Change detection in continuous tapping data

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Abstract: In this paper a novel algorithm is presented for computerized detection of changes in human motor responses. It is a sequential method based on the Maximum Likelihood approach. The method extends traditional approaches by considering changes with finite slope. This is a achieved by using a ramp-step model instead of a step model. The algorithm is successfully applied to position signals measured during tapping experiments.

Introduction

Tapping experiments are a common approach to investigate various effects of the human motor system [1]. In the traditional tapping setup the subject is sitting in front of a table tapping rhythmically with the index finger on a switch resulting in a periodic binary signal. In more sophisticated setups the movement of the index finger is monitored by an optical position sensor providing a continuous signal of the finger position. Thus the information decoded in the shape of the signal is preserved.

Figure 1 displays a section of the position signal comprising two subsequent tapping movements. The first tap reaches the table (indicated by the steady epoch from 100ms to 300ms). During the second tap the finger moves up again before the table is reached. Such extra taps sometimes occur in coordination experiments when the subject is asked to simultaneously perform concurrent motor tasks with both hands. These extra taps provide valuable information about the coordination strategies used by the human motor system but they would be masked when using a switch based tapping setup. On the other hand analysing the continuous signal demands a more sophisticated signal processing to locate the tapping movements.

Usually change detection methods assume a step-like change profile [2, 3, 4, 5]. This condition is not fulfilled in the present context which would result in biased change-point estimates. The signal is rather a series of changes with finite slope were the main interest of this paper is to estimate the starting point of each action. An action will be defined as a rapid movement of the finger either in upward (extension) or downward direction (flexion). An algorithm that regards finite slopes was presented by L. Charbonnier et al. [6]. They compute the slope in a heuristic post processing step. In contrast to this in this paper one action is modelled by a ramp-step function [7] depicted in Figure 2. It is a piecewise linear function composed of three straight lines that adjoin.

The ramp-step represents the simplest approximation to a signal comprising a change with finite slope. It is a model for a single action either a flexion or an extension movement.

In order to locate multiple tapping movements a sequential algorithm will be employed processing the signal from the beginning to the end. Furthermore the method is divided in two stages. The first stage is responsible to detect the next action. This is followed by a stage where the location of the detected action is estimated.

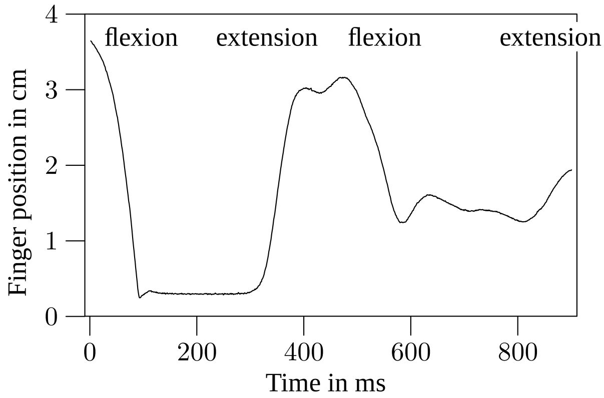

Figure 1: A typical tapping signal obtained in a coordination experiment. Note that the second flexion movement is interrupted before the finger has reached the table.

Method

A method is presented for detecting change-points in the time varying mean $u[n]$ of a signal $y[n]$ corrupted by additive white Gaussian noise $e[n]$. This means that each sample of $e[n]$ is Gaussian distributed with zero mean and variance σ^2 and is uncorrelated with all the other samples. The variance σ^2 is supposed to be constant.

$$y[n] = u[n] + e[n] \quad n = 1, 2, \ldots, N \tag{1}$$

The number of change-points and their position are unknown. A regression model is considered composed of K adjacent ramp-step functions.

A single ramp-step function appears in Figure 2. The function is defined on the interval $[a, b]$. The change-point k and the rise-time τ define the location and speed of a movement. Moreover the offset d and the magnitude h are introduced since neither the level before nor the level after the movement is known a priori.

