CONFIDENCE LIMIT COMPUTATION FOR EEG-EMG COHERENCE ESTIMATED WITH SEGMENT OVERLAPPING

R. Bortel* and P. Sovka*

* Faculty of Electrical Engineering, Czech Technical University, Technická 2, Prague, Czech Republic

bortelr@feld.cvut.cz, sovka@feld.cvut.cz

Abstract:
This paper suggests a formula for the computation of a confidence limit for a magnitude squared coherence (MSC) estimated with segment overlapping. So far, the confidence limit could be computed exactly only if MSC was estimated without segment overlapping. In case of overlapped segmentation the confidence limit could be only estimated through a quantile estimation, a procedure, which is shown to be imprecise. The suggested formula allows a solid evaluation of MSC estimated with segment overlapping, which provides more precise results. The confidence limit computation is intended for the cortico-muscular coherence analysis, which investigate coupling between the electroencephalogram (EEG) and the electromyogram (EMG). The formula is shown to be valid for signals with length typically used for EEG-EMG MSC computation – in particular the signal length assuring at least 100 degrees of freedom of an MSC estimate. Examples of EEG-EMG MSC were also used to show that the combination of the proposed formula and overlapped segmentation provides more precise MSC estimates, and can detect weaker EEG-EMG coupling.

Introduction

Coherence Function

The coherence function of two signals $u[n]$ and $v[n]$ is defined as [1]

$$\gamma(\Omega) = \frac{S_{uv}(\Omega)}{\sqrt{S_u(\Omega)S_v(\Omega)}}\,, \tag{1}$$

where $S_{uv}(\Omega)$, $S_u(\Omega)$ and $S_v(\Omega)$ are the cross-spectral and the power spectral densities of $u[n]$ and $v[n]$. More commonly a value of $|\gamma(\Omega)|^2$, termed as the mean squared coherence (MSC), is used.

MSC serves as a measure of linear coupling between two signals, being zero if these signals are uncorrelated and being one if signals are linearly coupled (i.e. one signal was created from another through linear filtration).

EEG-EMG Coherence

A significant coherence has been reported between a record of brain electrical activity, the electroencephalogram (EEG), and a record of electrical activity of a contracted muscle, the electromyogram (EMG). This coherence is most distinct for EEG recorded over the primary sensimotor area contralateral to the contracted muscle. It usually occurs in the beta band (15-30Hz) and occasionally in the low gamma band (30-60Hz) or the alpha band (6-12Hz) [2]. From a technical point of view the value of EEG-EMG MSC is quite low – about 0.1 only – however, it was proven to be significantly greater than zero, and so it confirms a positive coupling between EEG and EMG.

Statistical Evaluation of Coherence Significance

With stochastic nature of EEG and EMG signals, EEG-EMG MSC cannot be computed exactly but has to be estimated. Estimation, however, results in certain errors, which make the evaluation of EEG-EMG MSC estimate difficult. In particular, while for a true MSC, its being greater than zero suffice to expose EEG-EMG coupling, the random nature of an MSC estimate causes that it can be greater than zero even if the true MSC is not (that is when EEG and EMG are uncorrelated). Therefore, we need to find a value an MSC estimate has to exceed to confirm that the true MSC is greater than zero (i.e. EEG and EMG are correlated). Mathematically, we seek a value c, exceeding of which has a low probability α, provided that the true MSC is zero.

$$\mathrm{P}\left[|\widehat{\gamma}(\Omega)|^2 > c \,\middle|\, |\gamma(\Omega)|^2 = 0\right] = \alpha\,, \tag{2}$$

where $\mathrm{P}[.|.]$ denotes a conditional probability operator, and $|\gamma(\Omega)|^2$ and $|\widehat{\gamma}(\Omega)|^2$ are the true MSC and its estimate, respectively. The value of c is referred to as the $(1-\alpha)\cdot 100\%$ confidence limit (e.g. [3]), and is essential for evaluating EEG-EMG coherence.

Methods of MSC Estimation and Statistical Evaluation[1]

Estimation Procedure

To estimate EEG-EMG MSC the spectral densities in (1) need to be estimated. In almost all papers dealing with EEG-EMG MSC a direct estimation procedure [5] (a.k.a. the modified Welch method) was used. Namely, signals

[1] Only non-parametric methods of MSC estimation will be reviewed in this section. For parametric approaches see e.g. [4].

