MATHEMATICAL MODELING OF A NEW METHOD FOR LOCATION OF ISCHEMIC LESION WITH THE USE OF THE FRANK-M LEAD SYSTEM

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Abstract: Some results of mathematical modeling of a noninvasive method for locating ischemic lesion in the heart are presented. The initial data are electrocardiographic signals at the ST range of the cardiocycle measured by the Frank-M lead system providing unipolar signals. To take into account the spatial boundedness of the body as volume conductor, the potentials on the human chest surface are transformed into potentials on a homogeneous solid sphere model, using mathematical relations for the potentials generated by an equivalent dipole on the surface of the models considered. The position of a plane passing through the equivalent dipole generator of the ischemic lesion perpendicularly to the dipole moment is found, using the spherical model. This lesion midplane indicates the anatomical position of the ischemic zone. To estimate errors of the method, investigation was carried out on a simplified mathematical model of the chest with parallelepipedal shape. It is shown that for the overwhelming majority of the heart vector directions the midplane location error does not exceed 0,5 cm, so that this plane can indicate with sufficient accuracy the part of the heart invaded by acute ischemia.

Introduction

The dipole components of the cardioelectric generator, which are measured in the conventional orthogonal vectorcardiography, almost do not depend on the spatial position of the electrogenic zones in the heart, so that these components, in particular, do not provide location of acute ischemic lesions. The sensitivity of a vectorcardiographic lead system to the spatial position of the cardioelectric generator can be raised by using the unipolar leads. Therefore, in the method presented, we use a modification of the classical Frank lead system [1], namely Frank-M lead system with unipolar leads and with the same number of electrodes as the standard lead system, thus being convenient for practical applications. Taking into account scantiness of the information obtained by means of such lead system, as compared to multiple-electrode systems of electrocardiographic mapping, the method is intended only for determination of the most significant characteristic describing the spatial position of the acute ischemic lesion. This characteristic, called lesion midplane, approximates the boundary between the injured and healthy myocardium. As compared to the previous version of this approach [2], the presented method is based on more adequate mathematical relations, taking into account the boundedness of the body as volume conductor.

Materials and Methods

Averaged signals during the ST segment of the cardiocycle are considered, since the acute ischemia results in a stable shift of this segment. The electric generator set up by the local zone of acute ischemia is represented as an uniform electric double layer coinciding with the boundary between the regions of ischemia and healthy myocardium (Figure 1).

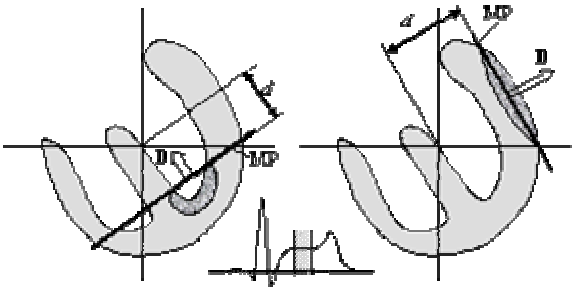

Figure 1: Subendocardial (left) and subepicardial (right) icshemic lesions in the heart cross-section. The midplane (MP) is indicated by the heavy line. The considered part of the cardiocycle is shown by darkened zone on the electrocardiogram below

The magnitude of the heart vector **D** (total dipole moment of the cardioelectric generator) directed from the healthy to ischemic region is proportional to the difference of the action potential amplitudes in the aforementioned regions and to the area of the double layer projection onto the plane perpendicular to the ischemic heart vector. The generator midplane situated

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with minimum deviation from the double layer boundary, is perpendicular to the ischemic heart vector and is displaced by a distance *d* from the heart geometric center, thus it characterizes the general position of the ischemic zone in the heart ventricles. The Frank-M lead system used for measurement includes 7 electrodes of the classical Frank vectorcardiographic lead system with additional electrodes R and L on the arms (Figure 2). The primary signals are the unipolar potentials of the individual Frank electrodes with respect to the Wilson terminal, while the signals proportional to the three components of the heart vector are also used.