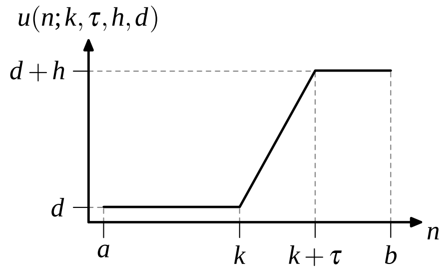

Figure 2: The ramp-step function which models a single action. The main interest is to estimate the change-point k.

The offset and magnitude are real valued where the change-point and rise-time are integer valued. The change-point is an element of the interval $[a, b-1]$ where the rise-time of the interval $[1, b-k]$ so that $k+\tau \leq b$. When $\tau = 1$ the ramp-step is equal to the step model and for $\tau = b-k$ it is equal to a pure ramp model.

The same pattern is used for modelling flexion ($h < 0$) and extension movements ($h > 0$).

The result of the method will be a list of intervals $[a^{(i)}, b^{(i)}]$ on which the ramp-step functions with parameters $k^{(i)}$, $\tau^{(i)}$, $h^{(i)}$ and $d^{(i)}$ are defined with $i = 1, 2, \ldots, K$; with the initial condition $a^{(1)} = 1$.

The algorithm starts with the first sample using a *Likelihood Ratio (LR)* test to detect the first action. Then a local estimate of the ramp-step parameters is done using the *Maximum Likelihood (ML)* method. These two steps are repeated until the end of the signal is reached. The aims of the two steps are different. Of great importance is a *reliable* detection that forms the basis of an *accurate* estimation.

In the remainder the different parts of the algorithm are explained in detail.

Detection of an action

The change detection algorithm used in this paper is a model based approach. These algorithms continuously monitors the difference of the change model with the condition of no change while scanning the signal. A change is detected when the difference is significant.

The detection algorithm used here is an approximated Generalised Likelihood Ratio (GLR) test [8] that uses a step model although, as stated above, a ramp-step is expected. Nevertheless, since the signal to noise ratio is high one can expect a reliable detection and the step model allows an efficient implementation.

The algorithm is a window based approach with L being the width of a sliding window. The ratio of the probability that a jump has happened at time $n-L$ to the no jump hypothesis is calculated and a change is detected if this ratio exceeds a threshold δ. In [8] the test statistic g_n is derived for the Gaussian case with

$$g_n = \frac{1}{2L} \left(\sum_{j=n-L}^{n} (y[j] - \mu_0) \right)^2 \tag{2}$$

where μ_0 is the mean before change. Since in the present context this mean is not known a priori it is replaced by its maximum likelihood estimate

$$\hat{\mu}_0 = \frac{1}{n-L-a^{(i)}} \sum_{j=a^{(i)}}^{n-L-1} y[j] \; . \tag{3}$$

The time instant n where $g_n > \delta$ serves as the upper bound of the interval $[a^{(i)}, b^{(i)}]$ that is used in the following estimation step.

Estimation of the ramp-step function

Once a change is detected the ML method is used to fit a ramp-step to the signal on the interval $[a^{(i)}, b^{(i)}]$. Introduce the vector $\boldsymbol{y} = (y[a^{(i)}], y[a^{(i)}+1], \ldots, y[b^{(i)}])^T$ and the vector $\boldsymbol{u} = (u[a^{(i)}], u[a^{(i)}+1], \ldots, u[b^{(i)}])^T$ with $u[n] = u(n;k,\tau,h,d)$ being the ramp-step function depicted in Figure 2. Moreover the parameters are collected in the vector $\boldsymbol{\theta} = (k, \tau, h, d)^T$ and the size of the interval $[a^{(i)}, b^{(i)}]$ is denoted by $c = b^{(i)} - a^{(i)} + 1$. Then the logarithm of the likelihood for $\boldsymbol{y}$ given $\boldsymbol{\theta}$ is equal to

$$\begin{aligned} \ln \mathrm{L}(\boldsymbol{y};\boldsymbol{\theta}) \\ &= \ln \left\{ \frac{1}{\left(\sqrt{2\pi}\sigma\right)^c} \exp\left[-\frac{1}{2\sigma^2} \sum_{j=1}^{c} (\boldsymbol{y}[j] - \boldsymbol{u}[j])^2 \right] \right\} \\ &= -c \ln\left\{\sqrt{2\pi}\sigma\right\} - \frac{1}{2\sigma^2} (\boldsymbol{y} - \boldsymbol{u})^T (\boldsymbol{y} - \boldsymbol{u}) \; . \end{aligned}$$