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$u[n]$ and $v[n]$, N samples long, are segmented into L segments $u_l[n]$ and $v_l[n]$

$$\begin{aligned} u_l[n] &= u[l(M-P)+n], \\ v_l[n] &= v[l(M-P)+n], \end{aligned} \tag{3}$$

where M is the length of a segment, and P is the segment overlap. Each segment is then weighted with a window $w_{\mathrm{d}}[n]$, and its Fourier transform is computed

$$U_l(\Omega) = \mathscr{F}\Big\{u_l[n]\cdot w_{\mathrm{d}}[n]\Big\}, \quad V_l(\Omega) = \mathscr{F}\Big\{v_l[n]\cdot w_{\mathrm{d}}[n]\Big\}. \tag{4}$$

The spectral densities and MSC are then estimated as

$$\widehat{S}_{\mathrm{u}}(\Omega) = \frac{1}{L}\sum_{l=1}^{L}|U_l|^2(\Omega), \quad \widehat{S}_{\mathrm{v}}(\Omega) = \frac{1}{L}\sum_{l=1}^{L}|V_l|^2(\Omega),$$

$$\widehat{S}_{\mathrm{uv}}(\Omega) = \frac{1}{L}\sum_{l=1}^{L}U_l^*(\Omega)V_l(\Omega), \tag{5}$$

$$|\widehat{\gamma}_{\mathrm{uv}}(\Omega)|^2 = \frac{|\widehat{S}_{\mathrm{uv}}(\Omega)|^2}{\widehat{S}_{\mathrm{u}}(\Omega)\cdot\widehat{S}_{\mathrm{v}}(\Omega)}. \tag{6}$$

We will now overview the variations of this estimation method with respect to the possibility of confidence limit computation or estimation.

Non-Overlapped Segmentation

In the majority of papers, dealing with EEG-EMG MSC, non-overlapped segmentation in (3) is used (i.e. $P = 0$).

The advantage of this approach is a known statistical distribution of MSC estimate (6) [6], which allows to derive an exact expression for the significance level c. Namely, if MSC is estimated from L non-overlapped segments the confidence limit is [3]

$$c = 1 - \alpha^{1/(L-1)}, \tag{7}$$

where the value of $2L$ is referred to as the number of degrees of freedom of an MSC estimate.

The disadvantage of the non-overlapped approach is that information contained in analyzed signals is somehow waisted. It is a known fact, that employing overlap into estimation procedures (5) and (6) increases their precision, lowering their variance and in the case of MSC even bias.

For better illustration, we computed bias and variance of MSC estimated with varying amount of segment overlap. For each value of overlap, MSC was computed in 170000 trials with formulas (5) and (6), using normally distributed random signals with true MSC of 0 and 0.2. The signal length was chosen to provide 32 segments in non-overlapped case. The estimated bias and variance in dependance on the segment overlap is shown in Figure 1, each row showing results for a different weighting window. Note, that at 70% overlap (i.e. $P = 0.7\cdot M$) the bias and variance of MSC is at least two times smaller than in the non-overlapped case[2].

[2]Similar illustration has already been used in [6].