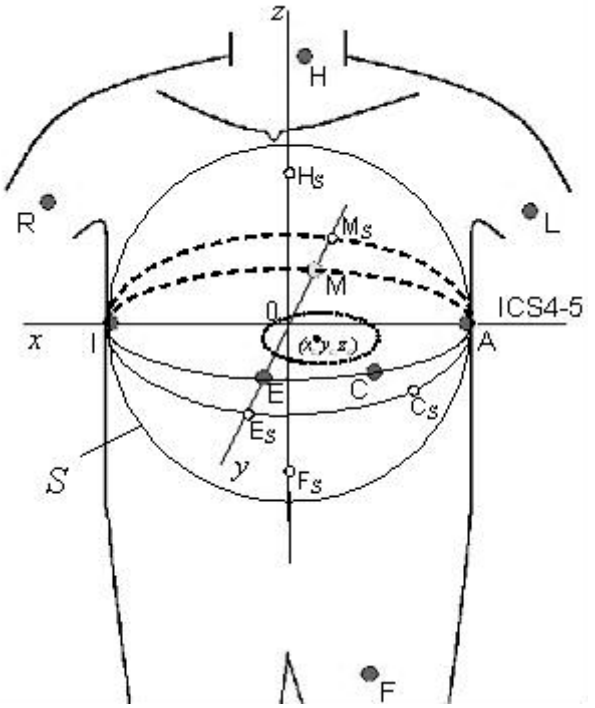

Figure 2: Positions of the model sphere *S* and Frank-M lead system electrodes with respect to the coordinate system *xyz* used. The heart region boundary in the horizontal plane, situated between the 4th and 5th intercostals spaces (ICS4-5), is indicated by the dotted line

The measuring electrodes A, I, M, E, H, and F of the unipolar leads are placed on the axes of the coordinate system *xyz* with the origin at the chest geometric center, and the electrode C in the horizontal (transversal) plane at equal angular distances from the semiaxes - *x* and y. The following main geometric (anthropometric) parameters of the chest are used: transversal diameter *a*, sagittal diameter *b*, and height *h*, as well as the relative parameters, namely, roundness index $\rho_b = b/a$ and prolateness index $\rho_h = h/a$. The position of the heart center with the coordinates x_c, y_c, z_c is assigned by the relative eccentricities $\varepsilon_x = x_c/a$, $\varepsilon_y = y_c/a$, $\varepsilon_z = z_c/a$ along the coordinate axes.

For calculation of the midplane displacement *d* sought for, a simplified electrodynamic model is used. This model takes into account such basic properties of the electrodynamic structure considered as prevalent dipolarity of the cardioelectric generator and total spatial boundedness of the chest volume conductor. The electric generator of ischemic lesion is approximated by a point dipole situated at the midpoint of the lesion, and the conducting medium by a homogeneous solid sphere surrounded by dielectric space and having its center at the geometric center of the chest and radius equal to the half transversal diameter of the chest.

As well, we use a simplified mathematical model of the chest, taking into account the most important deviation of the chest from the spherical shape, namely, its prolateness in the vertical direction and flatness in the sagittal direction. In particular, this simplified realistic model is a homogeneous parallelepipedal conductor surrounded by dielectric medium, with properly situated heart region and measuring points (Figure 3). The basic sizes of the model (lengthes of its edges) coincide with the anthropometric parameters *a*, *b*, *h*. The potential generated on the surface of such model can be expressed in analytical form [2]. In the model region considered as the ischemic lesion, the equivalent electric generator is specified in the form of a point dipole defining the given, or true, position of the midplane displaced over the distance *d* from the heart center.

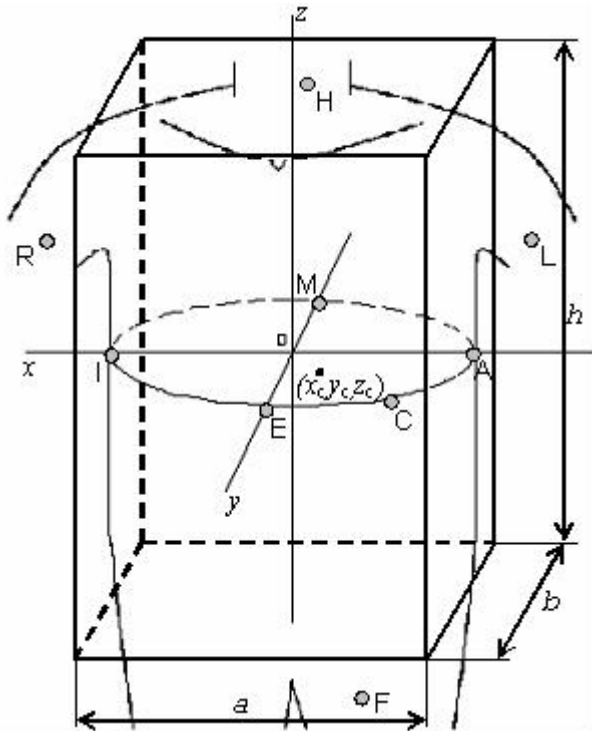

Figure 3: Simplified model of the chest in the form of a rectangular parallelepiped

