EVALUATION OF TORTUOSITY OF EYE BLOOD VESSELS USING THE INTEGRAL OF SQUARE OF DERIVATIVE OF CURVATURE

M. Patasius*, V. Marozas** A. Lukosevicius* and D. Jegelevicius*

* Kaunas University of Technology, Institute of Biomedical Engineering, Kaunas, Lithuania
** Kaunas University of Technology, Signal Processing Department, Kaunas, Lithuania

martynas.patasius@ktu.lt, vaidotas.marozas@ktu.lt, arunas.lukosevicius@ktu.lt, darius.jegelevicius@ktu.lt

Abstract: Tortuosity of eye fundus blood vessels is one of parameters that describe state of the blood vessels. It can be detected from fundus images. The increase in vessel tortuosity was observed in eyes of patients with advanced background diabetic retinopathy, papilloedema, even in some completely healthy eyes (in this case tortuosity does not change in time). Unfortunately, completely reliable definition and numerical estimation of tortuosity of line (blood vessel) does not exist, although there were some more or less successful attempts to define it. This study presents new way to estimate tortuosity.

Introduction

Tortuosity of eye blood vessels is one of the features by which vessels state can be described. It can be detected analyzing eye fundus images. The increase in vessel tortuosity was observed in eyes of patients with advanced background diabetic retinopathy, papilloedema, even in some completely healthy eyes (in this case tortuosity does not change in time) [1]. Unfortunately, completely reliable definition and numerical estimation of tortuosity of line (blood vessel) does not exist, although there were some more or less successful attempts to define it [2], [3]. This study presents new way to estimate tortuosity.

Materials and Methods

The preferred tortuosity estimate should posses these properties: resistance to noise, invariance to congruence transformations, possibly invariance to scale [2].

It is reasonable to assume that blood vessel is a planar curve that does not intersect itself. It can be defined parametrically. In computer memory it can be stored discretely (finite number of points). For every point of a continuous curve (and most of the points of discrete curve) it is possible to calculate curvature. For a parametric curve curvature can be calculated from derivatives of coordinates by formula [1]

$$k = \frac{x'y'' - y'x''}{\left(x'^2 + y'^2\right)^{\frac{3}{2}}} \tag{1}$$

If a curve is defined as dependency of one coordinate (y) from another (x), then curvature can be calculated:

$$k = \frac{y''}{\left(1 + y'^2\right)^{\frac{3}{2}}} \tag{2}$$

It is much harder to define tortuosity of a line. It is self-evident that a good measure of tortuosity has to be invariant of any congruence transformations (rotations, reflections) [1]. Opinions about desirability of its being invariant to scale transforms do vary [1]. Also, no consensus is achieved so far about relation between tortuosity of a line and tortuosities of parts of the same line [1][2].

There have been various attempts to define tortuosity of a line. One of the most simple methods is arc-chord ratio [1][2][3]. In this case, if length of the arc (curve) is L and distance between ends of curve (chord length) is S, it is possible to write:

$$V_1 = \frac{L}{S} \tag{3}$$

This method is simple, resistant to noise, easily generalized to three dimensional case. Furthermore, the measures like this are used in some of the other fields of science (tortuosity in geology, maneuver coefficient in military science etc.). Tortuosity calculated by this method is between one (for straight line) and infinity (for closed line). It is self-evident that scale has no influence to measurement in this case. Unfortunately, this method does not correspond to the understanding of tortuosity. For example, a circle is not thought to be a tortuous line, but its tortuosity by arc-chord ratio is infinite. Finally, no obvious way to generalize this method to network of blood vessels (lines) exists.

It is also possible to evaluate tortuosity as integral of absolute value or square of curvature:

$$V_{2a} = \int_{t_1}^{t_2} \left|k(t)\right| dt \tag{4}$$

$$V_{2b} = \int_{t_1}^{t_2} \left(k(t)\right)^2 dt \tag{5}$$

Both (4) and (5) measures can be divided by length of curve or chord. Also it is possible to filter coordinates of points that describe the curve [2]. By this method the tortuosity of straight line is zero, but tortuosity of circle is high.