The estimate of the vector parameter $\boldsymbol{\theta}$ is the argument which maximizes the likelihood function.

$$\hat{\boldsymbol{\theta}} = \arg \max_{\boldsymbol{\theta}} \ln \mathrm{L}(\boldsymbol{y}; \boldsymbol{\theta}) \tag{4}$$

Since the $\arg\max_{\boldsymbol{\theta}}$-operator is invariant for summands and positive factors not depending on $\boldsymbol{\theta}$ Equation 4 can be written

$$\hat{\boldsymbol{\theta}} = \arg \max_{\boldsymbol{\theta}} \Lambda \tag{5}$$

with

$$\Lambda = -(\boldsymbol{y} - \boldsymbol{u})^T (\boldsymbol{y} - \boldsymbol{u}) \; . \tag{6}$$

The parameters h and d are real valued. Their optimum can be derived by setting the partial derivatives to zero. For that reason the vector $\boldsymbol{p}$ is introduced so that $\boldsymbol{u} = h\boldsymbol{p} + d$ holds; consequently $\boldsymbol{p}$ is a unit ramp-step with magnitude 1 and offset 0.

Now the maximisation of the likelihood is simplified by substituting h and d by their estimates. The maximisation can then be expressed in terms of $\boldsymbol{y}^*$ and $\boldsymbol{p}^*$ as follows

$$\max_{\boldsymbol{\theta}} \Lambda = \max_{\boldsymbol{\theta}_2} \left(\boldsymbol{y}^{*T} \boldsymbol{p}^* \right)^2 \tag{7}$$

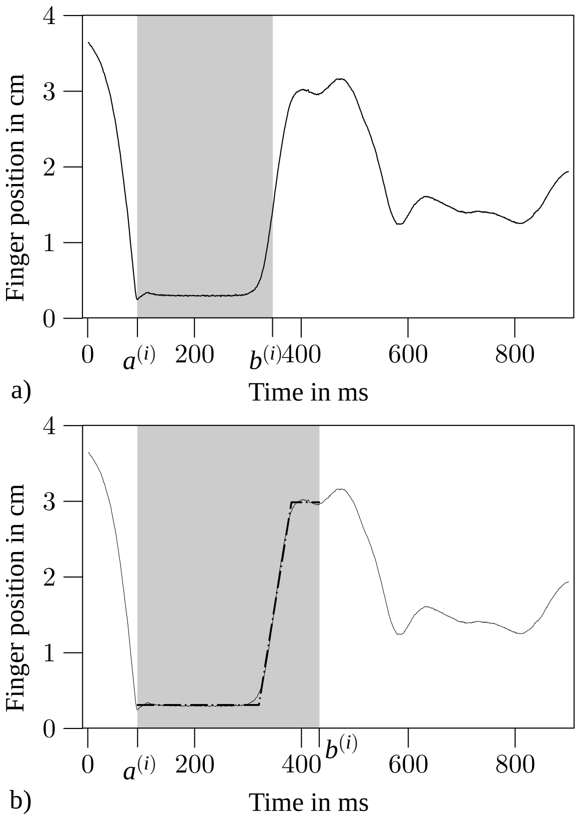

Figure 3: a) The end of the shaded epoch is the time where the action is detected. b) The complete movement is covered in the enlarged range $[a^{(i)}, b^{(i)}]$. The ramp-step function will be estimated correctly.

