ON THE USE OF THE CONSTANT PHASE TERM IN THE EEG-EMG TIME DELAY ESTIMATION

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Abstract:
This paper discusses the treatment of the constant phase term that is used to fit the phase of a cross-spectrum in the estimation of the time delay between the record of electrical activity of a brain, the electroencephalogram (EEG), and the record of electrical activity of a contracted muscle, the electromyogram (EMG). Commonly, when the EEG-EMG time delay is estimated as a slope of the phase of an EEG-EMG cross-spectrum the phase spectrum is fitted with a line given by a slope and an additive constant phase term. We point out that if analyzed signals are short the permanent use of the constant phase term may be unwise. First, based on the analysis of EEG-EMG data sets (with the record length of 2.5 minutes), we show that the constant phase term is often not significantly different from zero. Second, we illustrate that the use of the constant phase term provides results with much greater variance than without it. As a consequence, we suggest to limit the use of the constant phase term only to the cases where it is showed to be significantly different from zero, and we provide a statistical test to indicate these instances.

Introduction

The examination of the time delay between EEG and EMG signals provides additional information to their coherence analysis. While the coherence analysis reveals that there is coupling between EEG and EMG signals, the estimation of their mutual time delays sheds more light on the mechanism of this relation. In particular, the hitherto reported time delays suggest that EEG record precede EMG, which led to the conclusion that it is actually a cortical drive of muscles that the EEG-EMG coherence represents.

When choosing a time delay estimator (TDE) we have to ensure that it can sufficiently cover the relationship between examined signals (for instance it may have to deal with different time delays at different frequencies). If a TDE uses a too simple model to represent the relationship between analyzed signals, its results will be most probably biased. This bias can be reduced only by using a TDE with a model complex enough to fit the relation between signals. However, it is not necessarily true that the overall precision of the results increases with the complexity of a TDE. In fact, a more complex estimator with more parameters to estimate will provide results with greater variance. Or, in other words, with increasing complexity of a TDE, more data is needed to reach a given variance of the results. Therefore, there is a trade-off between the reduction of a bias (by increasing the complexity of the TDE) and the increase of the variance of the results. In this paper we show that for short EEG-EMG data records it may be worth reducing the complexity of the time delay estimator to obtain results with reasonable variance.

From the plethora of TDEs, the analysis of the time delay between EEG and EMG has mostly been based on a cross-correlation [1, 2, 3, 4, 5, 6, 7][1] and the phase of a cross-spectrum [2, 3, 4, 5, 8, 9, 10, 11][2]. There also have been a few attempts to use a more complex nonlinear TDE [12], but due to the complexity of the issue, we will limit ourselves only to the methods exploiting the second order statistics. In particular, we will concentrate on the time delay obtainable from the EEG-EMG cross-spectrum.

The phase of EEG-EMG cross-spectrum is said to be given by two factors [10] - a slope of the spectrum and an additive constant phase term. The former corresponds to the time delay while the latter arises when the relation between EEG and EMG cannot be described by a simple time delay model

$$u[n] = s[n] + \eta_1[n]\,,$$
$$v[n] = s[n-D] + \eta_2[n]\,, \qquad (1)$$

where $u[n]$ and $v[n]$ are the measured signals composed of a signal $s[n]$, its time delay version $s[n-D]$ and mutually uncorrelated noise $\eta_1[n]$ and $\eta_2[n]$. A complexity we would like to address is the additional constant phase term. We of course do not expect that EEG and EMG signals are merely time shifted signals with some noise. As signals in neural networks, EEG and EMG can be expected to have rather complex, most probably non-linear relationship. However, we believe that if the amount of

[1]So called back-averaging of EEG with respect to EMG, also used in the EEG-EMG time delay estimation, is basically a cross-correlation method, where EMG signal is represented by a series of unity pulses at the positions of EMG onsets

[2]Some of these works actually deal with MEG-EMG time delays (and coherence). MEG stands for the magnetoencephalogram - the record of magnetic activity of a brain. Methods of the MEG-EMG time delay estimation are essentially equivalent to those used for EEG-EMG records.

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recorded data is limited, it may be valuable to constrain our model of the EEG-EMG relationship to a simpler one. In fact, we show that if EEG-EMG relationship is modelled as a simple delay with additive noise (1), the obtained results may surpass those provided by more complex methods.