According to the method, developed by computer scientists of University of Padua [1], tortuosity is calculated in a more complex way. At first line is divided to N intervals, with constant sign of curvature (with hysteresis). Then arc-chord ratio is calculated for each of them. The tortuosity itself is calculated by formula

$$V_3 = \frac{N-1}{L} \cdot \sum_{i=1}^{N} \left(\frac{L_i}{S_i} - 1 \right) \qquad (6)$$

This method efficiently evaluates tortuosity of the real blood vessels. Tortuosity of the circle (and other curves, curvature of which doesn't change its sign) is zero according to this method. Unfortunately, this method is not logically simple (for example, it's difficult to give an obvious meaning to threshold of hysteresis). Also, it is hard to define how this measure depends on scale (because of hysteresis).

Both straight line and circle are commonly held to be non-tortuous lines. Because of that it is rational to expect that line is tortuous when its curvature changes (analogous fact is used in statistics [7]). In this case it is possible to describe tortuosity by derivative of curvature or (as sign of it might change) its square or absolute value. As it is sensible to characterize not the point, but the curve by tortuosity, it is possible to use integral of square of derivative of curvature from the beginning of the line to the end of it:

$$V_{pi} = \int_{t_1}^{t_2} \left(k'(t) \right)^2 dt \qquad (7)$$

It is worth to compare tortuosities of different lines. In such case this measure should be divided by length of the line:

$$V_{pn} = \frac{V_{pi}}{L} \qquad (8)$$

This measure can easily be generalized for several lines (network of blood vessels): when all individual tortuosities are calculated, they should be multiplied by length of their lines and sum of these values should be divided by sum of all lengths.

However, this measure is not resistant to noise (three successive numerical differentiations). Resistance to noise can be improved by using low-pass filters. That would also make possible to measure tortuosity more flexibly: not to evaluate small curves. It is possible to filter the coordinates or any of intermediate results (derivatives of coordinates, curvatures etc.). That (with possibility to change parameters of filters) would allow improving this method even further. Also, a smoothing spline can be used to approximate the line. Spline could be used to get points, equally spreaded and the points would be processed by the same algorithm, possibly with filters. It is also possible to differentiate the resulting spline and get the values of derivatives analytically.

Parameters of filter can be optimized by using lines (vessels) with known tortuosity. While it is hard to give any quantitative estimate "by eye", it is much easier to group lines into classes of similar tortuosity and to sort the groups so that tortuosity would increase with number of the group. Then tortuosity of each line can be estimated using the algorithm and differences between adjacent classes calculated (estimate of line from lower class is distracted from estimate from higher class). Ideally, all the differences should be positive, so the minimal difference could be a goal function to be maximized. Most of general optimization methods could be used for this task.

It is worth to examine characteristics of this measure. First, it is invariant to congruence transformations, as both curvature and length of line.

The value of this measure greatly depends on scale. From (1) and (7) it can be seen that doubling coordinates of all the points will make V_{pi} decrease 4 times. Length of the line will increase twice, so the whole measure (V_{pn}) will decrease 8 times. Because of that it is worth to adjust the measure if it is used for comparing images of different scale.

In tables thee methods are represented with these numbers:

1. Arc-chord ratio.
2. Arc-chord ratio with hysteresis when threshold of hysteresis is 0.5.
3. Integral of module of curvature, divided by length of the line.
4. Integral of square of curvature, divided by length of the line.
5. Evaluations using integral of square of derivative of curvature:
   - 5.1. Without filtering.
   - 5.2. Filtering coordinates with the third order low pass Butterworth filter with normalized cut-off frequency 0.1.
   - 5.3. Filtering derivatives of the coordinates with the sixth order low pass Butterworth filter with normalized cut-off frequency 0.16 (optimized).

Both synthetic models of blood vessels and real fundus images where used to examine these methods.

The synthetic models chosen were graphs of functions (later to be identified by numbers):

1) $y = x$,
2) $y = \sin(x)$,
3) $y = \sqrt{x}$,
4) $y = \sqrt{x} + 0{,}1 \cdot \sin(10x)$,
5) $y = \sqrt{x} + 0{,}01 \cdot \sin(10x)$.

Here x changes from 0 to 50 with increment 0.1. Of these lines the first is the least tortuous, the third and the fifth – a little more tortuous, the second – more tortuous and the fourth – the most tortuous.

Also, several fundus images were used (Fig. 1, Fig. 2, Fig. 3, Fig. 4). Lines representing the vessels were extracted from them manually. As this process is not very accurate, it was repeated 15 times for each vessel and the resulting evaluations statistically measured.

