a non parabolic one, which in recirculation flow (adverse flow) region the velocity vectors were not along the main stream. At the outlets the profiles were parabolic approximately.

*LDL concentration and shear stress*

Figure 3 shows the distribution of LDL concentration in the evaluated at upper recirculation flow regions. LDL concentration occurred at a blood-endothelial boundary layer where the fluid velocity was small.

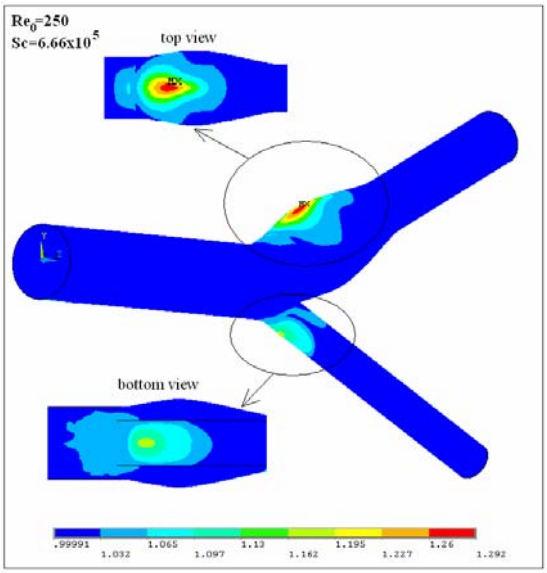


Figure 3: Normalized LDL concentration at the vessel wall

As obvious from the Figure 3, Concentration of LDL increased with decreasing the velocity value. The LDL concentration occurs within a thin boundary layer nearby the vessel wall, where fluid velocity is very low and also wall shear stress is low. In the vorticity region where separation and recirculation flow occurs, the LDL concentration is evaluated relative to the bulk value. Similar to the upper vorticity region occurs at the lower branch of the bifurcation. This LDL concentration evaluated region is smaller than the upper one.

Figure 4 shows the relationship between LDL wall concentration, $C_w/C_0$ and wall shear stress, $\tau_w$ at the vessel wall (Each point shows the luminal surface concentration against the wall shear stress at every node of the walls).

The wall concentration which corresponds to a certain wall shear stress value at a location is not only dependent on the wall shear stress value but also other factors such as geometry and global flow patterns affect the wall concentration. In general LDL wall concentration in not only a function of wall shear stress, but wall shear stress plays most important role in LDL concentration distribution. To show better the relationship between the LDL concentration and shear stress at the wall, a curve was fitted to these points. As it is evident from the figure the wall concentration increased with decreasing the wall shear stress and it increased very sharply as the wall shear stress decreased from 0.5 to 0.0 Pa.

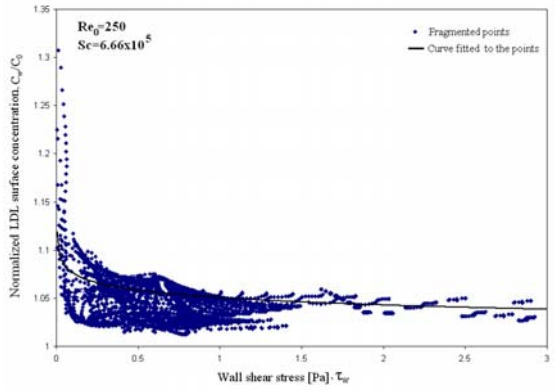


Figure 4: Normalized LDL luminal concentration against the wall shear stress

*The effect of the Reynolds and Schmidt numbers*

To study the effect of the Reynolds number on the LDL surface concentration distribution, the same model was solved with various flow rates in the physiologic range. The calculations were carried out with inflow rates, 375 and 475 mL/min which Reynolds number at these flow rates were, Re=340 and 430 respectively. Results show that in all three cases, LDL surface concentration was elevated at the separation region where wall shear rate was relatively low. It was found that the maximum value of LDL wall concentration occurred at the separation points and decreased in value with increasing the Reynolds number.

Table 1: Summarized calculations at different flow rates and different lipid diffusivity coefficients

| | | **Max. normalized wall concentration, $C_w/C_0$** |
|---|---|---|
| **Sc=6.66x10$^5$** | **Re=250** | 1.292 |
| | **Re=340** | 1.272 |
| | **Re=430** | 1.235 |
| **Re=250** | **Sc=6.66x10$^4$** | 1.061 |
| | **Sc=6.66x10$^5$** | 1.292 |
| | **Sc=6.66x10$^6$** | 1.672 |

In human plasma there are several different lipoproteins with various size, molecular weight and density, therefore the diffusivity of these molecules in the plasma are different. VLDL has larger size and HDL and


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VHDL have smaller size than LDL molecules [3]. In order to see the effect of molecule size, the calculation was repeated at two different diffusivity coefficient; one order smaller and one order larger than $D$=5x10$^{-12}$ m$^2$/s. Results show that there was much larger region with high concentration with increasing the size of molecules (decreasing the diffusivity coefficient), and the maximum wall concentration was increased. Table 1 shows the summery of the maximum LDL concentration and figure 5 shows the LDL wall contours nearby the vorticity 2 at different Reynolds and Schmidt numbers.