with

$$\boldsymbol{y}^* = \boldsymbol{y} - \bar{\boldsymbol{y}} \tag{8}$$

$$\boldsymbol{p}^* = \frac{\boldsymbol{p} - \bar{\boldsymbol{p}}}{\sqrt{(\boldsymbol{p} - \bar{\boldsymbol{p}})^T(\boldsymbol{p} - \bar{\boldsymbol{p}})}} \tag{9}$$

$$\boldsymbol{\theta}_2 = (k, \tau)^T . \tag{10}$$

The substitution $\boldsymbol{y}^*$ is the signal vector $\boldsymbol{y}$ minus its mean and $\boldsymbol{p}^*$ is a unit ramp-step with mean zero and norm one. In Appendix A a prove for Equation (7) is given.

Equation (7) simplifies the problem by reducing the number of parameters. The remaining parameters k and τ are integer valued. They are estimated by computing $\left(\boldsymbol{y}^{*T}\boldsymbol{p}^*\right)^2$ for each pair (k, τ) numerically and the estimate is the pair $(\hat{k}, \hat{\tau})$ that maximises $\left(\boldsymbol{y}^{*T}\boldsymbol{p}^*\right)^2$.

$$(\hat{k}, \hat{\tau}) = \max_{(k,\tau)} \left(\boldsymbol{y}^{*T}\boldsymbol{p}^*\right)^2 \tag{11}$$

If in addition the estimates of h and d are needed the equations in Appendix A can be used.

Recursive estimates

There is one aspect that has not been discussed yet. Since the signal to ratio is high, an action is usually detected while it is ongoing, which means the interval $[a^{(i)}, b^{(i)}]$ does not cover a whole action. Therefore the upper bound of the interval $[a^{(i)}, b^{(i)}]$ is increased recursively until it covers a whole movement.

The difference $\Delta s = b^{(i)} - (k^{(i)} + \tau^{(i)})$ serves as a decision criterion for this condition. Δs is the minimal value of the constant duration at the end of the movement. The recursion is performed by increasing $b^{(i)}$ and fitting the ramp-step on $[a^{(i)}, b^{(i)}]$ until Δs is greater than the positive integer Δs_{min}. This process is illustrated in Figure 3.

In Figure 3a the shaded epoch is the interval where the extension movement is detected. The window width $L = 40$ and the threshold $\delta = 0.001$ were used. As expected the action is detected while the movement is ongoing. Now the ramp-step function is estimated recursively on the growing window where the parameter $\Delta s_{min} = 50$ was used. The final interval $[a^{(i)}, b^{(i)}]$ is shown in Figure 3b. Furthermore the estimated ramp-step is depicted.

Initialise the next iteration

Finally the detection of the next change is initialised by setting $a^{(i+1)} = \hat{k}^{(i)} + \hat{\tau}^{(i)}$. Note that the adjacent ramp-steps overlap to improve the accuracy of the estimated mean $\hat{\mu}_0$ before change.

The complete algorithm is summarised in pseudo code in Figure 4.

```
a ← 1
while a < N do
    b ← detect action on [a, N]
    Δs ← estimate ramp-step on [a, b]
    while Δs < Δs_min do
        increase b
        Δs ← estimate ramp-step on [a, b]
    end
    update a
end
```

Figure 4: Sequential change detection with the ramp-step model

Results

The algorithm has three design parameters. The sliding window width L and the threshold δ for detection and for estimation there is just the minimal duration after change Δs_{min}. In Figure 5 the analysed signal of Figure 2 is depicted. Four actions are found and the corresponding ramp-steps are depicted (flexions with dashed lines and and extensions with dashed-dot lines). The overlapping of the ramp-steps can be seen especially between the first extension and the second flexion movement. The design parameters $L = 50$, $\delta = 0.001$ and $\Delta s_{min} = 50$ are used. The choice of the latter is done by a brief examination of the signal. It is set to the minimal expected

