COLLECTIVE NEURON DYNAMICS THROUGH CNN

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Abstract: In this paper, the extended dynamics and self-organized behaviors of biological neuron networks have been reproduced through a new analog computational architecture (the ACE16k VLSI chip). The system exactly maps the equations of the two-dimensional network of integrate-and-fire neurons. Experimental results on CNN chips are reported.

Introduction

The mathematical modeling of the neuron action potential related to a synaptic input current faces two fundamental aims: biological plausibility and implementation cost. Particularly, the integrate-and-fire (IF) neuron reproduces the main characteristics of the neuron evolution with a first order differential equation with a reset condition. This model presents the lower implementation cost in spite of a weak biological description. For an analogical implementation of an extended system consisting of biological coupled neuron the IF model represents a good trade-off.

In this work, the paradigm of Cellular Nonlinear Networks (CNNs), introduced by Leon O. Chua in 1988 [1] and used to emulate complex dynamics, has been selected to realize spatial-temporal biological neuron networks [2]. These are constituted by an array of locally coupled analog cells. Each elementary unit is a nonlinear first order circuit. The CNN identical cells are arranged in a rectangular grid and interact with their neighbors through the inputs, the outputs and the state signals. Each cell is mutually coupled with its nearest neighbors by means of voltage controlled current sources weighted by constant coefficients, known as the templates. Moreover, hardware implementations of CNNs have been developed and a chip constituted by 128×128 cells in 0.35 μm CMOS technology has been already designed and tested as result of the European project called DICTAM [3, 4]. In particular, this device is based on the dynamic evolution of an analog computational system programmable through a digital interface. The first phase of this work aims at giving an equivalent model of the IF neuron through a CNN unit. In the second phase of this study, an extended network of IF models has been considered by connecting neurons in square arrays where the coupling coefficient models the resistive synaptic connection towards the neuron action potentials. Finally, the CNN architecture has been adopted to model and to reproduce the spatio-temporal patterns characterizing the considered two-dimensional array of IF neurons.

IF model through CNN

In this section we first introduce the integrate-and-fire neuron model and then a network made up of IF cells. A rectangular grid, i.e. a network of identical cells arranged in a two-dimensional array with local connections, is taken into account. We have adopted this spatial configuration in order to match the well-known CNN paradigm as it is implemented on existing VLSI chips. After the introduction of the IF model, the CNN model is discussed and the equations to implement the IF model on a CNN are derived.

IF model

The integrate-and-fire neuron model is characterized by a first-order differential equation derived from the application of an input current $i(t)$ to an RC electrical circuit model. It describes the neuronal membrane potential dynamic behavior. As the neuron dynamic has a nonlinear behavior also the IF model needs the application of a nonlinearity to the state variable V. The IF model is described by the following equations:

$$C_{IF}\dot{V}(t) = -\frac{V(t)}{R_{IF}} + I \qquad (1)$$

with the resetting condition

$$\texttt{if } V(t^-) \geq V_{th} \Rightarrow V(t^+) = V_{rest} \qquad (2)$$

where $V_{th} = 30mV$, $V_{rest} = -70mV$, $\tau = R_{IF}C_{IF} = 10ms$, $C_{IF} = 1\mu F$, $I = 4\mu A$ In order to introduce a network of IF neurons, let us first define the neighborhood of a given cell as follows:

$$N_r(i,j) = \{C(k,l)\,|\max(|k-i|,|l-j|) \leq r\} \qquad (3)$$

Moreover, let us also define a neighborhood which does not include the cell itself, as follows:

$$\widetilde{N}_r(i,j) = \{C(k,l)\,|\max(|k-i|,|l-j|) \leq r, (k,l) \neq (i,j)\} \qquad (4)$$

We now consider a two-dimensional array of IF neurons. The array consists of $M \times N$ identical cells.

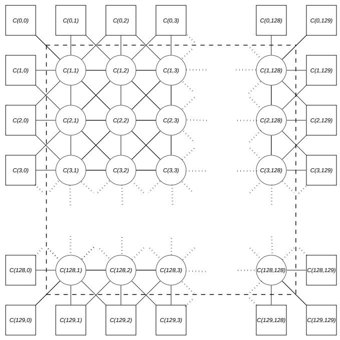


Figure 1: Two-dimensional array of 128×128 identical cells. The model is used both for the network of IF neurons and for the CNN cells.

