# VALIDATION OF IMPORTANCE SAMPLING IN GATE

J. De Beenhouwer*, S. Staelens*, A. Goedicke**, B. Schweizer**, Y. D'Asseler* and I. Lemahieu*

* Ghent University, Elis Department, Sint-Pietersnieuwstraat 41, B-9000 Ghent, Belgium
** Philips Research Laboratories,Weisshausstrasse 2, D-52066 Aachen, Germany

jan.debeenhouwer@ugent.be

**Abstract: Monte Carlo simulations of single photon emission computed tomography (SPECT) acquisitions generally require long computation times because the collimator stops most photons from reaching the detector crystal. The Monte Carlo platform GATE (Geant4 Application for Tomographic Emission) is capable of simulating an acquisition with accurate physical models [1] [2]. The downside of this high accuracy is an even longer computation time, taking up to several days for a single acquisition on a single CPU. A variance reduction technique known as importance sampling can be used to speed up this process. In this paper the validation results of GATE SPECT simulations with geometrical importance sampling are presented. The validation is based on the comparison of simulations with and without importance sampling. The examined properties include the energy spectra, the spatial resolution and sensitivity of typical SPECT setups based on $^{99m}$Tc and $^{67}$Ga isotopes. Overall results showed an excellent agreement between importance sampling simulations and analog simulations, with a 5 to 13-fold efficiency increase.**

## Introduction

Monte Carlo simulations are computationally very expensive. Geometrical importance sampling is besides analytical detector response modelling and more efficient tracking of particles through voxelized phantoms, a way to increase the efficiency of simulations. It is a variance reduction technique based on the crude criterion that only photons with a high detection chance should be tracked. Photons are increasingly split into exact copies with lowered weights as the distance to a detector decreases. Photon paths leading away from a detector are less likely to result in detection and therefore these photons are subjected to Russian roulette in order to increase the simulation efficiency. However, the technique introduces branches into the particle history. This results in a much more complicated pulse calculation when used for single photon emission computed tomography (SPECT) simulations. This paper is focused on the validation of SPECT simulations with importance sampling as [3] already explains how importance sampling is incorporated into GATE. As a first indication of validity of the particle history detangling approach, the energy spectra of both a low and medium energy isotope in air and in a water phantom are examined and compared with their analog counterparts. Further on, the spatial resolution is examined and the sensitivity is checked for a medium energy point source at different depths in a water phantom for a medium energy setup. In order to determine a figure of merit (FOM) indicating the efficiency increase through a variance reduction, a set of 100 simulations with and without importance sampling is examined.

## Materials and Methods

### *Efficiency estimation and variance behaviour*

The estimation of a FOM, indicating the efficiency increase over an analog simulation is of critical importance for any variance reduction technique. To this aim, a set of 100 simulations with importance sampling was compared with a set of 100 analog simulations in order to determine the behaviour of the variance of an estimate. Each simulation consisted of a uniform $^{99m}$Tc source of 3.5 MBq in a waterphantom for a realistic detector setup as in [4]. The phantom consisted of a 2 mm thin cylinder with a radius of 25 mm. A region of interest (ROI) of 30 mm x 30 mm was selected in each resulting projection in the photopeak window, and a size of 1mm was chosen for the projection bins. The score in each pixel was divided by the number of simulated events and thus it represents a flux through each pixel in the ROI. At this point it was possible to calculate the mean flux for each pixel in the ROI and the corresponding variance over the 100 simulations, for both the analog and non-analog case. In order to derive a FOM we compared the mean variances over all pixels in the ROI of both situations and scaled down the number of events in the importance sampling case until the mean variances agreed. Finally it was possible to compare the distribution of the variance around the mean variance over the ROI, and the distribution of the mean (in each pixel over the 100 samples) around the mean over the ROI between the analog and non-analog cases.

### *Energy spectra evaluation*

Energy spectra are a first indication of validity of the particle history detangling approach used for splitting and Russian roulette. Identical configurations were used each time for the importance sampling simulation and the ana-


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log simulation. First the energy spectrum of both $^{99m}$Tc and $^{67}$Ga in air was compared with the analog case. A $^{99m}$Tc point source with a radius of 1.5mm was used at a distance of 25cm from the collimator. The activity of the source was 150 MBq for the importance sampling case and 300 MBq for the analog case, with a total acquisition time of 30 seconds. The setup consisted of a single realistic detector head with a low energy high resolution (LEHR) collimator attached. The LEHR collimator was replaced by a medium energy general purpose (MEGP) collimator for the simulations with $^{67}$Ga. The point source with a radius of 1.5mm was placed at a distance of 25 cm from the collimator. The activity of the source was 36 MBq for importance sampling and 250 MBq for the analog simulation, with an acquisition time of 30 seconds. The activity values were chosen to result in a comparable number of detections, based on the number of detections per second during a short test run. As a final verification, the $^{67}$Ga point source was placed in the middle of a water phantom made of a cylinder with radius 12 cm and height 34.56 cm. An equal activity of 100 MBq for both cases was used over an acquisition time of 30 seconds. In this case the activity values were chosen equal in order to verify the improved statistics in the importance sampling simulation.