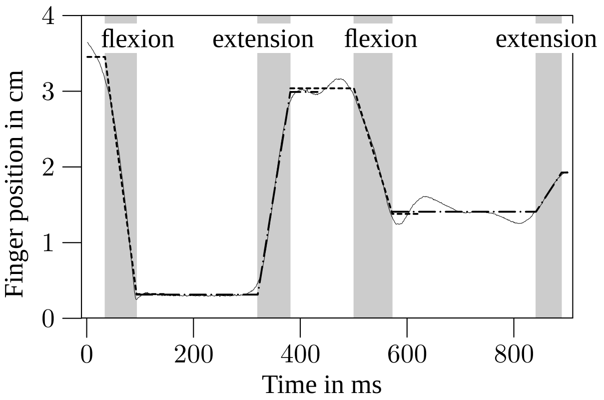

Figure 5: The analysed signal of Figure 1. The four found epochs are shaded. Furthermore the estimated ramp-step functions are depicted.

constant epoch between the movements, which is in general a good advice.

The signal in Figure 6a serves as a second example. The extension movement is prolonged by a short pause resulting in two sub-movements. The first analysis (Figure 6b is done with a sensitive detection. Two ramp-steps are fitted, one for each sub-movement. In the case of a large sliding window $L = 300$ the detection occurs during the second sub-movement. As a consequence one ramp-step is fitted to the extension movement.

When running the algorithm on larger datasets sub-movements can easily be found by checking the sign of the magnitude of subsequent ramp-steps.

Conclusions

Continuous tapping data has special demands on signal processing. A model based approach is presented for dividing a dataset in movements. It is shown by example that the presented algorithm based on a model of adjacent ramp-step functions is suitable for this task.

In future work the model will be improved so that it better represents the smooth nature of finger position data. This involves higher computational effort. That will follow a comparison of reliability, accuracy and computational costs of algorithms based on these two models.

A Proof of Equation (7)

In Equation (7) it is stated that the maximum of Λ can be expressed in terms of $\boldsymbol{y}^*$ and $\boldsymbol{p}^*$ as follows

$$\max_{\boldsymbol{\theta}} \Lambda = \max_{\boldsymbol{\theta}_2} \left(\boldsymbol{y}^{*T} \boldsymbol{p}^* \right)^2 \qquad (12)$$

with $\boldsymbol{u} = h\boldsymbol{p} + d$ and the parameter vectors $\boldsymbol{\theta} = (k, \tau, h, d)^T$ and $\boldsymbol{\theta}_2 = (k, \tau)^T$. In the following the proof is formulated as $\boldsymbol{p}$ being a unit ramp-step but $\boldsymbol{p}$ might in general be an arbitrary vector in $\mathbb{R}^c$.