Theoretical Background

To allow a solid discussion about methods used in the EEG-EMG time delay estimation, we first need to outline the principle of several TDEs and the spectral characteristics these TDEs are based on.

Definition and Estimation of Spectral Characteristics

The power and cross spectral densities of signals $u[n]$ and $v[n]$ are defined as [13]

$$S_{\mathrm{u}}(\Omega) = E\left[|U(\Omega)|^2\right], \quad S_{\mathrm{v}}(\Omega) = E\left[|V(\Omega)|^2\right],$$
$$S_{\mathrm{uv}}(\Omega) = E\left[U(\Omega)V^*(\Omega)\right], \tag{2}$$

where $U(\Omega)$ and $V(\Omega)$ are Fourier transforms of $u[n]$ and $v[n]$.

The magnitude squared function (MSC) is defined as [13]

$$|\gamma(\Omega)|^2 = \frac{|S_{\mathrm{uv}}(\Omega)|^2}{S_{\mathrm{u}}(\Omega)\cdot S_{\mathrm{v}}(\Omega)}. \tag{3}$$

To estimate these spectral characteristics we will use the modified Welch method [13]. Signals $u[n]$ and $v[n]$, N samples long, are first segmented into L segments $u_l[n]$ and $v_l[n]$, which are M samples long. The individual segments are then weighted with a window $w[n]$, and their Fourier transforms are computed

$$U_l(\Omega) = \mathscr{F}\{w[n]u_l[n]\} \quad V_l(\Omega) = \mathscr{F}\{w[n]v_l[n]\}. \tag{4}$$

Next, $\widehat{S}_{\mathrm{u}}(\Omega)$, $\widehat{S}_{\mathrm{v}}(\Omega)$ and $\widehat{S}_{\mathrm{uv}}(\Omega)$ are estimated as

$$\widehat{S}_{\mathrm{u}}(\Omega) = \frac{1}{L}\sum_{l=1}^{L}|U_l(\Omega)|^2, \quad \widehat{S}_{\mathrm{v}}(\Omega) = \frac{1}{L}\sum_{l=1}^{L}|V_l(\Omega)|^2,$$
$$\widehat{S}_{\mathrm{uv}}(\Omega) = \frac{1}{L}\sum_{l=1}^{L}U_l(\Omega)V_l^*(\Omega). \tag{5}$$

To setup the estimation procedure properly, we suggest the use of the Hamming window and at least 70% segment overlap.

The magnitude squared coherence is estimated as

$$|\widehat{\gamma}(\Omega)|^2 = \frac{|\widehat{S}_{\mathrm{uv}}(\Omega)|^2}{\widehat{S}_{\mathrm{u}}(\Omega)\cdot \widehat{S}_{\mathrm{v}}(\Omega)}. \tag{6}$$

Methods of Time Delay Estimation

In this section we outline basic TDEs, which are based on the second order statistics. We will show the variations of these TDEs with and without the additive constant phase term

Cross-Correlation: The most simple approach to estimate the time delay D is the cross-correlation method

$$\widehat{D} = \arg\max_{d} \widehat{R}_{\mathrm{uv}}[d], \tag{7}$$

where $\widehat{D}$ is the time delay estimate, and $\widehat{R}_{\mathrm{uv}}[m]$ is the estimate of the cross-correlation of $u[n]$ and $v[n]$

$$\widehat{R}_{\mathrm{uv}}[d] = \frac{1}{N}\sum_{n=0}^{N-d-1} u[n]v[n+d]. \tag{8}$$

This method is capable of dealing with the simple delay model (1), but is rather imprecise. It fails in presence of noise, disperse signal propagation or nonlinearities [13].

Phase of Cross-Spectral Density: A more sophisticated method utilizes the phase of the cross-spectrum of signals $u[n]$ and $v[n]$. Under the assumption of the simple time delay model (1), the phase of the cross-spectrum is a line with a slope given by the time delay D

$$\Phi(\Omega) = \arg S_{\mathrm{uv}}(\Omega) = -D\cdot\Omega. \tag{9}$$

Consequently, the time delay can be estimated by fitting the line (9) to the estimate of the phase spectrum $\widehat{\Phi}(\Omega)$.

The expected phase spectrum (9) can be somewhat generalized by adding a constant phase term Φ_0, which could account for deviations from the simple delay model (1)

$$\Phi(\Omega) = -D\cdot\Omega + \Phi_0. \tag{10}$$

The usefulness (or redundancy) of the constant phase term Φ_0 is what the latter sections of this paper will focus on.