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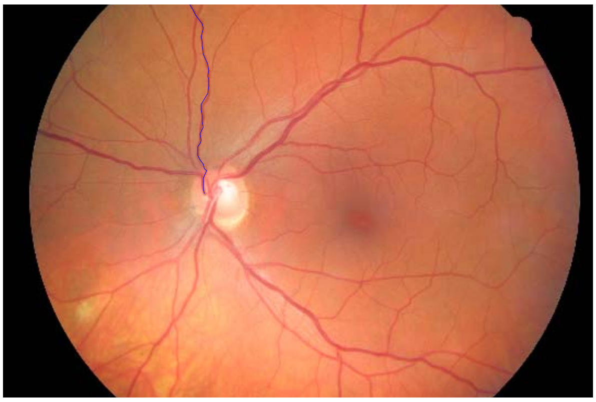

Figure 1: Image with relatively non-torturous vessels, almost straight vessel selected

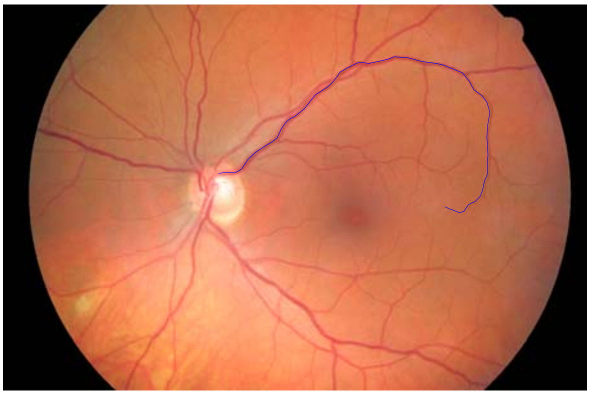

Figure 2: Image with relatively non-torturous vessels,: non-straight vessel selected

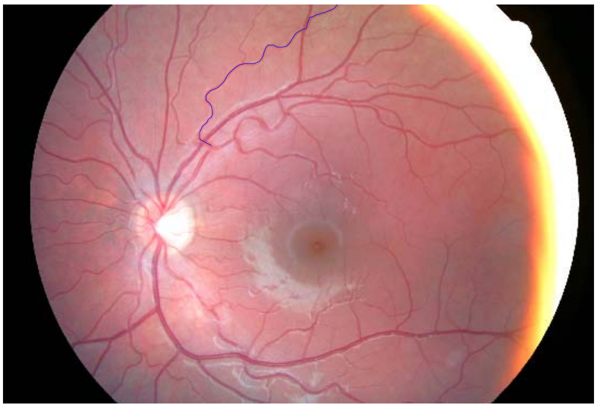

Figure 3: Image with torturous vessels, sample vessel selected

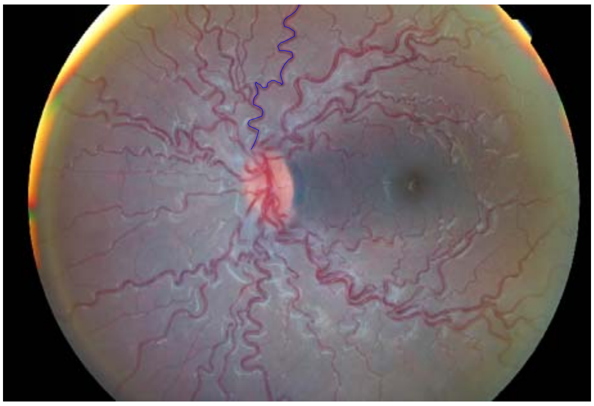

Figure 4: Image with very torturous vessels, sample vessel selected

Results

Some of methods to calculate tortuosity were implemented in MATLAB. It was tested how they find tortuosity of various lines.

At first, tortuosity of segment of straight line with or without noise (random values added to all the coordinates) was evaluated using different methods. Results are shown in Table 1. It can be seen that the proposed method without filtering is very sensitive to noise.

Table 1: Evaluations of tortuosity of straight line with of without noise using different methods

Method	Without noise	With noise
1.	1	1
2.	0	0.1908
3.	0	1.6176
4.	0	0.5861
5.1.	0	7.5815
5.2.	0	8.6537e-006
5.3.	0	9.9852e-006

Then tortuosities of more complex synthetic models were evaluated (Table 2). Of all methods, the versions of proposed method with filtering have shown the results that correspond best with the visual results.