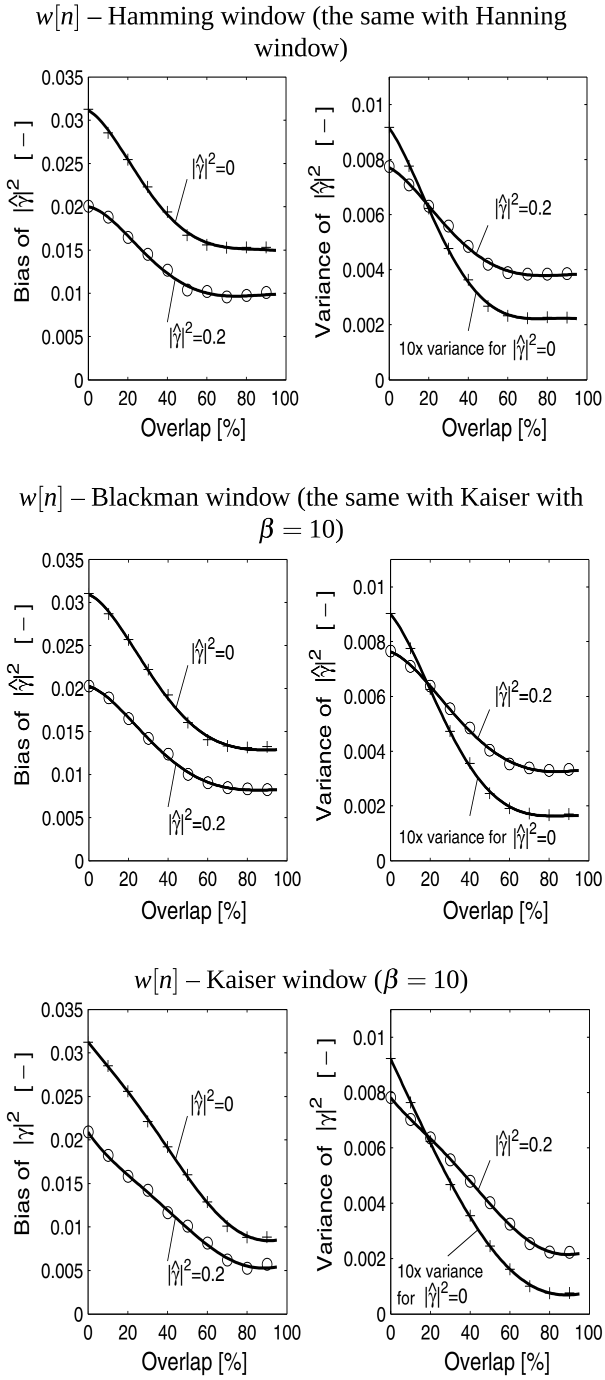

Figure 1: Bias and variance of an MSC estimate in dependance on the amount of segment overlap.

Overlapped Segmentation

To our knowledge there is only one work [7] dealing with MEG-EMG coherence (MEG is a record of magnetic field of a brain; MEG-EMG MSC is similar to EEG-EMG MSC, and methods of their estimation are identical), where overlapped segmentation is used.

The advantage of this method is high precision of the estimated MSC.

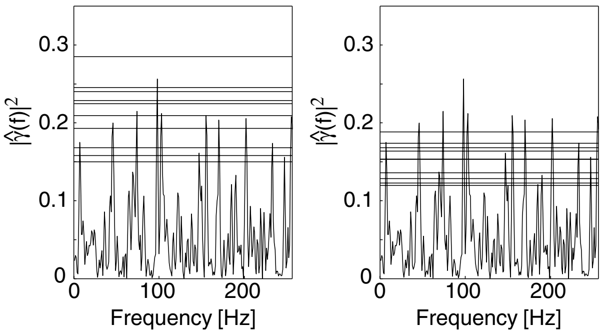

Figure 2: An example of the variation in 99% (left) and 95% (right) confidence limit estimated with (10). Figure shows ten confidence limits estimated from MSCs computed from uncorrelated signals with equal length.

The disadvantage of this method is an unknown statistical distribution of the MSC estimate. This makes the exact computation of the confidence limit impossible, and its becoming difficult to decide if an EEG-EMG MSC estimate indicates any EEG-EMG coupling.

This drawback has been partially solved (e.g. see [7]) by a procedure, which estimates the confidence limit approximately. Namely, the analyzed signals $u[n]$ and $v[n]$ are first shifted by d samples ($d > M$, where M is the segment length)

$$\begin{aligned} u'[n] &= u[n]\,, \\ v'[n] &= v[n+d]\,, \end{aligned} \tag{8}$$

to eliminate their true coherence. Then the MSC of $u'[n]$ and $v'[n]$ is estimated in M discrete points

$$\Omega_\mathrm{d} = 2\pi k/M, \qquad k = 0\dots M-1\,. \tag{9}$$

Finally, the confidence limit c is estimated as a value, under which lies $(1-\alpha)\cdot 100\%$ values of the estimated MSC

$$|\widehat{\gamma}_{\mathrm{u'v'}}(\Omega_\mathrm{d})|^2 < c \quad \text{at } (1-\alpha)\cdot 100\% \text{ frequencies } \Omega_\mathrm{d}\,. \tag{10}$$

This estimation procedure, though, is essentially a quantile estimation, and as such is rather imprecise, especially if α is close to one.