Now, the actual fitting of (9) or (10) (i.e. the time delay estimation) is performed with the weighted least squares method (ϕ_0 is set to zero if (9) is assumed)

$$[\widehat{D},\widehat{\Phi}_0] = \arg\max_{d,\phi_0} \sum_{\Omega_{\mathrm{d}}\in B} w_{\mathrm{LMS}}(\Omega_{\mathrm{d}})\left(\widehat{\Phi}(\Omega_{\mathrm{d}}) - \phi_0 + d\Omega_{\mathrm{d}}\right)^2, \tag{11}$$

where B denotes the frequency band of interest, $\widehat{\Phi}(\Omega_{\mathrm{d}})$ is the phase of $\widehat{S}_{\mathrm{uv}}(\Omega_{\mathrm{d}})$, and Ω_{d} denotes the points

$$\Omega_{\mathrm{d}} = \frac{2\pi k}{M}, \quad k = 1\ldots M/2-1, \tag{12}$$

where $\widehat{\Phi}(\Omega_{\mathrm{d}})$ is evaluated. The weights $w_{\mathrm{LMS}}(\Omega_{\mathrm{d}})$ were suggested to be disproportional to the variance of the phase spectrum estimate $\widehat{\Phi}(\Omega)$ [14]

$$w_{\mathrm{LMS}}(\Omega_{\mathrm{d}}) = \frac{|\widehat{\gamma}(\Omega_{\mathrm{d}})|^2}{1-|\widehat{\gamma}(\Omega_{\mathrm{d}})|^2} \sim \frac{1}{\mathrm{var}[\widehat{\Phi}(\Omega_{\mathrm{d}})]}. \tag{13}$$

A hitch is that any point of a phase spectrum, as a complex argument, is ambiguous, meaning that all values $\widehat{\Phi}(\Omega) + 2\pi k$, where k is integer, are equivalent. Therefore, the phase of $\widehat{S}_{\mathrm{uv}}(\Omega)$ can be unambiguously computed only if it is limited to the interval $(-\pi,\pi\rangle$ corresponding to the principal value of the arctangent function.

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However, the fitted expressions (9) and (10) are continuous lines, which are not limited to any interval. Therefore, to allow (9) and (10) to be properly fitted, the phase spectrum estimate must be transformed to form a continuous curve through a procedure called unwrapping.

The properties of the TDE based on the phase of a cross-spectrum are following.

The advantage is that it can focus on a chosen frequency band B, and so avoid frequency bands where the signal to noise ratio is poor or the propagation delay varies (as a result of disperse propagation). Additionally, in the chosen frequency band B the individual spectral lines can be weighted with (13) according to their variance, putting more emphasis on the spectral lines less effected by noise.

The disadvantage is the need of the phase unwrapping. Unwrapping procedures tend to fail in the distinct present of noise, which is the very case of EEG-EMG data sets, which have rather low MSC of about 0.1.

Generalized Correlation: The last of the reviewed TDEs is based on the generalized correlation. The generalized correlation is defined as [15]

$$R_{\mathrm{uv}}^{GC}[d] = \mathscr{F}^{-1}\left\{w_{\mathrm{GC}}(\Omega)\cdot S_{\mathrm{uv}}(\Omega)\right\}, \qquad (14)$$

and the time delay is estimated as

$$\widehat{D} = \arg\max_{d} \widehat{R}_{\mathrm{uv}}^{GC}[d], \qquad (15)$$

where $\widehat{R}_{\mathrm{uv}}^{GC}[d]$ is the estimate of $R_{\mathrm{uv}}^{GC}[d]$ obtainable from (14) when $S_{\mathrm{uv}}(\Omega)$ is replaced with its estimate $\widehat{S}_{\mathrm{uv}}(\Omega)$. The weights $w_{\mathrm{GC}}(\Omega)$ were suggested to be [15]

$$w_{\mathrm{GC}}(\Omega) = \frac{|\gamma(\Omega)|^2}{|\widehat{S}_{\mathrm{uv}}(\Omega)|\cdot(1-|\gamma(\Omega)|^2)}. \qquad (16)$$