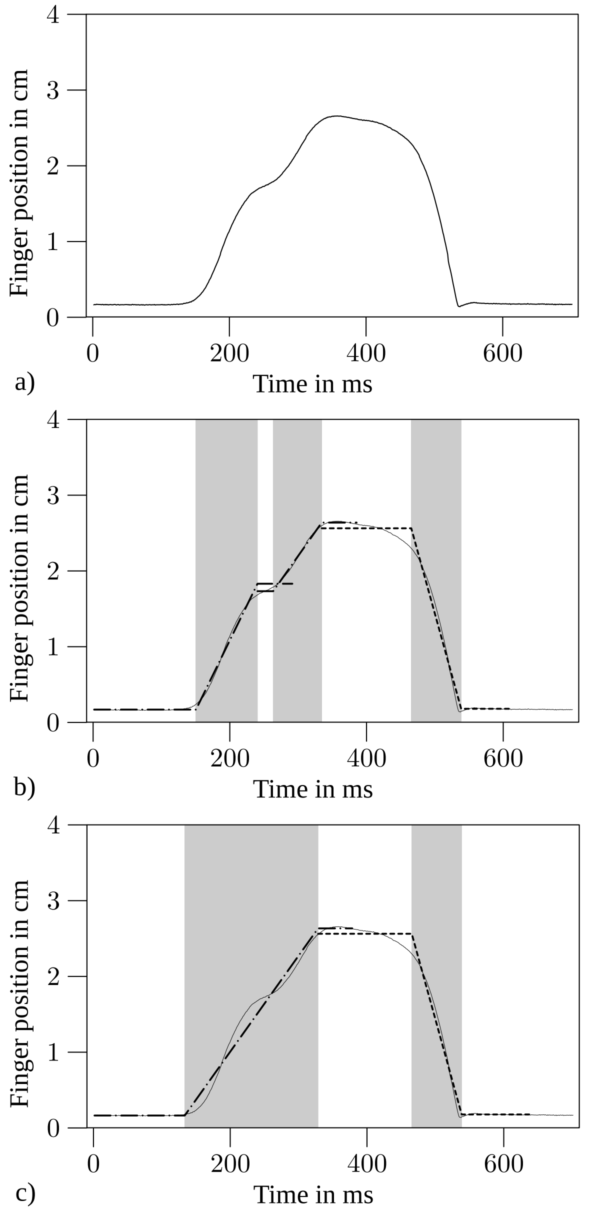

Figure 6: a) An extension movement comprising two sub-movements. b) Using the parameters $L = 50$, $\delta = 0.001$ and $\Delta s_{min} = 50$ the flexion is divided in two ramp-step functions. c) Using the parameters $L = 300$, $\delta = 0.001$ and $\Delta s_{min} = 50$ the sub movements are hidden because of the late detection.

It is a constructive proof starting with the definition of Λ. First the vector $\boldsymbol{u}$ is substituted by $h\boldsymbol{p} + d$.

$$\begin{aligned}
\Lambda &= -(\boldsymbol{y} - \boldsymbol{u})^T (\boldsymbol{y} - \boldsymbol{u}) \\
&= -\boldsymbol{y}^T\boldsymbol{y} + 2\boldsymbol{y}^T\boldsymbol{u} - \boldsymbol{u}^T\boldsymbol{u} \\
&= -\boldsymbol{y}^T\boldsymbol{y} + 2\boldsymbol{y}^T(h\boldsymbol{p} + d) - (h\boldsymbol{p} + d)^T(h\boldsymbol{p} + d) \\
&= -\boldsymbol{y}^T\boldsymbol{y} + 2h\boldsymbol{y}^T\boldsymbol{p} + 2cd\bar{\boldsymbol{y}} - h^2\boldsymbol{p}^T\boldsymbol{p} - 2cdh\bar{\boldsymbol{p}} - cd^2
\end{aligned} \qquad (13)$$

In the last term $\bar{\boldsymbol{y}}$ is a vector of length c which has equal elements. Each element is equal to the mean over the elements of $\boldsymbol{y}$. The vector $\bar{\boldsymbol{p}}$ is defined respectively.

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The maximisation of Λ in Equation (12) is divided in first maximising over h and d and secondly over the other parameters k and τ collected in the vector $\boldsymbol{\theta}_2 = (k,\tau)^T$. Since h and d are real valued, their optimum can be computed by setting the partial derivatives to zero.

From Equation (13) the partial derivatives with respect to h and to d are inferred to

$$\frac{\partial\Lambda}{\partial h} = 2\boldsymbol{y}^T\boldsymbol{p} - 2h\,\boldsymbol{p}^T\boldsymbol{p} - 2cd\,\bar{\boldsymbol{p}} \qquad (14)$$

$$\frac{\partial\Lambda}{\partial d} = 2c\bar{\boldsymbol{y}} - 2ch\,\bar{\boldsymbol{p}} - 2cd\,. \qquad (15)$$

The estimates of h and d are obtained by solving the following system of linear equations

$$\frac{\partial\Lambda}{\partial h} = 0$$

$$\frac{\partial\Lambda}{\partial d} = 0$$

and it follows

$$\hat{h} = \frac{\boldsymbol{y}^T\boldsymbol{p}}{\boldsymbol{p}^T\boldsymbol{p}} - c\hat{d}\,\frac{\bar{\boldsymbol{p}}}{\boldsymbol{p}^T\boldsymbol{p}} \qquad (16)$$

$$\hat{d} = \bar{\boldsymbol{y}} - \hat{h}\bar{\boldsymbol{p}}\,. \qquad (17)$$

Note that the estimate $\hat{h}$ depends on $\hat{d}$ and vice versa. This linear equation system can be solved with the final estimates in Equations (18) and (19).

