

## THE ANISOTROPIC VERSUS THE ISOTROPIC SPHERICAL HEAD MODEL IN THE PRESENCE OF NOISE

H. Hallez<sup>1,3</sup>, P. Van Hese<sup>1,3</sup>, B. Vanrumste<sup>2</sup>, P. Boon<sup>3</sup>, Y. D'Asseler<sup>1</sup>,  
I. Lemahieu<sup>1</sup> and R. Van de Walle<sup>1</sup>

<sup>1</sup> Medical Image and Signal Processing (MEDISIP), Department of Electronics and Information Systems  
(ELIS-MEDISIP), Ghent University, Ghent, Belgium

<sup>2</sup> Department of Electrical Engineering (ESAT-SCD), KU Leuven, Leuven, Belgium

<sup>3</sup> Laboratory for clinical and experimental neurophysiology, Department of Neurology, Ghent University  
Hospital, Ghent, Belgium

Hans.Hallez@UGent.be

**Abstract:** EEG source localization has proven to be a valuable tool in the presurgical evaluation of patients with epilepsy. However, the typical volume conductor models used for this purpose do not take into account the anisotropic properties of the skull and white matter in the brain. We investigated whether incorporating anisotropy in spherical head models is worthwhile taking into account the effect of additive Gaussian noise to the EEG. We compared the anisotropic head model with an isotropic head model in the presence of noise by means of the point of gravity of the dipole location estimates and the radius of the 68%-sphere of the dipole estimates. We can conclude that in order to have an accurate estimation of focal brain activity in spherical head models anisotropy has to be incorporated. Furthermore, we observed that a little noise has a big effect on the location of the dipole estimate. Noise suppression techniques have to be used to lower the noise level and make the dipole estimate more accurate.

### Introduction

When several neurons are active in a relatively small volume of the cortex, the resulting synchronous electrical activity can be represented by an equivalent current dipole with a certain location, orientation and magnitude. Furthermore, when the neurons depolarize and repolarize synchronously, potential differences will be registered between scalp electrodes. Starting from these potential differences one can estimate the source in a head model. This is called EEG source localisation. The current dipole consists of a current source and a current sink, with opposite current strength, located infinitesimally close to each other. The current dipole has three position parameters and three orientation parameters, called components.

In EEG dipole source localization two problems need to be solved: a forward problem and an inverse problem. The forward problem consists of calculating the potential difference between the electrodes for a given dipole. The mathematical formulation of the forward problem is a Poisson differential equation [1]. On the other hand, the inverse problem consists of finding the dipole which best represents the given potentials at the scalp electrodes.

It is known that the skull and white matter in the brain are anisotropic, which means that the electrical conductivity of these tissues is direction dependent (see figure 1). However, the head models commonly used assume isotropic conductors for these tissues. The skull is known to have an anisotropic conductivity with a ratio of up to 1:10 (radially:tangentially to the skull surface). The ratio of the anisotropic conductivity of white matter is 1:9 (transverse:longitudinal along the nerve fibre).

It was shown in [2] that neglecting skull anisotropy causes a dipole localization error of on average 20.16 mm for cortical test dipoles and 4.14 mm for test dipoles in the thalamic shell (representing deeper grey matter areas). Neglecting only white matter anisotropy causes an error of on average 4.23 mm for cortical dipoles and 17.05 mm for test dipoles in the thalamic shell. Neglecting both white matter anisotropy and skull anisotropy causes a dipole localisation error of 22.54 mm in the cortical region, while in the thalamic shell the error is on average 7.62 mm. Hence, the error caused by not taking into account the anisotropic conductivity is not negligible.

Dipole position errors also occur due to noise. In reality, noise is superposed to the focal EEG. Typical noise contributors in the EEG are environmental noise, instrumental noise and biological noise. We will focus on biological noise. It can be seen as a biological activity that is uncorrelated with focal brain activity. Some typical contributors are the heart, muscle and eye-movement artefacts. Also background EEG, generated by other brain areas, contributes to biological noise. We modelled this background EEG activity by additive Gaussian noise. We assumed in our simulations that the dipole localization error is only due to the presence of noise (unbiased estimator). This is in contrast dipole localization in the isotropic head model, which we assume to be a biased estimator: dipole position errors are due to noise but also due to the usage of isotropic conducting skull compartment and white matter compartment [3].