For computational purposes, (15) can be rewritten as [16]

$$\widehat{D} = \arg\max_{d} \sum_{\Omega_{\mathrm{d}}\in B} \widehat{w}_{\mathrm{GC}}(\Omega_{\mathrm{d}})\cdot\cos\left(\widehat{\Phi}(\Omega_{\mathrm{d}})+d\Omega_{\mathrm{d}}\right), \qquad (17)$$

$$\widehat{w}_{\mathrm{GC}}(\Omega_{\mathrm{d}}) = \frac{|\gamma(\Omega_{\mathrm{d}})|^2}{1-|\gamma(\Omega_{\mathrm{d}})|^2}. \qquad (18)$$

$\widehat{D}$ was shown to have asymptotically (i.e. for $N\to\infty$, $M\to\infty$ and $M/N\to 0$) normal distribution with variance [16]

$$\mathrm{var}[\widehat{D}] = \frac{M}{N\sum_{\Omega_{\mathrm{d}}\in B}\Omega_{\mathrm{d}}^2\widehat{w}_{\mathrm{GC}}(\Omega_{\mathrm{d}})}. \qquad (19)$$

Hence, the $\alpha\cdot 100\%$ confidence limit of $\widehat{D}$ can be estimated as

$$P\left[-\sqrt{\mathrm{var}[\widehat{D}]}\cdot z_{\frac{1+\alpha}{2}} < D-\widehat{D} < \sqrt{\mathrm{var}[\widehat{D}]}\cdot z_{\frac{1+\alpha}{2}}\right] = \alpha, \qquad (20)$$

where z_a denotes the $a\cdot 100\%$ quantile of the normal distribution.

In form (15) the generalized correlation estimates the delay D that determines the slope of the phase spectrum in (9), and is suitable for signals that correspond to the simple time delay model (1). Additionally, this method can be generalized to allow the estimation of both the slope of the phase spectrum and the constant phase term. In particular, according to [16]

$$[\widehat{D},\widehat{\Phi}_0] = \arg\max_{[d,\phi_0]} \sum_{\Omega_{\mathrm{d}}\in B} \widehat{w}_{\mathrm{GC}}(\Omega_{\mathrm{d}})\cdot\cos\Big(\widehat{\Phi}(\Omega_{\mathrm{d}})+d\Omega_{\mathrm{d}}-\phi_0\Big). \qquad (21)$$

During the computation, expression (21) does not have to be maximized simultaneously with respect to d and ϕ_0. It suffice [16] to first estimate the time delay as

$$\widehat{D} = \arg\max_{d} \left|\sum_{\Omega_{\mathrm{d}}\in B} \widehat{w}_{\mathrm{GC}}(\Omega_{\mathrm{d}})\cdot e^{j\left(\widehat{\Phi}(\Omega_{\mathrm{d}})+d\Omega_{\mathrm{d}}\right)}\right|, \qquad (22)$$

and then compute the constant phase therm

$$\widehat{\Phi}_0 = \arg\left(\sum_{\Omega_{\mathrm{d}}\in B} \widehat{w}_{\mathrm{GC}}(\Omega_{\mathrm{d}})\cdot e^{j\left(\widehat{\Phi}(\Omega_{\mathrm{d}})+\widehat{D}\Omega_{\mathrm{d}}\right)}\right). \qquad (23)$$

Additionally, $\widehat{D}$ and $\widehat{\Phi}_0$ were claimed [16] to have asymptotically (i.e. for $N\to\infty$, $M\to\infty$ and $M/N\to 0$) jointly normal distribution with the variance of $\widehat{D}$

$$\mathrm{var}[\widehat{D}] = \frac{M}{N\sum_{\Omega_{\mathrm{d}}\in B}(\Omega_{\mathrm{d}}-\overline{\Omega})^2\widehat{w}_{\mathrm{GC}}(\Omega_{\mathrm{d}})}, \text{ where} \qquad (24)$$

$$\overline{\Omega} = \sum_{\Omega_{\mathrm{d}}\in B}\Omega_{\mathrm{d}}\widehat{w}_{\mathrm{GC}}(\Omega_{\mathrm{d}}) \Big/ \sum_{\Omega_{\mathrm{d}}\in B}\widehat{w}_{\mathrm{GC}}(\Omega_{\mathrm{d}}), \qquad (25)$$

and the variance of $\widehat{\Phi}_0$

$$\mathrm{var}[\widehat{\Phi}_0] = \frac{M}{N}\cdot\frac{1}{\sum_{\Omega_{\mathrm{d}}\in B}\widehat{w}_{\mathrm{GC}}(\Omega_{\mathrm{d}})-\overline{\overline{\Omega}}}, \text{ where} \qquad (26)$$

$$\overline{\overline{\Omega}} = \sum_{\Omega_{\mathrm{d}}\in B}\Big(\Omega_{\mathrm{d}}\widehat{w}_{\mathrm{GC}}(\Omega_{\mathrm{d}})\Big)^2 \Big/ \sum_{\Omega_{\mathrm{d}}\in B}\Omega_{\mathrm{d}}^2\widehat{w}_{\mathrm{GC}}(\Omega_{\mathrm{d}}). \qquad (27)$$