$$\xrightarrow{(17)\,\text{in}\,(16)} \quad \hat{h} = \frac{\boldsymbol{y}^T\boldsymbol{p}}{\boldsymbol{p}^T\boldsymbol{p}} - c\left(\bar{\boldsymbol{y}} - \hat{h}\bar{\boldsymbol{p}}\right)\frac{\bar{\boldsymbol{p}}}{\boldsymbol{p}^T\boldsymbol{p}}$$

$$\Leftrightarrow \quad \hat{h}\,\boldsymbol{p}^T\boldsymbol{p} = \boldsymbol{y}^T\boldsymbol{p} - c\bar{\boldsymbol{y}}\,\bar{\boldsymbol{p}} + c\hat{h}\,\bar{\boldsymbol{p}}^2$$

$$\Leftrightarrow \quad \hat{h} = \frac{\boldsymbol{y}^T\boldsymbol{p} - c\bar{\boldsymbol{y}}\,\bar{\boldsymbol{p}}}{\boldsymbol{p}^T\boldsymbol{p} - c\bar{\boldsymbol{p}}^2} \qquad (18)$$

$$\xrightarrow{(18)\,\text{in}\,(17)} \quad \hat{d} = \bar{\boldsymbol{y}} - \frac{\boldsymbol{y}^T\boldsymbol{p} - c\bar{\boldsymbol{y}}\,\bar{\boldsymbol{p}}}{\boldsymbol{p}^T\boldsymbol{p} - c\bar{\boldsymbol{p}}^2}\,\bar{\boldsymbol{p}}$$

$$\Leftrightarrow \quad \hat{d} = \frac{\bar{\boldsymbol{y}}\,\boldsymbol{p}^T\boldsymbol{p} - c\bar{\boldsymbol{y}}\,\bar{\boldsymbol{p}}^2 - \bar{\boldsymbol{p}}\,\boldsymbol{y}^T\boldsymbol{p} + c\bar{\boldsymbol{y}}\,\bar{\boldsymbol{p}}^2}{\boldsymbol{p}^T\boldsymbol{p} - c\bar{\boldsymbol{p}}^2}$$

$$\Leftrightarrow \quad \hat{d} = \frac{\bar{\boldsymbol{y}}\,\boldsymbol{p}^T\boldsymbol{p} - \bar{\boldsymbol{p}}\,\boldsymbol{y}^T\boldsymbol{p}}{\boldsymbol{p}^T\boldsymbol{p} - c\bar{\boldsymbol{p}}^2} \qquad (19)$$

Now the parameters h and d can be replaced by their estimates in $\boldsymbol{u} = h\boldsymbol{p} + d$.

$$\boldsymbol{u} = \hat{h}\boldsymbol{p} + \hat{d} = \frac{\boldsymbol{y}^T\boldsymbol{p} - c\bar{\boldsymbol{y}}\,\bar{\boldsymbol{p}}}{\boldsymbol{p}^T\boldsymbol{p} - c\bar{\boldsymbol{p}}^2}\,\boldsymbol{p} + \frac{\bar{\boldsymbol{y}}\,\boldsymbol{p}^T\boldsymbol{p} - \bar{\boldsymbol{p}}\,\boldsymbol{y}^T\boldsymbol{p}}{\boldsymbol{p}^T\boldsymbol{p} - c\bar{\boldsymbol{p}}^2}$$