As shown in Fig. 1, two further rows and two further columns of *pseudo* cells are included in the model to account for boundary conditions. The state equation of the cell $C(i,j)$ of the IF network is the following:

$$C_{IF}\dot{V}_{ij}(t) = -\frac{V_{ij}(t)}{R_{IF}} + I_{IF} + D \sum_{C(k,l)\varepsilon \widetilde{N}_r(i,j)} [V_{kl}(t) - V_{ij}(t)] \tag{5}$$

$\forall i \ \varepsilon \ 1,2,\ldots,M$ and $\forall j \ \varepsilon \ 1,2,\ldots,N$ with the resetting condition:

$$\texttt{if} \ V_{ij}(t^-) \geq V_{th} \Rightarrow V_{ij}(t^+) = V_{rest} \tag{6}$$

CNN equations

A CNN is defined as a two-dimensional array of MxN identical cells arranged in a rectangular grid. Each cell mutually interacts with its nearest neighbors by means of constant coefficients $A(i,j;k,l)$ and $B(i,j;k,l)$ known as the *feedback and input cloning templates*, respectively. If they are equal for each cell, they are called *space-invariant*. If *B(i,j;k,l)=0* the CNN is said autonomous. A CNN is described by the state equations of all cells:

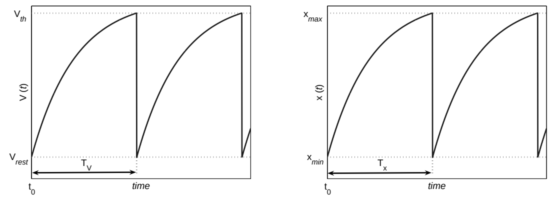
$$C_x \cdot \frac{dx_{ij}(t)}{dt} = -\frac{x_{ij}(t)}{R_x} + \sum_{C(k,l)\in N_r(i,j)} A(i,j;k,l)y_{kl}(t) + \\ + \sum_{C(k,l)\in N_r(i,j)} B(i,j;k,l)u_{kl}(t) + I \tag{7}$$

with $i = 1,.2,...,M$ and $j = 1,2,...,N$ where the neighborhood is defined as in equation (3) and the nonlinearity is

$$y_{ij} = 0.5(\left|x_{ij}+1\right| - \left|x_{ij}-1\right|) \tag{8}$$

The CNN algorithm for emulating IF dynamics

The general idea underlying the CNN-based model of the IF dynamics is to apply two different sets of templates to the CNN. The first one accounts for the linear behavior of the IF expressed by equation (5). The second set of templates implements the resetting mechanism expressed by equation (6). These templates will be derived by taking into account that in general the dynamics implemented on the CNN hardware and that of the IF neuron will have different dynamic ranges and typical time scales. In fact, the dynamic range of CNN devices is related to the typical supply voltage levels of the existing chips and also the time scale is connected to the proper working frequencies of the device. As shown in Fig. 2 the idea is thus to find appropriate parameters for the CNN model so that the dynamics implemented and that of the IF neuron are equivalent with the exception of dynamic range and time factor rescaling.



(a) (b)

Figure 2: Trend of the state variable $V(t)$ of a single IF neuron (a) and of the corresponding state variable $x(t)$ of an uncoupled CNN cell (b). For both models the dynamic range and the time scale are shown.

Let us first focus on the set of templates needed to implement equation (5). Let us consider an autonomous CNN (i.e. $\mathrm{B}(i,j;k,l) = 0$) and let us consider the following form for the feedback template:

$$\mathrm{A} = \begin{bmatrix} a_d & a_d & a_d \\ a_d & a_x & a_d \\ a_d & a_d & a_d \end{bmatrix} \tag{9}$$

Furthermore, let us assume that $|x_{ij}(t)| \leq 1 \ \forall t$, so that $y_{ij}(t) = x_{ij}(t) \ \forall t$. We can therefore rewrite the CNN equation (7) as follows:

$$\dot{x}_{ij}(t) = (\tfrac{a_x}{C_x} - \tfrac{1}{C_x R_x})x_{ij}(t) + \tfrac{I}{C_x} + \tfrac{a_d}{C_x} \sum_{C(k,l)\in \widetilde{N}_r(i,j)} x_{kl}(t) \tag{10}$$

In order to match equations (5) and (10) and in particular the dynamic range of the state variable $V_{ij}(t)$ and $x_{ij}(t)$, let us note that the dynamic range of $V_{ij}(t)$ is bounded by the minimum value V_{rest} and the maximum value V_{th} and let us define the minimum and maximum dynamic bounds of $x_{ij}(t)$ as X_{min} and X_{max}. In order to respect the condition $|x_{ij}(t)| \leq 1$, the condition $-1 \leq X_{min} < X_{max} \leq 1$ should hold. A linear relationship of the form

$$V_{ij} = m x_{ij} + p \tag{11}$$

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can be considered to equal the two dynamic ranges. It can be easily found that

$$m = \frac{V_{th} - V_{rest}}{X_{max} - X_{min}}$$

and

$$p = V_{rest} - mX_{min}.$$

We now apply a similar consideration to the time scales of the two models. The time scales of the two models are defined by considering the interspike period indicated as T_V and T_x (Fig. 2). Let us consider a time scale factor k_v defined as follows:

$$\dot{x}(t) = k_v \dot{V}(t) \tag{12}$$

The value of T_V is related to the IF model, whereas the value of T_x is imposed by hardware constraints. In fact, the hardware device used requires a minimum sampling time to acquire and store the analog data. Therefore, we fixed T_x in such a way that for each interspike period the waveform generated by the CNN device could be sampled with a sufficiently high number of samples.