Consequently, the $\alpha\cdot 100\%$ confidence interval of the delay estimate is given by (20), and Φ_0 is expected to differ from zero with $\alpha\cdot 100\%$ probability if

$$|\widehat{\Phi}_0| > \sqrt{\mathrm{var}[\widehat{\Phi}_0]}\cdot z_{\frac{1+\alpha}{2}}. \qquad (28)$$

The properties of the generalized correlation are quite favorable – it keeps all the advantages of the method using the phase of a cross-spectral density, but it needs no phase unwrapping. This can be deduced from the comparison of estimation formulas (11) and (21). They differ only in the function applied on the expression in the parenthesis – in (11) it is the quadratic function, in (21) it is the cosine function. Since cosine is locally quadratic in the proximity of zero, both procedures will give similar results when the fit is good and the expressions in the parenthesis are small. Additionally, the periodicity of the cosine function will provide the same behavior even if the expression in parenthesis is close to any integer multiply of 2π; therefore, there is no need to unwrap the phase spectrum. Ultimately, the generalized correlation was shown to be the maximum likelihood estimator, and so it provides the lowest possible variance for an unbiased estimator given by the Cramèr-Lao lower bound [15, 16].

Currently Used Methods for EEG-EMG Time Delay Estimation

In the published papers dealing with the EEG(MEG)-EMG time delay estimation mostly the cross-correlation and the phase spectrum approaches were used[3].

The cross-correlation was used in [1, 2, 3, 4, 5, 6, 7]. Except [1], the reported time delays were approximately comparable to those measured with the transcranial magnetic stimulation (TMS) of the motor cortex. Nonetheless, the correlation method uses data insufficiently, making no difference between useful information and noise, thus is not very suitable for the time delay estimation.

A more sophisticated phase spectrum approach was used in [2, 3, 4, 5, 8, 9, 10, 11]. In these papers the time delay was estimated using (11) with the constant phase term applied. Except the work [3], the results roughly corresponded to the time delays measured by TMS of the motor cortex. Nevertheless, several limitations were applied. First, the frequency band was limited to 15-40Hz (or higher), because 9-12Hz (or lower) band did not seem to provide tolerable results [8]. Next, the time delay computation was sometimes limited to the EEG-EMG data sets with the strongest coherence [4, 10] or to those with a linear phase only [9].

We suppose that fitting of the phase of an EEG-EMG cross-spectrum with a straight line is quite sensible. It was repeatedly reported that the phase of EEG-EMG cross-spectrum manifest a frequency band where the phase is linear. Thus it appears reasonable to try deducing the time delay from its slope. However, we find it rather questionable if the fitted line must always include the additive constant phase term. In fact, according to the published papers, the constant phase term was either directly measured in the proximity of 0 (e.g. see results in [11]) or it was close to $\pm\pi$ [5, 9, 10], which only means that it would be close to zero again if the polarity of either EEG or EMG was switched. Therefore, the constant phase term often appears redundant, its including into the method (11) appears unnecessary, and, as a consequence, the variance of the time delay estimates is needlessly big.

To our knowledge, there is no paper that would use the generalized correlation method for the EEG-EMG time delay estimation.

Suggested Method

Since both the cross-correlation and the method based on the phase of a cross-spectrum are inferior to the generalized correlation TDE, we suggest the generalized correlation to be used for the EEG-EMG time delay estimation[4].

To deal with the constant phase term we suggest the use of the following procedure (based on the suggestions in [16]). First use formulas (21), (24), (20) and (23) to estimate the time delay $\widehat{D}$, its variance, confidence interval and the constant phase term $\widehat{\Phi}_0$. Then use the test (28) to check if Φ_0 significantly differs from zero. If not, recompute the time delay estimate with formula (15), and compute its confidence limits with (19) and (20).