In the present study noise values are added to the electrode potentials generated with the anisotropic

model. The question that we want to answer is: is it worth to use spherical head models with anisotropic conducting compartments, given the noise level of the EEG-signal? The dipole location error in the presence of noise is compared for the isotropic spherical head model and the anisotropic spherical head model with (a) only the skull shell as anisotropic conducting compartment, (b) only the white matter shell as anisotropic conducting compartment and (c) both the skull shell and white matter shell as an anisotropic conducting compartment.

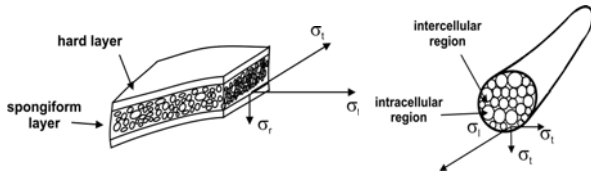


Figure 1: The anisotropic conductivities of skull (left) and white matter (right).

## Materials and Methods

### A. Head model

Spherical head models have the advantage that an analytical expression for the forward problem exists. Here we use a 5 shell spherical head model (see figure 2) with outer radius of the scalp compartment at 92 mm (eccentricity 1), the skull compartment at 86 mm (eccentricity 0,93), the cortical compartment at 70 mm (eccentricity 0,87), the white matter compartment at 70 mm (eccentricity 0,76) and the thalamic compartment at 20 mm (eccentricity 0,22). The scalp shell, cortical shell and thalamic compartment have an isotropic conductivity of 0.33 S/m. The skull and white matter compartment can be made isotropic or anisotropic. The isotropic conductivity is 0.020 S/m and 0.33 S/m, respectively. The anisotropic conductivity of the skull compartment is 0.043 S/m in the radial direction and 0.43 S/m in the tangential direction to the skull surface. The white matter has been modelled as follows: the nerve fibres start from the thalamic compartment and go in the radial direction to the cortical compartment. In this way the white matter compartment also has a radial and a tangential conductivity of 1.42 S/m and 0.15 S/m, respectively. The values for the radial and tangential direction can be calculated from the isotropic conductivity according to the volume constraint [4].

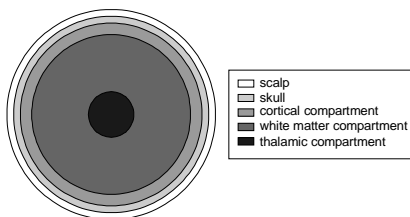


Figure 2: A 2D representation of the 5 shell spherical head model. The different compartments are shown as a different greyscale.

The electrodes were placed according to the international 10-20 system, with 6 extra electrodes located at the temporal region, resulting in a total of 27 electrodes. The placement of the electrodes is also shown in figure 3.

### B. Forward Problem

The forward problem calculates the electrode potentials at the scalp electrodes, given a dipole location and orientation and a head model. In this problem, quasi-static Maxwell equations can be used, because the time delays between source and measurements are

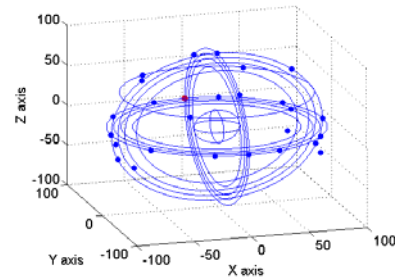


Figure 3: the 5 shell spherical head model and the electrode placement.

supposed to be negligible [3]. Therefore the potentials at the electrodes reflect the instantaneous parameters of the dipole. The potential field can be described by Poisson's equation:

$$\nabla \cdot (\bar{\sigma} \cdot \nabla V) = \nabla \cdot \mathbf{J}, \quad (1)$$

where  $V$  is the potential field distribution in the head model.  $\mathbf{J}$  represents the current density of the source.  $\bar{\sigma}$  is the matrix representation of the conductivity tensor at a specific location. The Poisson equation is accompanied by a Dirichlet and Neumann boundary condition at each interface between two shells.

The electric potentials at each electrode are calculated as follows: due to the 5 shell spherical head model, the potentials can be calculated by an analytical formula. The analytical formula uses an infinite sum of Legendre terms. It was presented by de Munck et al. in [5]. This formula leaves room for the incorporation of anisotropic conductivities, but limited to radial and tangential direction of the conductivity only.