The value of k_v depends on T_V and T_x. It can be proved that:

$$k_v = \frac{T_V}{mT_x} \tag{13}$$

The last step in the derivation of the CNN-based model is to find the parameters of the CNN templates (i.e. a_x, a_d and I) as a function of those of the IF model.

Since $\dot{x}_{ij}(t) = k_v \dot{V}_{ij}(t)$, substituting equation (5) and equating the expression with equation (10), the parameters of the template can be found:

$$\begin{aligned} a_x &= -mk_v C_x\left(\tfrac{1}{\tau} + 8\tfrac{D}{C_{IF}}\right) - \tfrac{1}{C_x R_x} \\ I &= k_v C_x\left(\tfrac{I_{IF}}{C_{IF}} - \tfrac{p}{\tau}\right) \\ a_d &= mk_v D \tfrac{C_x}{C_{IF}} \end{aligned} \tag{14}$$

As concerns the resetting mechanism, it can be implemented considering another set of templates. However, the CNN VLSI chip used in this work provides a very effective way to implement this mechanism. A writing mask can be applied to selectively reset the state values of the CNN cells. The cells to be reset are selected by comparing their state values with the threshold value.

Implementation of the IF model on the ACE16k chip

The ACE16k chip is a chip of 128×128 first-order CNN cells. The peculiarity of the chip is to have an analog core entirely interfaced with a digital circuitry so that it is able to implement a CNN Universal Machine (CNN-UM). The CNN-UM processes on a parallel analog hardware a sequence of templates, thus executing a spatio-temporal algorithm.

The chip has several working modes. Depending on the working model used, the dynamical equations of a single cell are slightly different (in particular, different parameters can be set in each working mode). In the adopted working mode the CNN is autonomous (i.e. $B = 0$) and the template A has the following form:

$$A_{hw} = \begin{bmatrix} v_{w(0)} & v_{w(1)} & v_{w(2)} \\ v_{w(3)} & 2(v_{w(4)} + v_{wx2} + v_{wx3}) & v_{w(5)} \\ v_{w(6)} & v_{w(7)} & v_{w(8)} \end{bmatrix} \tag{15}$$

Moreover, in this working mode the bias is given by

$$I = 2Kv_{w(9)}v_{xmax} \tag{16}$$

In our case, some parameters of the template A_{hw} can be considered equal each other. In particular, we fixed $v_{w(j)} = v_{wd}$, $\forall j = 0, 1, \ldots, 8,\ j \neq 4$ and $v_{w(4)} = v_{wx2} = v_{wx3} = v_{wx}$. Therefore, the cell state equations assume the following form:

$$\dot{v}_{ij} = \tfrac{6Kv_{wx}}{C_x} v_{ij} + \tfrac{2Kv_{w(9)}}{C_x} v_{xmax} + \tfrac{Kv_{wd}}{C_x} \sum_{C(k,l)\in\widetilde{N}_r(i,j)} v_{kl} \tag{17}$$

where $K = 1.28\mu A/V^2$ and $C_x = 1.26pF$ are constant parameters and their value is that implemented in ACE16k chip. Equation (17) is equal to equation (10), if the parameters of ACE16k are chosen as follows:

$$\begin{aligned} v_{wx} &= -mk_v \tfrac{C_x}{6K}\left(\tfrac{1}{\tau} + 8\tfrac{D}{C_{IF}}\right) \\ v_{w(9)} &= k_v \tfrac{C_x}{v_{xmax}}\left(\tfrac{I_{IF}}{C_{IF}} - \tfrac{p}{\tau}\right) \\ v_{wd} &= mk_v \tfrac{C_x}{K}\tfrac{D}{C_{IF}} \end{aligned} \tag{18}$$

All the parameters which characterize the evolution of the ACE16k CNN-UM are stored in digital memories and then used in analog multipliers after a digital to analog conversion. For this reason, quantized values of the parameters are effectively implemented on the ACE16k CNN-UM. The quantization error associated with this analog to digital conversion is in the order of magnitude of 0.15% and does not significantly influence the performance of the system.