Table 2: Evaluations of tortuosity of various lines using different methods

Method	2)	3)	4)	5)
1.	1.2148	1.0091	1.2031	1.0113
2.	0.5371	0	19.1539	3.3618
3.	3.7290	0.1566	49.7108	9.2161
4.	2.5859	0.0476	504.6718	10.8655
5.1.	0.0404	0.0054	1.4779e+003	16.8224
5.2.	0.0408	0.0017	1.0143	0.0123
5.3.	0.0393	0.0196	0.1766	0.0215

Tables 3, 4, 5 and 6 show statistical measures of evaluations of tortuosity of vessels, manually extracted from real fundus images. From them it is possible to see

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that of all tested methods the version of proposed method with filtering of derivatives of coordinates gives the best results: minimal evaluation of torturous vessels are higher than maximal evaluations of non-torturous vessels.

Table 3: Statistical measures of evaluations of tortuosity of vessel shown in Fig. 1 using different methods

Method	Mean	Std	Min	Max
1.	1.0622	0.0052	1.0519	1.0715
2.	2.3101e-04	4.7857e-04	0	0.0012
3.	0.0023	0.0014	9.8022e-04	0.0054
4.	0.0020	0.0044	7.8816e-05	0.0170
5.1.	0.0131	0.0287	6.3238e-04	0.1110
5.2.	4.8863e-06	9.9983e-06	5.3130e-12	4.0217e-05
5.3.	1.6320e-08	2.3729e-08	1.0339e-09	9.1777e-08

Table 4: Statistical measures of evaluations of tortuosity of vessel shown in Fig. 2 using different methods

Method	Mean	Std	Min	Max
1.	2.0552	0.0974	1.7173	2.1455
2.	1.4691e-04	3.0446e-04	0	7.6470e-04
3.	0.0016	0.0017	4.7052e-04	0.0070
4.	0.0065	0.0180	1.8812e-04	0.0703
5.1.	0.0398	0.1158	2.0562e-04	0.4442
5.2.	1.2824e-07	1.3092e-07	3.0358e-08	4.8482e-07
5.3.	8.4210e-09	7.4043e-09	1.9319e-09	2.7776e-08

Table 5: Statistical measures of evaluations of tortuosity of vessel shown in Fig. 3 using different methods

Method	Mean	Std	Min	Max
1.	1.2684	0.0059	1.2576	1.2768
2.	3.0486e-04	6.7804e-04	0	0.0023
3.	0.0027	0.0016	0.0014	0.0069
4.	0.0061	0.0179	3.0040e-05	0.0687
5.1.	0.0103	0.0214	9.1671e-04	0.0839
5.2.	5.0189e-06	2.2599e-06	1.2176e-06	8.0926e-06
5.3.	6.1893e-07	4.0699e-07	1.1520e-07	1.4284e-06

Table 6: Statistical measures of evaluations of tortuosity of vessel shown in Fig. 4 using different methods

Method	Mean	Std	Min	Max
1.	1.5933	0.0159	1.5655	1.6198
2.	6.9263e-04	0.0015	0	0.0057
3.	0.0039	0.0035	0.0015	0.0155
4.	0.0109	0.0318	9.1633e-05	0.1215
5.1.	0.0704	0.2046	4.7983e-04	0.7829
5.2.	6.4317e-06	2.7480e-06	2.9980e-06	1.2716e-05
5.3.	6.0036e-06	8.9251e-06	7.2596e-07	3.2093e-05

Discussion

As tortuosity of retinal blood vessels is generally thought to be caused by increased blood pressure or blood viscosity [1], it seems reasonable to assume that a good tortuosity estimation might be adapted to be used not only directly for diagnostics of various eye diseases, but also for blood pressure (or blood viscosity) estimation. It seems likely that such method would help to estimate not the momentary blood pressure (or blood viscosity), but average blood pressure in given time interval, what might be useful in some circumstances (for example, in veterinary, as preparations for blood pressure measurement themselves might cause stress to animal and increase the momentary blood pressure).

Conclusions

The proposed definition of tortuosity of the blood vessels in eye fundus images is intuitively understandable and gives acceptable results usually better than definitions under comparison.

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