After factorisation, the terms $-c\bar{\boldsymbol{y}}\,\bar{\boldsymbol{p}}^2 + c\bar{\boldsymbol{y}}\,\bar{\boldsymbol{p}}^2$ are added to the numerator. Resorting leads then to the equations

$$\boldsymbol{u} = \frac{\boldsymbol{y}^T\boldsymbol{p}\,(\boldsymbol{p} - \bar{\boldsymbol{p}}) - c\bar{\boldsymbol{y}}\,\bar{\boldsymbol{p}}\,(\boldsymbol{p} - \bar{\boldsymbol{p}}) + \bar{\boldsymbol{y}}\,(\boldsymbol{p}^T\boldsymbol{p} - c\bar{\boldsymbol{y}}\,\bar{\boldsymbol{p}}^2)}{\boldsymbol{p}^T\boldsymbol{p} - c\bar{\boldsymbol{p}}^2} = \frac{(\boldsymbol{y}^T\boldsymbol{p} - c\bar{\boldsymbol{y}}\,\bar{\boldsymbol{p}})(\boldsymbol{p} - \bar{\boldsymbol{p}}) + \bar{\boldsymbol{y}}\,(\boldsymbol{p}^T\boldsymbol{p} - c\bar{\boldsymbol{y}}\,\bar{\boldsymbol{p}}^2)}{\boldsymbol{p}^T\boldsymbol{p} - c\bar{\boldsymbol{p}}^2}\,.$$

Finally this equation is split up in the following way.

$$\boldsymbol{u} = \frac{\boldsymbol{y}^T\boldsymbol{p} - c\bar{\boldsymbol{y}}\,\bar{\boldsymbol{p}}}{\sqrt{\boldsymbol{p}^T\boldsymbol{p} - c\bar{\boldsymbol{p}}^2}}\,\frac{\boldsymbol{p} - \bar{\boldsymbol{p}}}{\sqrt{\boldsymbol{p}^T\boldsymbol{p} - c\bar{\boldsymbol{p}}^2}} - \bar{\boldsymbol{y}} \qquad (20)$$

Comparing this equation with the definition of $\boldsymbol{y}^*$ and $\boldsymbol{p}^*$ in Equations (8) and (9), it turns out, that the first factor is equal to $\boldsymbol{y}^{*T}\boldsymbol{p}^*$ and the second factor is equal to $\boldsymbol{p}^*$. Hence

$$\boldsymbol{u} = \boldsymbol{p}^*\,\boldsymbol{y}^{*T}\boldsymbol{p}^* + \bar{\boldsymbol{y}}\,. \qquad (21)$$

The final step of this proof is to replace $\boldsymbol{u}$ in by $\boldsymbol{p}^*\,\boldsymbol{y}^{*T}\boldsymbol{p}^* + \bar{\boldsymbol{y}}$ in the definition of Λ.

$$\begin{aligned}\Lambda &= -(\boldsymbol{y} - \boldsymbol{u})^T(\boldsymbol{y} - \boldsymbol{u})\\ &= -\left(\boldsymbol{y} - \left(\boldsymbol{p}^*\,\boldsymbol{y}^{*T}\boldsymbol{p}^* + \bar{\boldsymbol{y}}\right)\right)^T\left(\boldsymbol{y} - \left(\boldsymbol{p}^*\,\boldsymbol{y}^{*T}\boldsymbol{p}^* - \bar{\boldsymbol{y}}\right)\right)\\ &= -\left(\boldsymbol{y}^* - \boldsymbol{p}^*\,\boldsymbol{y}^{*T}\boldsymbol{p}^*\right)^T\left(\boldsymbol{y}^* - \boldsymbol{p}^*\,\boldsymbol{y}^{*T}\boldsymbol{p}^*\right)\\ &= -\boldsymbol{y}^{*T}\boldsymbol{y}^* + 2\boldsymbol{y}^{*T}\boldsymbol{p}^*\;\boldsymbol{y}^{*T}\boldsymbol{p}^* - \left(\boldsymbol{y}^{*T}\boldsymbol{y}^*\right)^2\\ &= -\boldsymbol{y}^{*T}\boldsymbol{y}^* + \left(\boldsymbol{y}^{*T}\boldsymbol{p}^*\right)^2\end{aligned}$$

Since the term $-\boldsymbol{y}^{*T}\boldsymbol{y}^*$ does not depend on any parameter it can be cancelled when the maximisation over Λ is performed. For this reason Equation (7) is proven to be correct.

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