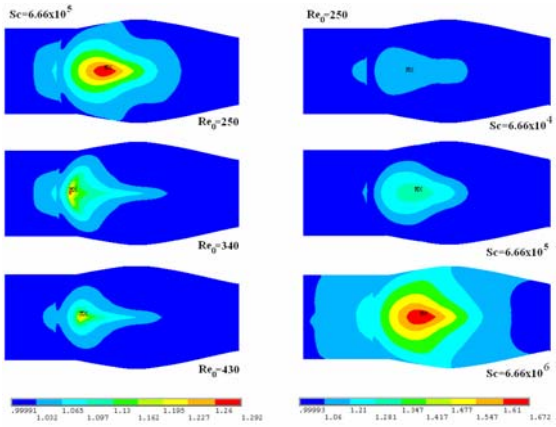


Figure 5: Normalized LDL luminal concentration contours nearby the vorticity 2 at different Reynolds and Schmidt numbers

## Conclusion

The purpose of this study was to find out how the geometry and the blood flow patterns affect the LDL concentration distribution. The numerical simulation shows that the LDL concentration occurs within a thin boundary layer nearby the vessel wall, where fluid velocity and wall shear stress are low. In regions where separation and recirculation flow occurs this boundary layer is thicker and the evaluated LDL luminal surface concentration occurs near the vorticites.

Calculations were carried out at three different flow rates and three different lipid diffusivity coefficients. It was concluded that the maximum value of the LDL wall concentration decreases with Reynolds increasing and increases with Schmidt increasing.

The study was carried out by assuming that the blood flow was steady. If the flow was pulsatile, the degree of LDL concentration distribution may vary. Therefore the study will be extended to a three dimensional model with a pulsatile inlet flow.

## References

[1] WADA S., and KARINO T. (1999): 'Theoretical study on flow-dependent concentration polarization of low density lipoproteins at luminal surface of a straight artery', *Biorheology,* 36, pp. 207-223

[2] KU D.N., GIDDENS D.P., ZARINS C.Z., and GLAGOV S. (1985): 'Pulsatile flow and atherosclerosis in the human carotid bifurcation', *Atherosclerosis,* 5, pp. 293-302

[3] WADA S., and KARINO T. (2002): 'Theoretical prediction of low-density lipoproteins concentration at the luminal surface of an artery with a multiple bend', *Annals of Biomedical Engineering,* 30, pp. 778-791

[4] LUTOSTANSKY E.M., KARNER G., RAPPITSCH G., KU D. N., and PERKTOLD K. (2003): 'Analysis of hemodynamic fluid phase mass transport in a separated flow region', *Journal of Biomechanical Engineering,* 125, pp. 189-196

[5] CHILDERS M.A., HISLEY K. C., and EGGLETON C.D. (2001): 'The internal carotid artery as a low pass filter', Proc. of ASME Bioengineering Conference, 50, pp. 519-520.

[6] ETHIER C.R. (2002): 'Computational modeling of mass transfer and links to atherosclerosis', *Annals of Biomedical Engineering,* 30, pp. 461-471.

[7] STANGEBY D.K., and ETHIER C.R. (2002): 'Computational Analysis of Coupled Blood-wall arterial LDL transport', *Journal of Biomechanical Engineering,* 124, 2002, pp. 1-8.

[8] KAAZEMPUR-MOFRAD M.R., and ETHIER C.R. (2001): 'Mass transport in an anatomically realistic human right coronary artery', *Annals of Biomedical Engineering,* 29, pp. 121-127

[9] FATOURAEE N., DENG X.Y., DE CHAMPLAIN A., and GUIDOIN R. (1998): 'Concentration polarization of low density lipoproteins (LDL) in the arterial system', *Annals of New York Academy of Sciences,* 858, pp. 137-146

[10] FATOURAEE N., DENG X.Y., DE CHAMPLAIN A., and GUIDOIN R. (2004): 'Blood Flow and Lipid Transport in a 45 Degree Bifurcation', Proc. of the 28th Canadian Medical and Biological Engineering Society Conference (CMBEC28), Quebec, Canada

[11] DENG X.Y., FATOURAEE N., and GUIDOIN R. (2004): 'Numerical simulation of low density lipoproteins transport in arterial stenoses', Proc. of the 2nd IASTED International Conference on Biomechanics, Honolulu, Hawaii, USA, August 23-25, pp. 94-96

[12] AMILI O. and FATOURAEE N. (2005): 'Low density lipoproteins transport from the blood flow to the arterial wall in the human carotid bifurcation', Proc. of the 3rd IASTED International Conference on Biomechanics, Benidorm, Spain, pp. 139-144

[13] PARKER P.M., and PARKER J.N. (2003): 'Atherosclerosis', (ICON Group International Inc.)


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