To illustrate imprecision of confidence limit estimation (10) we estimated MSC of uncorrelated normally distributed signals. Signals were $N = 5000$ samples long, segmented into $M = 256$ samples long segments, which were weighted with the Hamming window and overlapped by 70%. Then, we estimated 95% and 99% confidence limits. This experiment was repeated 10 times providing 10 different confidence limit estimates, which are shown in Fig. 2 together with one of the MSC estimates. Note that the individual confidence limit estimates vary considerably even though there is only one true confidence limit. For that reason we conclude that the confidence limit estimation procedure (10) is imprecise.

K	20	50	100	200	500
L_{CL}	74.2	187.9	377.7	759.0	1894.0
$\mathscr{L}_{PSD}$	74.0	187.4	376.8	756.7	1900.6
Δ[%]	0.27	0.27	0.24	0.30	-0.35

Table 1: Estimated numbers of degrees of freedom of MSC and PSD. The last line of the table shows their percentual relative difference.

Suggested Method

In this section we derive a formula that provides a value of the confidence limit deterministically without any random fluctuations.

We have noticed that in both non-overlapped and overlapped cases there is a relationship between the number of degrees of freedom $2L$ in formula (7) and the number of degrees of freedom of χ^2 distribution that approximates the distribution of the power spectral densities (PSD) in (5). In the non-overlapped case the numbers of degrees of freedom are obviously equal. In the overlapped case the equality seems to hold too, at least approximately. To illustrate the latter case we computed 500 MSC estimates of normally distributed spectrally white uncorrelated signals, segmented into $M = 1024$ samples long segments, which were overlapped by 70% and weighted with the Hamming window. The signal length was $N = M\cdot K$, where $K =$20, 50, 100, 200, 500. For each MSC estimate we estimated a 95% confidence limit using (10). The individual confidence limit estimates were then averaged, providing one precise confidence limit estimate. This estimate was substituted into formula (7), which provided the number of degrees of freedom $2L_{CL}$ needed to reach this confidence limit. Concurrently, we estimated the number of degrees of freedom $2\mathscr{L}_{PSD}$ of χ^2 distribution that best fits PSD used for MSC estimation, using maximum likelihood method

$$\mathscr{L}_{PSD} = \arg\max_{L} \prod_{\mathrm{i}} f_{\chi^2}(\widehat{S}_i|2L)\,, \tag{11}$$

where $f_{\chi^2}(.,2L)$ is the probability density function of χ^2 distribution with $2L$ degrees of freedom and $\widehat{S}_\mathrm{i}$ are the individual estimates of PSD used for the computation of MSCs. The estimated L_{CL} and $\mathscr{L}_{PSD}$, shown in Table 1, seem to be very close. Therefore, in case of overlapped segmentation, we suggest the confidence limit to be computed with (7), but with the number of degrees of freedom equal to those of χ^2 distribution that best fits PSD in (5). The question remaining is how to obtain the value of $\mathscr{L}_{PSD}$ without using rather cumbersome maximum likelihood estimation.

Suppose that $S_\mathrm{u}(\Omega)$, $S_\mathrm{v}(\Omega)$ and $S_\mathrm{uv}(\Omega)$ are estimated through the indirect procedure [5]

$$\begin{aligned} \widehat{S}_\mathrm{u} = \mathscr{F}\left\{\widehat{R}_\mathrm{u}[k]\cdot w_\mathrm{ind}[k]\right\}, \ \widehat{S}_\mathrm{v} = \mathscr{F}\left\{\widehat{R}_\mathrm{v}[k]\cdot w_\mathrm{ind}[k]\right\}, \\ \widehat{S}_\mathrm{uv} = \mathscr{F}\left\{\widehat{R}_\mathrm{uv}[k]\cdot w_\mathrm{ind}[k]\right\}, \end{aligned} \tag{12}$$