### C. Inverse Problem

Solving the inverse problem consists of finding the parameters of the dipole source that best explain a set of measured potentials. We find the optimal dipole position  $\mathbf{r}_{opt}$  and components  $\mathbf{d}_{opt}$  for the input potentials  $\mathbf{V}_{in} \in \mathfrak{R}^{k \times 1}$  at  $k$  scalp electrodes. This is done by minimizing the relative residual energy (RRE)[6]:

$$RRE = \frac{\|\mathbf{V}_{in} - \mathbf{V}_{model}(\mathbf{r}, \mathbf{d})\|_2^2}{\|\mathbf{V}_{in}\|_2^2} + C(\mathbf{r}), \quad (2)$$

where  $\mathbf{V}_{\text{model}} \in \mathfrak{R}^{k \times 1}$  are the average referenced potentials obtained from the forward evaluation in the inverse problem.  $\|\cdot\|$  indicates the  $L_2$ -norm.  $C(\mathbf{r})$  is zero for dipole positions in the brain compartment (cortical shell, white matter shell and thalamic shell) and is set to a high value elsewhere. This additional term will restrict the solution of the inverse solver to the brain compartment. The Nelder-Mead simplex method is used to find the global minimum of the RRE, because of its simplicity and robustness against local minima [7].

### C. Noise

The noise values were Gaussian distributed with zero mean and standard deviation  $\sigma$ . We assume that the Gaussian noise is not correlated between the different scalp electrodes. Furthermore the noise values at successive time samples are also not correlated.

An important measure is the noise level. It gives us an idea of the amount of noise that occurs in the EEG signal. The noise level ( $nl$ ) reads:

$$nl = \frac{\sigma}{V_{RMS}}, \quad (1)$$

where  $V_{RMS}$  denotes the root-mean-square (RMS) value for the average referenced.

For spikes the noise level typically equals 0.2. The noise level for averaged spikes is typically 0.1.

### D. Simulation Setup

In the aforementioned head model we placed 9 test dipoles: one in the centre (eccentricity 0), four in the thalamic grey matter (eccentricities 0.05, 0.1, 0.15 and 0.2) and four in the cortical grey matter (eccentricities 0.75, 0.775, 0.8 and 0.825). All simulations were done according to the flowchart of figure 4. Below we focus on the different simulation set-ups.

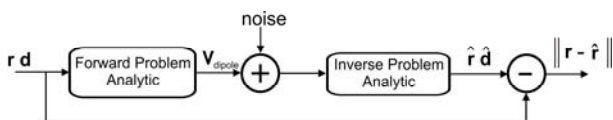


Figure 4: The flowchart of the simulations.

**Building a reference.** First we considered isotropic head models in order to have a reference for comparison. For each dipole the forward problem was solved in an isotropic head model. Then noise instances were generated and added to the obtained electrode potential. The noise level varied from 0 (no noise) to 0.5 in steps of 0.1. For each dipole and for each noise level the number of instances was 500, yielding 500 sets of electrode potentials superimposed with noise.

The noisy electrode potentials were then used to solve the inverse problem in an isotropic head model, yielding 500 dipole location estimates per test dipole.

**Omitting anisotropic conducting compartments.** Next, we considered simulations when anisotropic conducting compartments were assumed to be isotropic

in the presence of noise. For each dipole we calculated the forward problem in an anisotropic head model with the skull and white matter compartment being anisotropic.

Then noise instances were generated and added to the obtained electrode potential for each dipole. The noise level was again varied from 0 (no noise) to 0.5 in steps of 0.1. For each dipole and for each noise level the total number of noise added electrode potentials was 500.

The noisy electrode potentials were then used to solve the inverse problem in the head model with the anisotropic compartment, used in the forward calculations, set to an isotropic compartment. This yielded 500 dipole estimates for each test dipole.

### Dipole localization errors in anisotropic head models in the presence of noise.

Here we wanted to investigate the dipole localization error in anisotropic head models in the presence of noise. For each dipole the forward problem was solved in an isotropic head model. Then noise instances were generated and added to the obtained electrode potential. The noise level varied from 0 (no noise) to 0.5 in steps of 0.1. For each dipole and for each noise level the number of instances was 500, yielding 500 sets of electrode potentials superimposed with noise.