Experimental results

In order to verify the proposed approach, the IF neurons network model and the CNN based network implemented on ACE16k has been compared. A network of IF neurons as described by equations (5) and (6) has been considered. The interspike period of this model ($T_v \simeq 24ms$) can be easily calculated by the solution of equation (5) fixing V_{th}. Fig. 3 shows the time series related to nine network cells obtained by integrating the model with a simulation time $T_{sim} = 100ms \simeq 4 \cdot T_v$ and an integration step $\Delta t_{IF} = 0.2ms$.

In order to obtain the same behavior for the CNN network models the value of the interspike period T_x have been setup. The minimum sampling time of the ACE16k chip available to acquire the signal generated by its analog core is about $10ns$, then the integrating step of the CNN model has fixed as $\Delta t_{CNN} = 10ns$. The number of

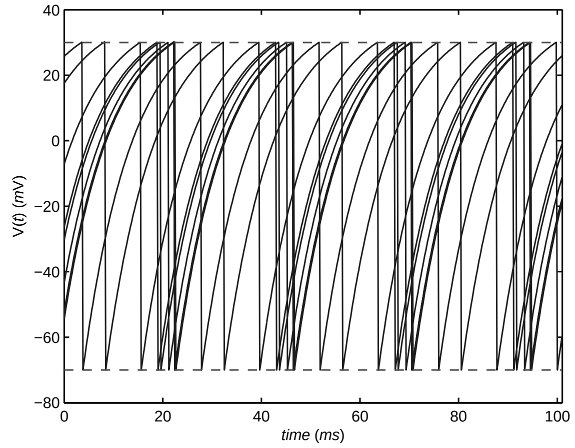

Figure 3: Time series related to nine cells of the IF network model simulation .

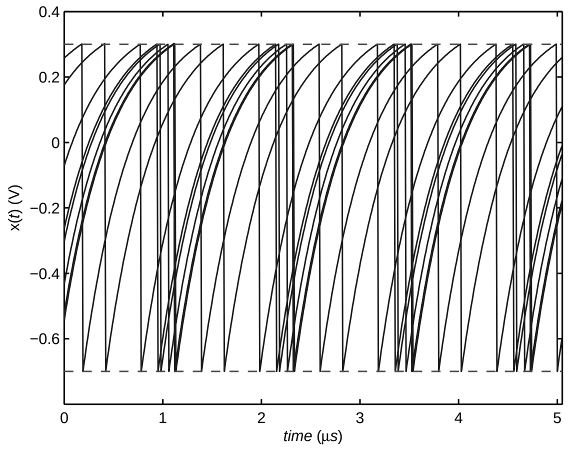

Figure 4: Time series relative to nine cells of the CNN network model simulation.

samples in an interspike period is coherent between the models choosing

$$T_x = \frac{\Delta t_{CNN}}{\Delta t_{IF}} \cdot T_v \simeq 1.2\mu s \tag{19}$$

The CNN dynamic range has been imposed fixing $X_{max} = 0.3V$ and $X_{min} = -0.7V$, therefore, applying the CNN algorithm for emulating IF dynamics, proper values for the CNN template parameters a_x, a_d and I can be found. By simulating the CNN network model (10) with the imposition of the same non-linearity of IF neuron model, we obtain the behavior shown in Fig. 4.

The software simulations has confirmed the mathematical equivalence between the two models. Further results have been found comparing CNN network simulation with hardware implementations measurements. IF network behavior can be emulated by ACE16k chip applying template parameters fixed by equation (18). Fig. 5 shows the dynamical evolution of nine cells of the ACE16k chip.

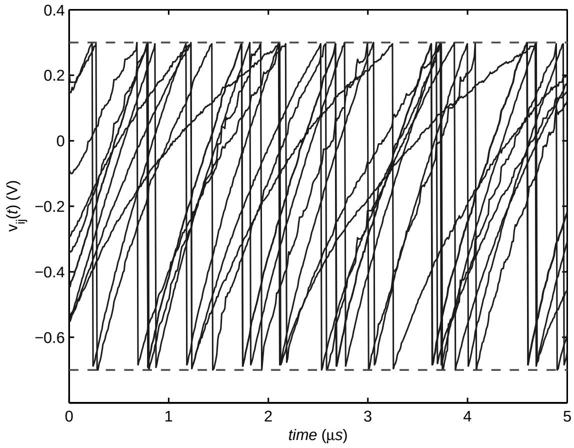

Figure 5: Evolution of nine cells of the ACE16k chip emulating IF network model.

Conclusions

In this paper extended dynamics of coupled neurons have been reproduced by using an analog architecture based on the CNN paradigm. The nonlinear neuron dynamics have been described by a CNN-cell model that maps the integrate-and-fire behavior. The spatio-temporal system have been simulated on chip constituted by 128×128 CNN cells in 0.35 μm CMOS technology. Several collective dynamics have been reproduced and integrate and fire neuron complex patterns recognized.

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