where $w_{\mathrm{ind}}[k]$ is a weighting window (scaled so that $w_{\mathrm{ind}}[0] = 1$ [8]), and $R_{\mathrm{u}}[k]$, $R_{\mathrm{v}}[k]$, $R_{\mathrm{uv}}[k]$ are the auto and cross-correlation functions of $u[n]$ and $v[n]$ defined as

$$\widehat{R}_{\mathrm{u}}[k] = \frac{1}{N}\sum_{n=0}^{N-k-1} u[n]\cdot u[n+k]\,, \tag{13}$$

$$\widehat{R}_{\mathrm{v}}[k] = \frac{1}{N}\sum_{n=0}^{N-k-1} v[n]\cdot v[n+k]\,, \tag{14}$$

$$\widehat{R}_{\mathrm{uv}}[k] = \frac{1}{N}\sum_{n=0}^{N-k-1} u[n]\cdot v[n+k]\,. \tag{15}$$

According to [8], $S_u(\Omega)$ and $S_v(\Omega)$ have χ^2 distribution with $2\mathscr{L}$ degrees of freedom

$$2\mathscr{L} = \frac{2N}{\sum_k w_{\mathrm{ind}}^2[k]}\,. \tag{16}$$

Additionally, it can be shown that the indirect approach (12) is equivalent to the direct one (5) if overlap $P = M-1$ is used, $M-1$ zeros are padded before and behind the signals $u[n]$ and $v[n]$, and

$$w_{\mathrm{ind}}[n] = \sum_{m=1}^{M} w_{\mathrm{d}}[m]w_{\mathrm{d}}[m+n]\,, \tag{17}$$

(see the appendix for derivation). Therefore, with this setting, the the number of degrees of freedom of MSC estimated with (6) can be approximated with formula (16) and the confidence limit is

$$c = 1-\alpha^{1/(\mathscr{L}-1)}\,. \tag{18}$$

Moreover, the extensive overlap $P = M-1$ can be reduced. The first row of Figure 1 indicates that for the Hamming and Hanning window MSC variance and bias does not change any more after exceeding 70% overlap. Hence, we tried decreasing the overlap to 70%, and we obtained MSC estimates, confidence limits of which are still given by formula (18) – this is shown using statistical tests in the next section. Additionally, from the second and third row of Fig. 1 we similarly deduced that the segment overlap can be reduced to 80% for the Blackman and Kaiser window with parameter 10 and to 90% for the Kaiser window with parameter 20 (high amount of overlap, however, increases the computational cost; therefore, we find Hamming and Hanning window to be the most suitable choice, as they provide low leakage and require lower amount of overlap).

Testing

To test the suggested method, we estimated MSC of uncorrelated signals in 1000 trials. We used normally distributed signals segmented into M=1024 samples long segments, which were weighted with the Hamming, Blackman or Kaiser ($\beta = 10$) window and overlapped accordingly. The signal length was chosen to obtain $2\mathscr{L}$ =100, 200, 500 and 1000 degrees of freedom[3]. In each trial we computed a confidence limit estimate (10). Finally, the individual estimates were averaged, providing one precise confidence limit estimate $\bar{c}$. Next, we compared the value of $\bar{c}$ to the confidence limit provided by the formula (18), computing their relative difference

$$\Delta = \frac{\bar{c}-\widehat{c}}{\bar{c}}\cdot 100\%\,. \tag{19}$$

These differences are shown in Table 2.

$w[n]$ – Hamming window

α \ $\mathscr{L}$	100	200	500	1000	2000
0.01	1.9	1.4	0.7	0.7	0.9
0.05	1.2	0.7	0.3	0.0	0.2
0.10	1.0	0.7	-0.2	-0.2	0.0

$w[n]$ – Blackman window

α \ $\mathscr{L}$	100	200	500	1000	2000
0.01	1.8	0.2	1.0	-2.6	-0.6
0.05	0.4	0.4	-0.6	-2.0	-0.3
0.10	0.2	0.6	0.1	-1.7	-0.5

$w[n]$ – Kaiser window ($\beta = 10$)

α \ $\mathscr{L}$	100	200	500	1000	2000
0.01	2.3	1.5	0.8	0.1	-0.3
0.05	-0.1	0.5	0.4	0.2	0.3
0.10	-0.2	1.2	0.6	0.0	0.4

Table 2: Relative errors between precisely estimated confidence limits and confidence limits computed with formulas (16) and (18).