The noisy electrode potentials were then used to solve the inverse problem in an anisotropic head model, yielding 500 dipole location estimates per test dipole.

For simulations applying the unbiased model the forward and inverse problem were solved utilizing the same volume conductor model. For the simulations carried out with the biased model, the forward calculations were performed with an anisotropic compartment, while the inverse problem was solved by assuming that compartment isotropic.

**Error Measures.** To evaluate the fitted dipoles a set of error measures are needed. First we define the point of gravity (see figure 4) as the sum of  $x$ ,  $y$  and  $z$  coordinates of the estimates divided by the number of estimates:

$$p_x = \frac{1}{500} \sum_{i=1}^{500} x_i \quad (2)$$

where  $x_i$  is the  $x$ -coordinate of the  $i$ -th estimate and  $p_x$  is the  $x$ -coordinate of the point of gravity. The same holds for the  $y$  and  $z$  coordinates for the point of gravity. We can calculate the Euclidian distance from that point of gravity  $\mathbf{p}$  to the original test dipole  $\mathbf{d}_{\text{test}}$ . We will note this as  $DPG$ . This is schematically represented in figure 5.

A second error measure is the 68% sphere. It roughly can be seen as a standard deviation of the estimates. It indicates that 68 percent of the estimates are within the boundary of the sphere (see figure 4). The sphere is specified by the point of gravity and the radius  $r$ :

$$r^2 = \frac{1}{500} \sum_{i=1}^{500} \|\mathbf{r}_i^{\text{estim}} - \mathbf{p}\|^2 \quad (3)$$

where  $\mathbf{p}$  is the vector denoting the point of gravity and  $\mathbf{r}_i^{\text{estim}}$  is the vector denoting the  $i$ -th estimate of the dipole location.  $\|\cdot\|$  is the L2-norm.

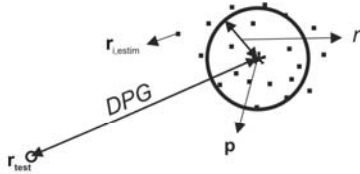


Figure 5: this figure shows the error measures and the dipole estimates  $\mathbf{d}_{i,\text{estim}}$  ( $\cdot$ ). The  $DPG$  is the Euclidian distance between the test dipole  $\mathbf{d}_{\text{test}}$  ( $\circ$ ) and the point of gravity  $\mathbf{p}$  ( $*$ ). The radius  $r$  is given if formula 3.

## Results

### A. Dipole localization errors in isotropic head models in the presence of noise

Figure 6 shows an example of the dipole estimates. In a noiseless case the mean distance from the test dipole to the point of gravity of the estimates is zero as the models for solving the forward and inverse problem are equal. For thalamic dipoles the mean distance was below 1 mm, while for cortical dipoles an error up to 3.28 mm exists for noise levels up to 0.5. For all test dipoles the radius of the 68% sphere rises logarithmically from 0 till 3.5 mm.

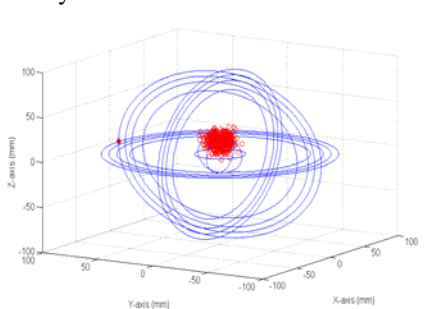


Figure 6: the dipole estimates plotted in a isotropic spherical head model in the presence of noise. The test dipole was at eccentricity 0.15 and the noise level was 0.2. The estimates show a sphere around the test dipole.

### B. Dipole localization errors when omitting anisotropic conducting compartments in the presence of noise

Figure 7 shows the mean DPG in function of the noise level when only the skull compartment was set isotropic when solving the inverse problem while all other compartments remained equal (case B-skull). The results show the average localization error for the test dipoles located in each region. We have also added results for a dipole in the center of the spherical model. We can see that for deep dipoles and the center the DPG remains constant. For cortical sources the DPG diminishes with increasing noise level, indicating that the point of gravity closes in on the original test dipole.