*Spatial resolution validation*

The spatial resolution for low and medium energy simulations was compared with the analog case. For $^{99m}$Tc a 30 MBq point source with a radius of 0.5 mm was placed at 3.65, 13.65, 24.65 and 38.65 cm from the detector with LEHR collimator in the importance sampling case. The analog case consisted of a 20 MBq line source with a radius of 0.5 mm at 5, 15, 26 and 40 cm from the LEHR collimator as in [4]. The medium energy setups consisted of a 1.5 mm point source filled with 20 MBq $^{67}$Ga and placed at 3.65, 13.65, 24.54 and 38.65 cm from the detector with MEGP collimator attached for both the analog and the importance sampling case. For the $^{99m}$Tc simulations a photopeak window at 129-151 keV was used, while for $^{67}$Ga two photopeak windows were used : at 83.7-102.3 keV and at 171.1-198.8 keV.

*Sensitivity validation*

The absolute sensitivity (in cps/MBq) was evaluated for a medium energy setup. A point source filled with 20 MBq (importance sampling) and 100 MBq (analog) $^{67}$Ga was placed at different depths in a cylindrical water phantom with a radius of 12 cm and height 34.65 cm : 1, 5, 10, 15 and 20 cm depth. The activity values were chosen to result in a comparable number of detections and to limit the total computing time. The total acquisition time was 30 seconds and two photopeak windows were used : at 83.7-102.3 keV and at 171.1-198.8 keV. A realistic detector was used with a MEGP collimator attached. Dead time correction was not necessary because of the resulting low count rate at the detector.

## Results

*Efficiency estimation*

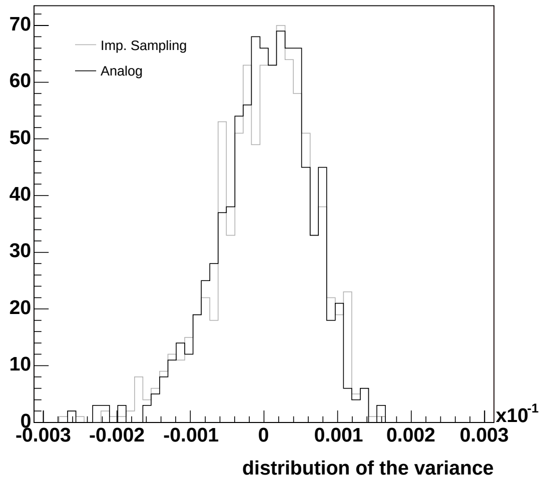


Figure 1: Distribution of the variance over the ROI around its mean value.

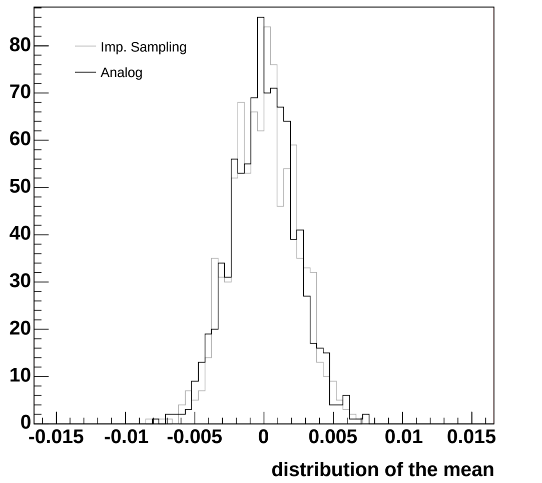


Figure 2: Distribution of the mean over the ROI around its the mean value.

Figure 1 shows the distribution of the variance for all pixels (based on 100 simulation samples) in the ROI around its mean value for both the analog and the importance sampling case. The distribution of the variance with importance sampling shows no constant bias when compared to the analog distribution. Figure 2 shows the distribution of the mean for all pixels (based on 100 simulation samples) in the ROI around its mean value for both cases. Again the distrubution shows no constant bias against the analog distribution. In order to compare the distributions, a factor of 12.5 fewer detections were used for importance sampling. The number of detections per second were 0.36 (analog) and 4.73 (importance sampling). As a


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crude FOM, the division of the number of detections per second for both cases results in a factor 13.2 increase in efficiency.