Conversion of the measured potentials from the realistic model to the potentials on the spherical model surface is carried out by using the linear transformation of the vector $\bar{\varphi}$ of unipolar lead potentials on a realistic model to the respective vector $\bar{\varphi}_S$ of unipolar potentials on the spherical model,

$$\bar{\varphi}_S = [K] * \bar{\varphi} \,, \qquad (1)$$

where $\bar{\varphi}_S$ and $\bar{\varphi}$ are the vectors with components φ_{iS} and φ_i, respectively, the subscript i = I, E, C, A, M, H, F indicating the corresponding lead, and [K] is the matrix of dimension 7x7. The elements of this matrix are calculated as optimal solution of a system of equations similar to Equation (1) for a set of dipoles with various positions inside the heart region, while their moment vectors pass through the heart center, thereby taking into account the dominating spatial position of the ischemic heart vector.

The potential generated at the measuring point (x,y, z) on the surface of the model sphere with the radius r_S and specific conductivity σ by a dipole with the moment components D_x, D_y, D_z and position coordinates x_D, y_D, z_D, situated at the distance $r_0<r_S$ from the sphere center is defined by the following expression [2, 3]:

$$\varphi = \frac{1}{2\pi\sigma r_S^2 \rho}\sum_{i=1}^{3} D_i\left[\frac{C_{Li}-C_{Gi}}{\rho^2}+\frac{(1+\rho)C_{Li}-C_{Gi}}{(1+\rho)^2-f^2}\right], \qquad (2)$$

where $D_1= D_x$, $D_2= D_y$, $D_3= D_z$, $C_{G1}= x_D/r_S$, $C_{G2}= y_D/r_S$, $C_{G3}= z_D/r_S$, $C_{L1}= x/r_S$, $C_{L2}= y/r_S$, $C_{L3}= z/r_S$, $f = r_0/ r_S$ is the eccentricity of the dipole, and $\rho = (1/r_S)\sqrt{(x-x_D)^2+(y-y_D)^2+(z-z_D)^2}$ is the relative distance between the dipole and the measuring point.

The position of the midplane is defined by the displacement d of the dipole with respect to the heart center in the direction collinear with the dipole moment vector. This displacement is expessed as

$$d = (x_D - x_C)\sin\theta\cos\psi + (y_D - y_C)\sin\theta\sin\psi + (z_D - z_C)\cos\theta, \qquad (3)$$

where θ and ψ are the angular spheric coordinates (polar distance and longitude) of the heart vector.

The input data used in the electrocardiographic investigation for determination of the lesion midplane are the following: the measured unipolar lead signals φ_I, φ_E, φ_C, φ_A, φ_M, φ_H, φ_F, and anthropometric parameters a, b, h; the mean constant value σ is also used.

The corrected values of the lead potentials are calculated by Equation (1). Then, by equating these values to the expression (2) for each lead, a system of 7 equations for 6 variables, D_x, D_y, D_z, x_D, y_D, z_D, is obtained. The optimal values of these unknown variables, which satisfy the system of equations with the minimum error, are found by means of successive approximations of the dipole moment components and dipole position coordinates. Here the initial approximation is assigned as the moment components determined by the Frank formulas and coordinates of the dipole situated at the heart geometric center. The resulting values are used in Eq. (3) for calculation of the midplane displacement d.

The main sources of the errors in determination of d are the neglect of the internal heterogeneity of the chest as volume conductor and inaccuracy of the measured signal transformation for transition to the spherical model. The error of the first kind have systematic character, its effect on the determined parameters of the equivalent dipole generator of the heart is estimated in [3]. To estimate the error of the second kind, which affects most significantly the obtained results, the described above parallelepipedal model was used. For a given dipole with the midplane displacement d the potentials φ_I, φ_E, φ_C, φ_A, φ_M, φ_H, φ_F at the lead electrodes are obtained by solving the forward electrodynamic problem for this model. With the use of the above-mentioned coefficients, these potentials are transformed to fit the spherical model, and inverse electrodynamic problem is solved as described above. The final result obtained is a midplane displacement value d_S, while its variation from the true value d characterizes the error of problem solution.

Results

The presented technique for evaluating the accuracy of determination of the ischemic lesion midplane was realized with the use of a set of 200x7=1400 dipole generators with various directions of moment and positions uniformly distributed over the heart region at the distances 0 cm, 1 cm, 2 cm, and 3 cm from the heart center. For each such dipole, the forward problem was first solved on the parallelepipedal chest model with the standard parameters a = 30 cm, ρ_b = 0.7, ρ_h = 1.4, ε_x = = ε_y = 0.1, ε_z =0, and the matrix [K] of potential conversion was obtained by the aforementioned approach. Then, using the transformed lead potentials, the inverse problem was solved for each dipole on the spherical model.