Figure 8 shows the mean DPG for each noise level when only the white matter compartment is set isotropic when solving the inverse problem (case B-white). Also, here the results are shown as graphs indicating the mean DPG in the center, the thalamic region and cortical region. Also in this case the DPG is

almost constant for thalamic and center sources. For cortical dipoles, the DPG increases and becomes constant at noise level 0.3.

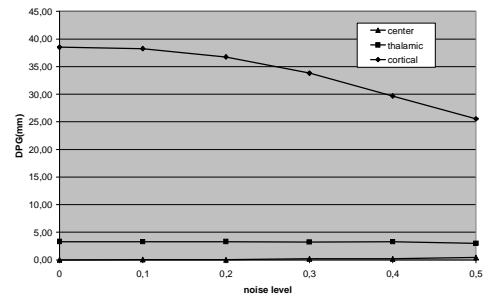


Figure 7: the mean distance from the testdipoles to the point of gravity in function of the noise level when the skull compartment is assumed isotropic (case B-skull). The values obtained from the 4 cortical dipoles are averaged into one value to obtain a clearer representation. The same holds for thalamic dipoles. The center dipole is shown apart (eccentricity 0).

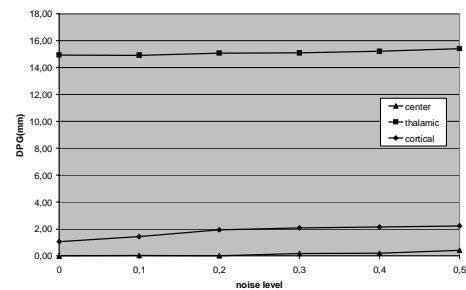


Figure 8: The mean distance from the testdipoles to the point of gravity in function of the noise level when the white matter compartment is assumed isotropic (case B-white). The 4 thalamic dipoles and 4 cortical dipoles are averaged. The center dipole is shown apart.

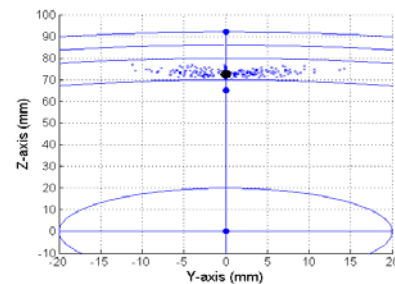


Figure 9: Dipole estimates from a test dipole located in the cortical region (eccentricity 0.775, z-coordinate 71,3 mm) when only white matter anisotropy was assumed isotropic (case B-white). The noise level was set to 0.1. The center of the big black dot denotes the original dipole location.

Figure 9 shows some dipole estimates when the test dipole is located in the cortical region when the white matter compartment is set to isotropic. The estimates have a disk-like shape instead of a spherical shape in the case of section A.

Figure 10 shows the mean DPG as a function of the noise level when both white matter and skull compartment is set isotropic when solving the inverse problem (case B-skull-white). The results are shown in the same manner as figure 7 and 8. Figure 11 displays the estimates plotted in the 5 shell spherical head model

in this case. The red star denotes the original dipole location (eccentricity 0.775).

In all cases the 68% sphere had the same values as in the simulations with the isotropic head model in the presence of noise. The changes regarding the reference results were not significant. Therefore these results are not shown.

*C. Dipole localization errors in anisotropic head models near the edge of an anisotropic conducting compartment with an isotropic conducting compartment in the presence of noise*

Figure 12 shows the DPG of the test dipoles with eccentricity 0.15 and 0.2 when applying an unbiased estimator (i.e. the models used to solve the forward and inverse calculations are equal and contain anisotropic compartments). The DPG from the other test dipoles were in the same order as the error in isotropic head models. Note that for noise level equal to zero the DPG measure is also zero. We want to focus the two mentioned eccentricities. The test dipoles with the above mentioned eccentricities are in the very near proximity of an edge between an anisotropic conducting compartment and an isotropic conducting compartment.

The 68% sphere was in all cases in the same order as with isotropic head models.

Figure 13 shows the dipole estimates in the 5 shell anisotropic spherical head model in the presence of noise. The figure was zoomed in for clarity puposes.