*Energy spectra evaluation*

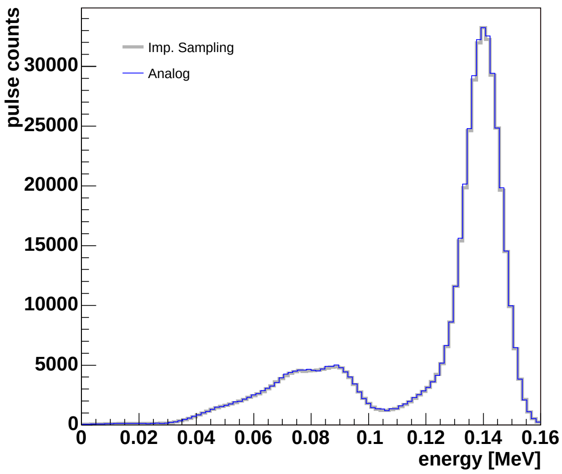


Figure 3: $^{99m}$Tc spectra for a point source in air with and without importance sampling

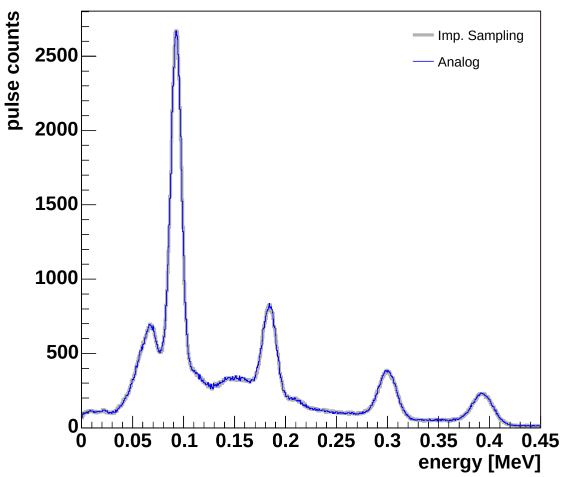


Figure 4: $^{67}$Ga spectra for a point source in air with and without importance sampling

Figure 3 shows the energy spectrum of a $^{99m}$Tc point source in air both with and without importance sampling. Figure 4 shows the same for $^{67}$Ga. An excellent agreement was found between the analog and the importance sampling simulations for the spectra of both isotopes. Figure 5 shows the comparison between the energy spectra of analog and importance sampling simulations for a $^{67}$Ga point source in a water phantom. Table 1 shows the simulation details. A clearly lower variance can be observed for the importance sampling case. Division of the

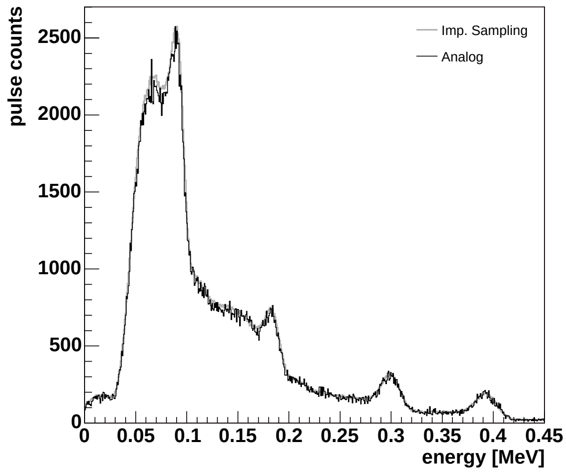


Figure 5: $^{67}$Ga spectra for a point source in a water phantom with and without importance sampling

Table 1: The simulation and acquisition time is shown with and without importance sampling for an equal number of starting events, together with the number of detections per second.

| | imp. sampling | analog |
|---|---|---|
| Activity | 100MBq | 100MBq |
| Acq. time | 30s | 30s |
| Sim. time | 5,600,000s | 1,954,000s |
| Detections/s | 0.93 | 0.17 |

number of detections per second for both cases gives an indication of the relative efficiency, being 5.5 in this case.

*Spatial resolution validation*

The spatial resolution for a low energy isotope ($^{99m}$Tc) and a medium energy isotope ($^{67}$Ga) is shown in figure 6. A linear was drawn through the experimental values obtained from [4] and the simulation values in the analog case. In both the low and medium energy case a good agreement was found.

*Sensitivity validation*

Figure 7 shows the absolute sensitivity results for a $^{67}$Ga point source in a water phantom at different depths. The results with and without importance sampling are in excellent agreement.