Table 2 shows that the relative difference between the precisely estimated confidence limit (the best approximation to the true one) and the value provided by formula (18) is small. We therefore conclude that the formula (18) provides a good mean for the confidence limit computation of MSC estimated with overlapping[4].

[3]The range of degrees of freedom is deduced from the requirement on the precision of an EEG-EMG MSC estimate. Since the true EEG-EMG MSC is often smaller than 0.1, the confidence limit should be 0.02 at maximum. Such value of confidence limit requires at least 298 degrees of freedom (follows from (7)).

[4]We noticed, however, that the computation of confidence limit will deviate for degrees of freedom smaller than 50. Even though so small number of degrees of freedom is not used in the EEG-EMG MSC computations, it is used in other applications. Therefore, we will address this problem in our future work.

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Examples

This section illustrates how advantageous it is, when a known confidence limit formula allows the use of overlapped segmentation in the MSC estimation.

Presented MSCs were computed using EEG and EMG signals measured during an isometric extension of an index finger. Signals were 2.5 minutes long (sampling at 512Hz gives $N \approx 80000$), segmented into M=512 samples long segments, which were weighted with a Hamming window. EEG-EMG MSCs were computed with both zero and 70% overlap.

Estimated EEG-EMG MSCs are shown in Figure 3. MSCs estimated without overlapping are shown in the left column, while the corresponding MSCs estimated with 70% overlap are on the right.

Note that in the first three rows EEG-EMG MSC estimated without overlapping exceeds the confidence limit weakly, while the overlapped segmentation has revealed much clearer EEG-EMG coupling. In the last three rows EEG-EMG MSCs already distinguishable without overlapping are computed with less random variation when overlapping was employed. Thus, when the confidence limit formula allows the use of overlapped segmentation, the final EEG-EMG MSC estimate can reveal weaker EEG-EMG coupling and the resulting EEG-EMG MSCs are more precise.

Moreover, if the precision attained with non-overlapped segmentation is already sufficient the suggested method of MSC estimation can provide the same precision with half the data. This follows from the comparison of the number of degrees of freedom of MSC estimates obtained with and without overlapping. In the non-overlapped case, the signal length $N_{\text{no}-\text{ovrlp}}$ provides $2L = 2N_{no-ovrlp}/M$ degrees of freedom. In the overlapped case, the signal length N_{ovrlp} provides the $2\mathscr{L}$ degrees of freedom given by formula (16). The precision of MSCs estimated with and without overlapping will be the same if $L = \mathscr{L}$, which will happen when

$$N_{\text{ovrlp}} = N_{\text{no}-\text{ovrlp}} \frac{\sum_k w_{\text{ind}}^2[k]}{M}\,, \qquad (20)$$

which for Hamming window $w_{\text{d}}[k]$ gives

$$N_{\text{ovrlp}} = 0.523 \cdot N_{\text{no}-\text{ovrlp}}\,. \qquad (21)$$

Thus, the overlapped segmentation provides the precision of non-overlapped segmentation with only half the data.

Conclusion

The procedure suggested for EEG-EMG estimation appears to be superior to the previously used methods. First, the deterministic formula for confidence limit computation gives more accurate values than the hitherto used method based on an imprecise quantile estimation. Second, the precise knowledge of confidence limit allows the MSC to be estimated with overlapped segmentation, that provides more precise MSC estimates. Additionally, it was illustrated, that this increased precision allows to detect weaker EEG-EMG coupling and that estimated EEG-EMG MSCs have less random variance.