**Discussion**

For isotropic head models (case A) we notice that the dipole estimates are distributed symmetrically around the test dipole and ideally, the point of gravity of the fitted dipoles and the test dipole position coincide. By solving the inverse problem, the dipoles are restricted to the brain compartments (i.e., thalamic, white matter and cortical grey matter compartment). Hence, the symmetrical distribution is lost for fitted dipoles in the cortical compartment close to the skull boundary.

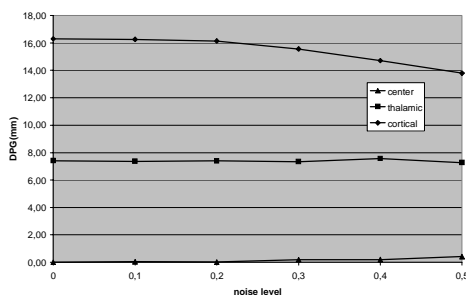


Figure 10: the mean distance from the testdipoles to the point of gravity in function of the noise level when both white matter and skull compartement is assumed isotropic in the presence of noise (case B-skull-white). The 4 thalamic dipoles and 4 cortical dipoles are averaged. The center dipole is shown apart.

When applying isotropic instead of anisotropic compartments to solve the inverse problem (cases B-skull, B-white and B-skull-white), and when no noise is

added to the electrode potentials of the dipole, the localization error is merely due to wrongly assuming skull and white matter to be isotropic conducting compartments. The errors are comparable to the study made in [2].

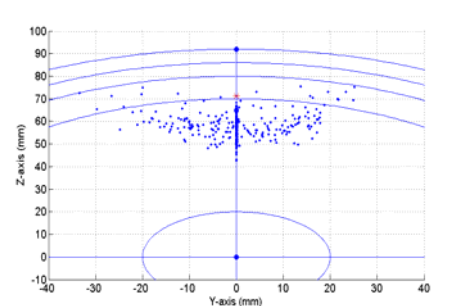


Figure 11: The estimates plotted in the 5 shell spherical head model when both skull and white matter compartements are assumed isotropic in the presence of noise (case B-skull-white). The red star indicates the original test dipole location at eccentricity 0.775. The noise level was 0.3.

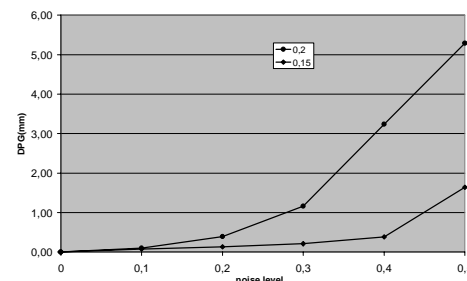


Figure 12: the DPG in function of the noise level in anisotropic head models in the presence of noise of testdipoles with excentricity 0.15 and 0.2.

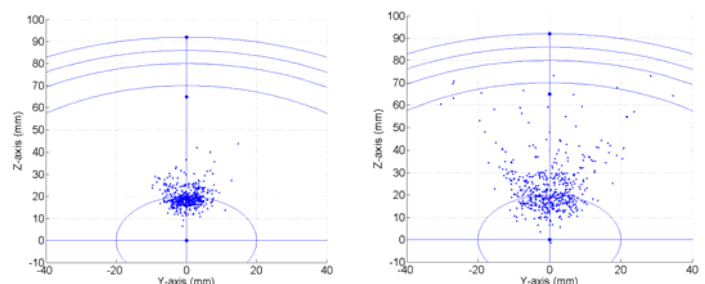


Figure 13: A plot of the dipole estimates projected on the YZ plane. The test dipole was placed at excentricity 0.2 (coordinate (0,0,18.4)) and the noise level was 0.3 (left) and 0.5 (right). The figure was zoomed in for clarity puposes.

For the case B-white, the thalamic dipoles and center dipoles are not much affected by the noise, as illustrated by the constant curve in figure 8. Although, Thalamic dipoles have a big error due to assuming the white matter compartment to be isotropic, an increasing noise level does not affect the constant curve.

For the cortical test dipoles we can see from figure 9 that the estimates are not sphere-shaped anymore, but disc-shaped. This explains the slope at low noise levels in the graph denoting the mean DPG of the cortical dipoles (figure 8). After noise level 0.3, many dipoles are estimated along the brain boundary, due to the

search space restriction. The DPG does not change when the noise level is larger.

