

# THE TENDON SLACK LENGTH CALCULATION FROM THE MAXIMAL AND MINIMAL EXCURSION OF MUSCULOTENDON

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**Abstract:** Many researchers and biomedical engineers dealt with developing of a general method for musculotendon (MT) actuator force calculation. In the equation, which expresses dynamic conditions of muscle and tendon loading, are some input parameters, which are difficult to measured or obtained otherwise. One of these input parameters is the tendon slack length  $L_s^T$ . From relationships between maximum and minimum MT length, pennation angle and normalized muscle length, has been derived relation for tendon slack length calculation. The values from this finding are close to experimentally estimated and published data.

## Introduction

In the equations, which express dynamic conditions of musculotendon loading, are some input parameters, which are difficult to measured or simply obtain otherwise. One of these input parameters is a tendon slack length  $L_s^T$ . Tendon slack length is the length on elongation at which tendon just begins to develop force [1]. It is very difficult to find the complete experimentally obtained data about human muscles. Therefore, some authors develop simulation methods for estimation of this value. Manal and Buchanan [2] use the numerical optimization method based on the fact, that the tendon slack length,  $L_s^T$ , is constant value when the musculotendon length,  $L^{MT}$ , and muscle length,  $L^M$ , are different in a different joint angle and the musculotendon length is measurable. In opposite, Garner and Pandy [3] use the two phase nested optimization technique for estimation of the tendon slack length,  $L_s^T$ , together with other muscle parameters as optimal fiber length,  $L_0^M$ , and isometric muscle force,  $F_0^M$  within a group of muscles. In the Garder and Pandy's method, the obtained parameters are not independent and an error in one parameter can denote error in other parameter and it is not possible to say which musculotendon parameter is true and which is false. The Manal and Buchanan's method is always for one muscle, it means that musculotendon parameters are independent between muscles, and this method assumes true values of inputs. For the necessity of this study, the own simulation method for estimation of the tendon slack length was derived, which is close to Garder and Pandy's method but musculotendon parameters are independent between muscles.

From the relationship between maximum and minimum length of musculotendon (MT), pennation angle, normalized muscle length and tendon slack length has been deduced relation for the tendon slack length calculation. After optimizing the interval of effective operating range of muscle length the values from this finding are close to experimentally measured and published data.

## Materials and Methods

Calculation of the length tension is in this case based on next statement: "When tendon slack length is large, muscle-fiber length is small; thus, muscle excursion will be small. Conversely, when tendon slack length is small, muscle-fiber length is large, and muscle excursion will be large. The muscle excursion is defined as the difference between the maximum physiological length  $L_{max}^{MT}$  and the minimum physiological length  $L_{min}^{MT}$  of the musculotendon [4]. Minimum and maximum physiological lengths can be calculated from position of MT actuator attachments and from joint angle value. Generally, the rela-

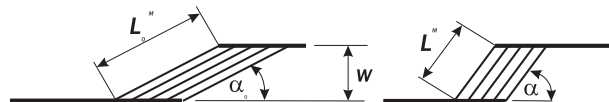


Figure 1: Relation between pennation angle  $\alpha$  and muscle fiber length  $L^M$  and optimal pennation angle  $\alpha_0$  and optimal muscle fibre length  $L_0^M$ , eq. (1).

tion between pennation angle and muscle fiber length and their optimal values is shown at Figure 1 and given by expression

$$w = L_0^M \sin(\alpha_0) = L^M \sin(\alpha). \quad (1)$$

The total musculotendon length is given

$$L^{MT} = L^T + L^M \cos(\alpha) \quad (2)$$

where

$$\cos(\alpha) = \sqrt{1 - \sin^2(\alpha)} = \frac{L_0^M}{L^M} \sqrt{(\tilde{L}^M)^2 - \sin^2(\alpha_0)} \quad (3)$$

is the relationship between cosine and sine of the pennation angle. A dimensionless muscle properties are used, proposed by Zajac [1]. The  $\tilde{L}^M$  is the normalized muscle fiber length, which is the muscle fiber length divided by the optimum muscle fiber length,  $\tilde{L}^M = L^M / L_0^M$ .

Table 1: Calculated values of  $L_s^T$  by eq's. (6) and (7) with the theoretical values  $\tilde{L}_{min}^M = 0.5$  and  $\tilde{L}_{max}^M = 1.5$ .