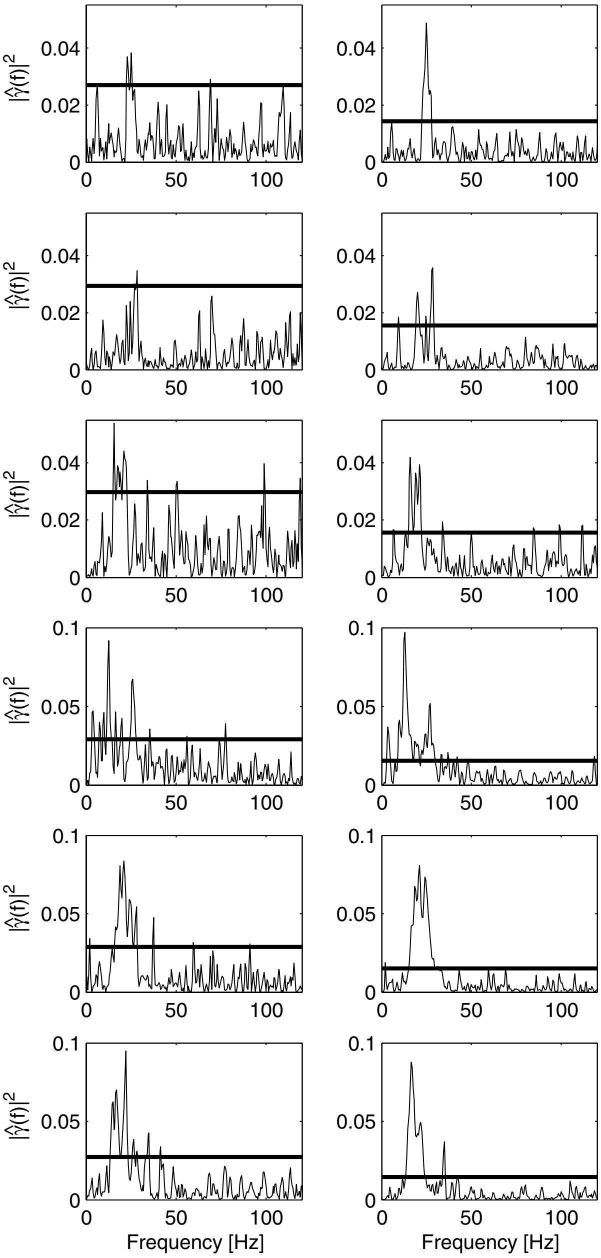

Figure 3: EEG-EMG MSCs. The left collum shows MSCs estimated without overlapping. Corresponding MSCs estimated with overlapping are in the right collum. The horizontal lines represent 99% confidence limits.

Appendix

Comparison of Direct and Indirect Estimation of Spectral Characteristics

This appendix analyzes the equivalence of a direct and indirect estimation of spectral characteristics. It is shown that under certain conditions these two approaches provide equivalent results.

We will analyze an estimate of a cross-spectral density $S_{\mathrm{uv}}(\Omega)$ of two signals $u[n]$ and $v[n]$ (the same derivation can be done for auto-spectral densities). First suppose that $S_{\mathrm{uv}}(\Omega)$ is estimated as $\widehat{S}_{\mathrm{uv}}(\Omega)$ using direct approach (5) with a signal length N, segment length M, segment overlap $P = M-1$ and weighting window $w_{\mathrm{d}}[n]$.

To outline the relationship between the direct estimate (5) and the indirect estimate (12) we will analyze the inverse Fourier transform of $\widehat{S}_{\mathrm{uv}}(\Omega)$.

$$\begin{aligned}
&\mathscr{F}^{-1}\{\widehat{S}_{\mathrm{uv}}\} = \frac{1}{L}\sum_{l=1}^{L}\mathscr{F}^{-1}\{U_l^* V_l\} = \\
&= \frac{1}{L}\sum_{l=1}^{L}\big(u_l[-n]w_{\mathrm{d}}[-n]\big)*\big(v_l[n]w_{\mathrm{d}}[n]\big) \overset{(3)}{\downarrow}{=} \\
&= \frac{1}{L}\sum_{l=1}^{L}\sum_{m=0}^{M-1} u[l-n+m]w_{\mathrm{d}}[-n+m]v[l+m]w_{\mathrm{d}}[m] = \\
&= \sum_{m=0}^{M-1} w_{\mathrm{d}}[m]w_{\mathrm{d}}[m-n]\frac{1}{L}\sum_{l=1}^{L}u[l-n+m]v[l+m]\,. \quad (22)
\end{aligned}$$