The errors of the lesion midplane position determined by the solution of the inverse problem are illustrated in Figures 4 and 5.

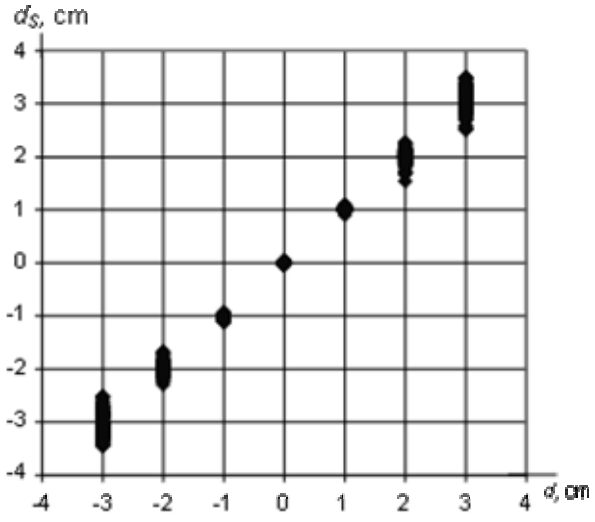

Figure 4: Dependence between the true (d) and calculated (d_S) displacements for the sets of 1400 radial dipoles distributed in the heart region at the

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distances 0 cm, 1 cm, 2 cm, and 3 cm from the heart center. The heavy lines indicate the distribution range of 95% test dipoles

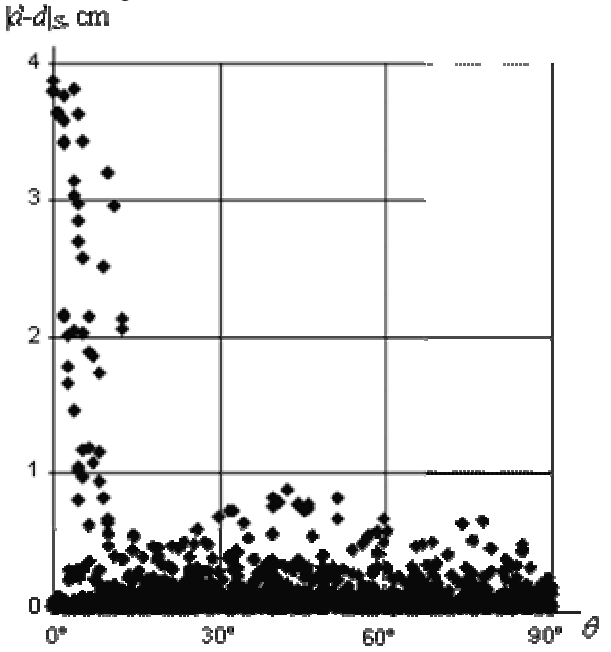

Figure 5: Dependence of the absolute midplane displacement error on the angular distance between the heart vector and *z* axis. Each dark patch corresponds to a test dipole

It is found that for the vast majority ($\approx 95\%$) of the dipoles used as the testing set for the cardiogenerator, the midplane displacement error does not exceed 0.5 cm. The maximum values of the error lying in the range 0.5 ÷ 3.8 cm, occur for the heart vector directions approaching the positive and negative *z* semiaxes, that is at the polar distances $0° \le \theta \le 15°$, $165° \le \theta \le 180°$.

Discussion

As soon as the resulting errors of the midplane position usually do not exceed the mean thickness of the free ventricular walls, determination of the midplane allows at least recognition of the basic section of ventricles invaded by the acute ischemia.

The accuracy of the proposed method could be further improved with the use of more sophisticated mathematical model of the chest [4] for transformation of the true volume conductor to the spherical model.

Conclusions

The models and quantitative description of the ischemic region structure with the use only of the lesion midplane position are adequate to the limited information, which can be obtained by means of 7 signals of the Frank-M lead system, practical application of which is not more complicated as compared to the procedure of standard electrocardiography. At the same time, here we can considerably extend the information on the ischemic lesion as compared to the ordinary vectorcardiographic data, which are almost insensitive to the spatial position of the lesion and indicate only the magnitude and angular orientation of its dipole moment. Incorporation of the midplane determination method into the DECARTO technique developed previously [2] provides its efficient improvement. As a consequence, noninvasive recognition and map-like visualization of the local acute ischemic lesion are achieved. It is possible not only to estimate the lesion size characterized by the heart vector magnitude at the shifted ST segment of the electrocardiogram, but also to recognize the part of the ventricles where this lesion is situated.

Acknowledgements

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