Muscle:	BRD [cm]	BRA [cm]	BIC [cm]	TRI [cm]
eq.(6)	21.90	3.19	23.45	25.22
eq.(7)	21.89	3.19	23.26	25.15
pandy	6.04	1.75	22.98	19.05
winters	7.00	3.00	20.50	19.33

Based on equations (2) and (3), for the maximal and minimal muscle length can be written

$$L_{min}^{MT} = L_s^T + L_0^M \sqrt{(\tilde{L}_{min}^M)^2 - \sin^2(\alpha_0)} \quad (4)$$

$$L_{max}^{MT} = L_s^T + L_0^M \sqrt{(\tilde{L}_{max}^M)^2 - \sin^2(\alpha_0)} \quad (5)$$

where tendon length  $L^T$  is in this case replaced by tendon slack length value  $L_s^T$ . This difference can be neglected because maximal strain of tendon, when tendon is interrupted is about  $\varepsilon^T = 4\%$ . For expression of  $L_0^M$  and  $L_s^T$  in terms two quantities  $\tilde{L}_{max}^M$  and  $\tilde{L}_{min}^M$  were used and represent the minimum and maximum physiological muscle lengths normalized by  $L_0^M$ .

From the equations (4) and (5) can be derived for tendon slack length

$$L_s^T = \frac{L_{max}^{MT} \sqrt{(\tilde{L}_{min}^M)^2 - \sin^2(\alpha_0)} - L_{min}^{MT} \sqrt{(\tilde{L}_{max}^M)^2 - \sin^2(\alpha_0)}}{\sqrt{(\tilde{L}_{min}^M)^2 - \sin^2(\alpha_0)} - \sqrt{(\tilde{L}_{max}^M)^2 - \sin^2(\alpha_0)}} \quad (6)$$

and for muscles where optimal pennation angle is small,  $\alpha_0 < 5^\circ$  and  $\sin(\alpha_0) \doteq 0$ , can be written the more simple form

$$L_s^T = \frac{L_{max}^{MT} \tilde{L}_{min}^M - L_{min}^{MT} \tilde{L}_{max}^M}{\tilde{L}_{min}^M - \tilde{L}_{max}^M} \quad (7)$$

The theoretical and ideal case the nominal region where muscle can produce active force is  $0.5 \leq \tilde{L}^M \leq 1.5$  [5]. For the minimal and maximal theoretical values of normalized muscle fiber length, which are  $\tilde{L}_{min}^M = 0.5$  and  $\tilde{L}_{max}^M = 1.5$ , can be written

$$L_s^T = 1.5L_{min}^{MT} - 0.5L_{max}^{MT} \quad (8)$$

the most simple relationship between tendon slack length  $L_s^T$  and the muscle excursion.

The equations (6) and (7) were used for the practical calculations. The simple musculoskeletal elbow problem with elbow actuators was used for the first validation of this finding. The musculotendon excursion of elbow muscles (Brachioradialis (BRD), Brachialis (BRA), Biceps brachii (BIC), Triceps brachii (TRI)) was calculated from the anatomical positions of muscle attachments and elbow angle. The range of elbow angle during full flexion and extension of elbow was  $0^\circ$  and  $145^\circ$ . Theoretical values of  $L_s^T$  are shown in Table 1.

The difference between results given by equations (6) and (7) and published data are in some cases more than

Table 2: Optimized values of  $\tilde{L}_{min}^M$  and  $\tilde{L}_{max}^M$ , and newly estimated  $L_s^T$  by eq's. (6) and (7)

Muscle:	BRD	BRA	BIC	TRI
optimized $\tilde{L}_{min}^M$	0.84	0.46	0.7	0.62
optimized $\tilde{L}_{max}^M$	1.17	1.54	1.3	1.38
new $L_s^T$ [cm] by (6)	10.90	3.96	21.92	20.22
new $L_s^T$ [cm] by (7)	10.87	3.86	21.91	19.30

100% , see table 1 - Brachioradialis. Here is offer to say, the real interval of effective operating range of muscle length is different but close to the  $\tilde{L}_{min}^M = 0.5$  and  $\tilde{L}_{max}^M = 1.5$  values. Assumed values of  $\tilde{L}_{min}^M$  and  $\tilde{L}_{max}^M$  do not valid for every muscle, evidently.

One of the possible way for elimination of differences in  $L_s^T$  is to optimize interval when muscle can produce active force, the values for normalized maximal and minimal muscle length,  $\tilde{L}_{max}^M$  and  $\tilde{L}_{min}^M$  respectively, when muscle can produce active force [6]. Optimization problem was to calculate new  $\tilde{L}_{max}^M$  and  $\tilde{L}_{min}^M$  parameters for each of muscle, based on minimization of the errors between new normalized parameters and assumed parameters,  $\tilde{L}_{min}^M = 0.5$  and  $\tilde{L}_{max}^M = 1.5$ , and the knowledge of the optimal muscle length  $L_0^M$  for calculated muscles taken from [7]. The optimization was constrained by the equation

$$L_0^M = \frac{L_{max}^{MT} - L_{min}^{MT}}{\sqrt{(\tilde{L}_{max}^M)^2 - \sin^2(\alpha_0)} - \sqrt{(\tilde{L}_{min}^M)^2 - \sin^2(\alpha_0)}} \quad (9)$$

derived from (4) and (5).