If the inner sum in (22) was not dependent on m it would denote the cross-correlation of $u[n]$ and $v[n]$. To get rid of this dependance, we will require $M-1$ zeros to be padded before and behind $u[n]$ and $v[n]$

$$\begin{aligned}
\dot{u}[n] &= \underbrace{0,\ldots,0}_{M-1}, u[n], \underbrace{0,\ldots,0}_{M-1}, \\
\dot{v}[n] &= \underbrace{0,\ldots,0}_{M-1}, v[n], \underbrace{0,\ldots,0}_{M-1}\,. \quad (23)
\end{aligned}$$

When $\dot{u}[n]$ and $\dot{v}[n]$ (with length $\dot{N}$ and number of segments $\dot{L}$) are used instead of $u[n]$ and $v[n]$, the shift introduced by adding m in the inner sum of (22) will not have any effect on the summation and

$$\begin{aligned}
&\frac{1}{\dot{L}}\sum_{l=1}^{\dot{L}}\dot{u}[l-n+m]\dot{v}[l+m] = \frac{1}{\dot{L}}\sum_{l=1}^{\dot{N}}\dot{u}[l-n]\dot{v}[l] = \\
&= \frac{\dot{N}}{\dot{L}}\widehat{R}_{\dot{\mathrm{u}}\dot{\mathrm{v}}}[n] = \frac{N}{\dot{L}}\widehat{R}_{\mathrm{uv}}[n]\,. \quad (24)
\end{aligned}$$

Thus, (22) will be

$$\begin{aligned}
\mathscr{F}^{-1}\{\widehat{S}_{\dot{\mathrm{u}}\dot{\mathrm{v}}}\} &= \frac{\dot{N}}{\dot{L}}\widehat{R}_{\dot{\mathrm{u}}\dot{\mathrm{v}}}[n]\sum_{m=0}^{M-1}w_{\mathrm{d}}[m]w_{\mathrm{d}}[m-n] = \\
&= \frac{\dot{N}}{\dot{L}}\widehat{R}_{\dot{\mathrm{u}}\dot{\mathrm{v}}}[n]\cdot MR_{\mathrm{w_d}}[n]\,, \quad (25)
\end{aligned}$$

and so $\widehat{S}_{\dot{\mathrm{u}}\dot{\mathrm{v}}}$ can also be computed as

$$\begin{aligned}
&\widehat{S}_{\dot{\mathrm{u}}\dot{\mathrm{v}}} = \frac{\dot{N}}{\dot{L}}\mathscr{F}\{MR_{\mathrm{w_d}}[n]\cdot\widehat{R}_{\dot{\mathrm{u}}\dot{\mathrm{v}}}[n]\} = \\
&= \frac{\dot{N}}{\dot{L}}\mathscr{F}\{w_{\mathrm{ind}}[n]\widehat{R}_{\dot{\mathrm{u}}\dot{\mathrm{v}}}[n]\} = \frac{N}{\dot{L}}\mathscr{F}\{w_{\mathrm{ind}}[n]\widehat{R}_{\mathrm{uv}}[n]\}\,, \quad (26)
\end{aligned}$$

which, up to a multiplicative constant, is the indirect estimate of S_{uv} (the constat $\dot{N}/\dot{L}$ cannot be incorporated into the weighting window $w_{\mathrm{ind}}[n]$ because this window has to meet the condition $w_{\mathrm{ind}}[0] = 1$ [1, 8], which is accomplished exactly when $w_{\mathrm{ind}}[n] = MR_{\mathrm{w_d}}[n]$).

In conclusion, the direct and indirect approaches are equivalent up to a multiplicative constant, if the adjacent segments are mutually shifted by one sample only, $M-1$ zeros is padded before and behind the signals, and the weighting function $w_{\mathrm{ind}}[n]$ of the correlation estimate $\widehat{R}_{\dot{\mathrm{u}}\dot{\mathrm{v}}}[n]$ is given by

$$w_{\mathrm{ind}}[n] = \sum_{m=1}^{M} w_{\mathrm{d}}[m]w_{\mathrm{d}}[m+n]\,. \quad (27)$$

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