For case C, the location of the dipole estimate is shifted away from the original test dipole location as illustrated in figure 12. Due to the anisotropic compartments, small additive values have already large effect on the location of the dipole estimate, resulting in considerable dipole localization errors.

With this observation we can also explain the results in figure 7 and 10. In these figure we can see a drop in the mean DPG for cortical dipoles. Because of the omission anisotropic conductivities of the skull compartment the dipoles are estimated toward the centre [2]. Because of the noise, the dipoles are estimated more outwards in the direction of a cone-shape (see figure 11). This results in figure 9: the dipoles are estimated outwards, bringing them on a closer distance to the original test dipole.

The results from the 68%-sphere were the same in all simulation and have a maximum of 3.5 mm at noise level 0.5. This value is much lower than the mean DPG when skull and white matter are assumed to be isotropic conducting compartments.

This indicates the distance between the point of gravity and the original test dipole location is much larger than the distance between the individual dipole estimates and the point of gravity.

We have to remark that using the error measures we assumed, the locations of the estimates of the dipoles are Gaussian distributed. However, figure 13 shows that, because of the anisotropy, the estimates of the dipoles cannot be modelled by a 3D Gaussian distribution.

We are aware that the anisotropic spherical head model is an over-simplification of the human head. However, this work was intended as an indicative study. Further research needs to be done in realistic head models.

## Conclusions

We can conclude that perturbations of the electrode potentials caused by Gaussian noise, have a considerable influence on dipole location estimation. Incorporation of anisotropy causes an increase in the sensitivity to noise. This effect is prominent for dipoles located near an anisotropic compartment. This indicates that the estimate of a dipole location near an edge of an isotropic and anisotropic compartment is not reliable. By means of noise suppression techniques the noise level can be made lower, making the dipole localization estimation more accurate.

Furthermore, the mean dipole localization error - of omitting an anisotropic conducting compartment in the presence of noise - is bigger than the mean dipole localization error in an anisotropic head model in the presence of noise, suggesting that the noise has less influence than omitting an anisotropic conducting compartment. The distance between the point of gravity and the original test dipole location is much bigger than

the distance between the individual dipole estimates and the point of gravity. However, it is not entirely clear whether the incorporation of anisotropy is beneficial. Due to the increased sensitivity to noise it may occur that the systematical error due to not incorporating anisotropy, is surpassed.

## Acknowledgments

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## References

- [1] SARVAS J., "Basic mathematical and electromagnetic concepts of the biomagnetic inverse problem", *Physics in Medicine and Biology*, vol. 32, nr. 1, pp. 11-22, 1987
- [2] HALLEZ H., VAN HESE P., VANRUMSTE B., BOON P., D'ASSELER Y., LEMAHIEU I., VAN DE WALLE R., "Dipole Localization Error due to not Incorporating Compartments with Anisotropic Conductivities: Simulation Study in a Spherical Head Model", *Proceedings of the joint 5<sup>th</sup> Int. Conference on Bioelectromagnetism and 5<sup>th</sup> Int. Symposium on Non-invasive Functional Source Imaging, 2005, in press.*
- [3] VANRUMSTE B., "EEG dipole source analysis in a realistic head model", Ph. D. dissertation, Ghent University, Ghent, Belgium, 2001.
- [4] WOLTERS C. H., "Influence of Tissue Conductivity Inhomogeneity and Anisotropy on EEG/MEG based Source Localisation in the Human Brain", Ph. D. dissertation, University of Leipzig, Leipzig, Germany, 2003.
- [5] DE MUNCK J.C., PETERS M.J., "A Fast Method to Compute the Potential in the Multisphere Model", *IEEE Transactions on Biomedical Engineering*, vol. 40, nr. 11, pp. 1166-1174, November 1993.
- [6] VANRUMSTE B., VAN HOEY G., VAN DE WALLE R., D'HAVÉ M., LEMAHIEU I., BOON P., "Dipole location errors in electroencephalogram source analysis due to volume conductor model errors", *Medical & Biological Engineering & Computing*, Vol. 38, pp. 528-534, 2000
- [7] PRESS W. H., TEUKOLSKY S. A., VETTERLING W.T., FLANNERY B.P., "Numerical Recipes in C", Cambridge, Cambridge University Press, 1995, pp. 430-443