The Optimized values of  $\tilde{L}_{min}^M$  and  $\tilde{L}_{max}^M$ , and newly estimated  $L_s^T$  by eq's. (6) and (7) of elbow problem mentioned above are shown in Table 2.

## Results

Next task in tendon slack length estimation was application of this approach on the lower extremity muscles: Semimembranosus (SM), Biceps femoris long head (BFL), Biceps femoris short head (BFS), Tensor fasciae latae (TFL), Gracilis (GRC), Rectus femoris (RF), Vastus medialis (VM), Vastus Intermedius (VI), Vastus Lateralis (VL). The input data as positions of muscle attachments, pennation angle, optimal muscle fibre length and measured values of tendon slack length  $L_s^T$  were taken from the lower extremity musculoskeletal model including in SIMM software [8]. The maximal and minimal musculotendon length were derived from the combination of the knee and hip flexion/extension angles. The derived tendon slack lengths and optimized interval of effective operating range of muscle length  $\tilde{L}_{min}^M$  and  $\tilde{L}_{max}^M$  for selected lower extremity actuators are shown in Table 3.

Table 3: Tendon slack length values calculated for knee actuators from the optimized interval of effective operating range of muscle length  $\tilde{L}_{min}^M$  and  $\tilde{L}_{max}^M$ . The  $L_s^T*$  values are taken from the SIMM musculoskeletal lower extremity model [8].

Muscle	$\alpha_0 [^\circ]$	$\tilde{L}_{min}^M$	$\tilde{L}_{max}^M$	$L_s^T$ by (6) [cm]	$L_s^T*$ [cm]
SM	15	0.26	2.03	33.15	35.9
BFL	0	0.04	1.96	32.20	34.1
BFS	23	0.71	1.23	5.62	10
TFL	3	0.16	1.85	37.47	42.5
GRC	3	0.85	1.15	9.21	14
RF	5	0.11	2.06	33.05	34.6
VM	5	0.52	1.48	12.65	12.6
VI	3	0.51	1.49	13.82	13.6
VL	5	0.50	1.50	15.84	15.7

## Discussion

These values are of course only theoretical, because the difference between the tendon length  $L^T$  and the tendon slack length value  $L_s^T$  was neglected. This method reflect fact, that the people with the same anatomical proportions and the same positions of muscle attachments can have different tendon slack length because the maximum and minimum length of musculotendon depend strongly on the joint mobility.

It can be discussed, that the tendon slack length value must be greater than zero and from equation (8) can be derived

$$L_s^T \geq 0; \quad 1.5L_{min}^{MT} - 0.5L_{max}^{MT} \geq 0 \quad (10)$$

and obtained condition is:

$$\Rightarrow 3L_{min}^{MT} \geq L_{max}^{MT}. \quad (11)$$

This condition (11) corresponds with maximal and minimal sarcomere length, see for example [9].

## Conclusions

The necessary constraints for muscle modeling are input data about muscle physiological and morphology properties as positions of muscle attachments and significant points of body segments, PCSA, pennation angle  $\alpha_0$ , tendon slack length  $L_s^T$ , optimum muscle length  $L_0^M$ , maximal isometric force  $F_0^M$ , etc. The most of these parameters are experimentally estimated (measured) possibly. Especially in the intention of using the muscle forces as inputs to general calculations of FEM stress analysis of bones is adequate to use data from literature.

The problem is occurred in tendon slack length  $L_s^T$  and operating range of muscle length,  $L_{min}^M - L_{max}^M$ , the muscle length when muscle can produce active force. In general, theoretical operating range of muscle length for active muscle force is between  $0.5L_0^M$  and  $1.5L_0^M$  but real

active operating range of muscle length and  $L_s^T$  for concrete muscle is very hard to estimate or measure.

Here, the simple simulation method for estimation of the tendon slack length values, based on maximal and minimal length of musculotendon was derived. The second product from this simulation is the operating range of active muscle length for concrete muscle. The obtained  $L_s^T$  values are close to published measured data and is shown, that the operating range of active muscle length can be different from theoretical values. This method can be used for first approach and can help investigators, who can not directly measure this value.

**Acknowledgement:** This study was supported by the interdisciplinary research project No.: MSM 6840770012.

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