This procedure should limit the use of the constant phase term only to those cases, where it is clearly necessary. Thus the procedure will provide results with smaller variance for the EEG-EMG data sets with the constant phase term indistinguishable from zero.

[3] There has also been one work [12] dealing with MEG-EMG coherence that used a non-linear TDE based on the phase of envelope of analyzed signals.

[4] Nonetheless, the conclusions related to the use of the constant phase therm Φ_0 hold true even for the method based on the phase of a cross-spectrum thanks to its similarity to the generalized correlation.

Results

To test the proposed method we analyzed 88 EEG-EMG data sets.

Data were measured on 11 subjects that performed extension and flection of an index finger with four different weights. EEG was measured with 82 electrode system that covered frontal, paretial, temporal and occipital areas. Surface EMG was measured over the muscles extensor indicis (during extension) and flexor superficialis (during flexion). The recorded signals were 2.5 minutes long and sampled at 512Hz.

After the rectification of EMG, the EEG signals were spatially filtered with the Taylor series based surface Laplacian filtration [17], and the electrode providing the highest EEG-EMG MSC was chosen. Then, EEG-EMG MSC was computed with (6). The EEG-EMG MSC was considered significant if it exceeded its confidence limit by 1.3 times at no less than one frequency in the band 14-35Hz (for the confidence limit computation see [18] in these proceedings). Otherwise, the EEG-EMG time delay was not computed. The time delay was estimated with the proposed generalized correlation method with and without the constant phase term. The frequency band used for the estimation was limited to 14-35Hz. The obtained EEG-EMG delays and their 95% confidence limits are shown in Table 1. Asterisks indicate the cases where the constant phase term was found to be significantly different from zero with 95% probability.

Discussion

From the 71 EEG-EMG data sets which manifested significant coherence, the constant phase term was found significantly different from zero with 95% probability in 15 cases. This is 21% of 71 cases so it cannot be caused by the errors of the test. Therefore, the non-zero constant phase term does occur. On the other hand, its occurrence is not so frequent. In 56 out of 71 EEG-EMG data sets the constant phase term was not found significantly different from zero.

Another detail worth noticing is the variance of the obtained time delays, which is reflected in the width of their confidence intervals. While the estimation without the constant phase term provided quite sensible variance and confidence intervals with relative width raging from

	Subject Number	
	$\Phi_0 = 0$	$\Phi_0 \neq 0$
extension	delays [ms]	delays [ms]
flexion	delays [ms]	delays [ms]

	Subject #1	
1.	**5.6±1.7**	17.8±15.7
2.	**6.1±1.6**	11.0± 5.5
3.	**4.9±1.4**	7.0± 4.8
4.	**4.6±1.9**	2.2±12.5
1.	**4.8±2.5**	32.8±41.9
2.	**4.2±1.8**	2.5±11.5
3.	**6.8±2.0**	2.2±20.3
4.	**6.4±1.6**	22.2± 9.6(*)

	Subject #2	
1.	**8.9±0.7**	3.1± 4.3(*)
2.	**9.9±0.5**	0.3± 3.5(*)
3.	**9.6±0.7**	2.6± 4.7(*)
4.	**7.5±1.0**	8.2± 4.6
1.	**8.7±1.5**	8.6± 6.3
2.	**7.0±1.2**	9.5± 5.9
3.	**8.3±0.8**	9.5± 4.6
4.	**11.4±1.5**	-12.2±13.9(*)

	Subject #3	
1.	**8.0±3.3**	-60.0±11.4
2.	**10.8±3.3**	-60.0±57.9
3.	**7.5±3.2**	-3.5±15.4
1.	**11.5±2.6**	-60.0±16.6
2.	**14.3±2.5**	-17.7± 8.9
3.	**54.0±4.1**	-19.5±15.9
4.	**13.0±6.6**	-15.8±57.6

	Subject #4	
1.	**17.1±2.3**	25.0±17.6
2.	**15.8±2.7**	7.5±12.7
3.	**20.6±2.9**	17.0±12.3
4.	**24.3±2.4**	46.7±13.1(*)
1.	**17.0±3.8**	-60.0±17.0
2.	**16.7±2.3**	-38.5±15.7
4.	**15.8±3.0**	40.0±14.5(*)