## Discussion

In order to study the influence of geometrical importance sampling on the variance of an estimate, a set of 100 analog and 100 non-analog simulations were run. With such a large number of simulations it was not feasible to run very high count studies. A relatively small phantom was

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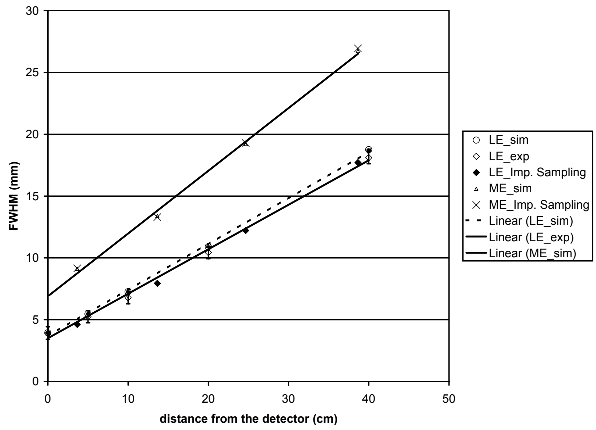


Figure 6: Spatial resolution comparison for low and medium energy setups. For $^{99m}$Tc both simulations (with and without importance sampling) and experimental values are shown. For $^{67}$Ga the results with and without importance sampling are shown. A linear was drawn through both simulation (analog only) and experimental values.

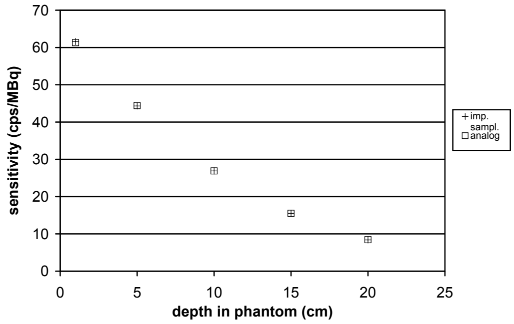


Figure 7: Absolute sensitivity for a $^{67}$Ga point source in a water phantom at different depths in the phantom. Results for simulations with and without importance sampling are shown.

chosen however at a close distance from the detector. A distribution of the variance and mean values was calculated and no constant bias could be detected for the importance sampling values when compared to the analog case. By comparing the crude FOM based on the number of detections per second (13.2) with the factor by which the number of events in the importance sampling simulations was decreased (12.5), it can be concluded that the FOM can be used as an indication of effiency for this particular type of geometrical importance samplng.

In theory, the maximum efficiency of this technique is inversely related to the sensitivity of the detector. The larger septa of a MEGP collimator result in a larger angle of acceptance. This allows for a larger flux through the collimator holes and reduces the benefit of particle splitting. This lower efficiency can be observed for the simulations with a MEGP collimator with an efficiency increase of 5.5 compared to those with a LEHR collimator such as the simulations in the variance comparison with an efficiency increase of 12.5. The tracking overhead resulting from the increased number of particles to be tracked is besides the sensitivity the most important efficiency limiting factor. Despite the detangling of each detected photon history and the increased tracking overhead, geometrical importance sampling can result in a 5 to 13-fold increase over analog SPECT simulations.

The method of binary splitting used in these simulations (see [3]) combined with the relatively simple layout of the importance maps results in the fact that all particles have equal weight. The weight only depends on the importance region where the particle ends up and is independent from the path by which the particle reached that final region. A more complex importance map based on production values could be used, which disciminates certain regions on the detector. This would likely end in a weight distribution depending on the complexity of the importance map. A weight distribution resulting from either a more complex importance map or a different splitting algorithm could increase the efficiency further. Care has to be taken however as increased efficiency comes with higher risks compared to the current conservative approach: a higher degree of variation of the weights in a single importance region increases the variance of the tallies in that region.

The results shown for the energy spectra verify that the detangling of the photon histories results in correct pulse height tallies. The spectrum in figure 5 is a typical example of how geometrical importance sampling results in lower variance compared to the analog case, for an equal amount of simulated events. The spatial resolution and the sensitivity results all show an excellent agreement compared to analog simulations. Further work will focus on the inclusion of forced detection in GATE and the evaluation of other splitting algorithms which result in a weight distribution and possibly higher efficiency.

## Conclusions

The importance sampling techniques of splitting and Russian roulette for GATE have been validated with regard to energy spectra, spatial resolution, and sensitivity. The variance of the flux through a detector was estimated with large set of simulations in order to compare its behaviour with and without importance sampling. A figure of merit was thus verified and it was shown that despite the detangling and increased tracking overhead, these techniques can result in a 5 to 13-fold increase over analog simulations.


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