	Subject #5	
3.	**20.2±3.2**	-3.0±26.0
4.	**10.3±2.0**	18.0± 9.0

	Subject #6	
1.	**16.8±1.8**	15.0±16.4
2.	**17.6±1.4**	16.7±15.9
3.	**18.7±1.6**	17.7±15.9
4.	**14.1±1.8**	-4.0±22.0
1.	**15.3±3.2**	-5.1±39.1
2.	**17.5±1.9**	7.0±23.2
3.	**18.2±2.1**	21.2±21.5
4.	**19.0±2.0**	12.6±16.5

	Subject #7	
1.	**9.0±1.5**	-17.1± 6.8(*)
2.	**17.3±1.1**	-7.6± 6.4(*)
3.	**6.8±1.7**	6.7±12.9
4.	**5.9±1.5**	-4.7± 8.2(*)
1.	**8.0±2.0**	-2.5± 8.3(*)
2.	**6.1±1.4**	0.8± 8.3
3.	**4.5±2.2**	-28.5±15.6
4.	**8.9±2.2**	-8.3±12.3(*)

	Subject #8	
1.	**15.5±2.8**	-3.6± 8.7(*)
2.	**14.0±3.5**	26.0±23.7
3.	**13.4±2.4**	-22.1±17.4
4.	**12.4±2.4**	9.8±15.7
1.	**12.6±3.9**	-46.7±22.2
2.	**12.6±2.6**	3.1±15.5
3.	**14.6±3.9**	9.3±17.0
4.	**22.5±3.0**	-60.0±26.1

	Subject #9	
1.	**10.5±2.8**	11.2±36.0
3.	**9.9±1.6**	3.8±17.7
4.	**5.7±1.4**	-8.1±12.7(*)
1.	**13.3±3.5**	12.0±37.7
2.	**5.9±3.3**	-34.5±22.2

	Subject #10	
1.	**7.3±1.7**	13.6±12.9
2.	**7.1±1.2**	21.0±10.1(*)
3.	**4.6±1.0**	1.7± 8.1
4.	**5.8±0.5**	8.1± 3.9
1.	**18.7±3.3**	35.2±19.5
2.	**23.5±2.1**	-52.3±26.2
4.	**21.0±2.6**	60.0±21.2

	Subject #11	
3.	**14.8±2.9**	5.5±18.0
4.	**30.5±3.2**	-50.0±17.7
1.	**15.7±2.8**	-60.0±34.9

Table 1: The time delay estimates. As indicated in the top left template, each table contains the time delay estimates for one subject. The first four lines show the time delays for index finger flexion and the last four lines for its extension. The increasing numbering of each row corresponds to the increasing weight loading an index finger. The left hand side of each table contains the time delays estimated using the generalized correlation with no constant phase term, the corresponding time delays obtained with the constant phase term are on the right. For each estimate 95% confidence intervals are given. The asterisks indicate cases when the constant phase term was found to be significantly different from zero with 95% probability. Missing values mean that no significant MSC was distinguished.

±10% to ±40%, the use of the constant phase term increased the variance and widened the confidence intervals to ±50% ∼ ±500%. It can bee seen that up to some outliers, the time delays estimated without the constant phase term are fairly consistent for each subject; however, the time delays estimated with the constant phase term vary greatly even to negative values. Therefore, it seems that the use of the constant phase term worsen the quality of the time delay estimates beyond tolerable precision (this would of course improve if longer EEG and EMG records were used).

The last point to evaluate is how the estimated time delays correspond to the expected values (now we concentrate to the time delays estimated without the constant phase term only). It seems that in some subjects (e.g. 8,6,4) the time delays are close to the value of about 15ms, which is a time delay typical for muscle extensor indicis obtained with TMS of the motor cortex [12] (the muscle flexor superficialis should have a very similar time delay). In other subjects (e.g. 1,2) the estimated time delays are somewhat shorter. The shorter delays were encountered before [9, 10], and they were explained by slow signal conduction through the scull. We also believe that the non-linearity of neural networks can be accounted for biasing of the results.

Conclusion

We demonstrated that for signals with length as short as 2.5 minutes the automatic inclusion of the constant phase term into EEG-EMG time delay estimation is unwise. First, it turns out that for this short signal length the constant phase term is, according to the provided statistical test, often undistinguishable from zero. Second, the variance of the results without the constant phase is much better than with it. Third, even though the assumed simple delay model (1) may not fully correspond to the relationship between the EEG and EMG signals, the bias resulting from a simpler TDE does not damage the results so much as the huge variance provided by the addition of the constant phase term. We therefore conclude that for short EEG-EMG records the use of the constant phase term should be limited as much as possible.

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