THE EFFECT OF ADDITIONS ON APPARENT VISCOSITY OF BLOOD

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Abstract: The expression and numerical values for effective shear viscosity of a dilute suspension of spherical microparticles in blood are obtained. If blood reveals properties of a polar fluid during hydrodynamic interaction with suspended particles, then the Cowin polar fluid should be used for the rheological modeling of blood. The above-mentioned statement is true even in the cases of blood flow in large blood vessels or in channels of different devices, in which blood, in the absence of suspended spherical particles, behaves as the Newtonian fluid.

Introduction

A dilute suspension in blood of rigid microspheres of the same size possessing zero buoyancy is considered in this paper.

Suspensions in blood can arise [1] on addition of particles of contrast agents for the purposes of X-ray visualization of blood vessels, on addition of particles of medical substances with the aim of delivery of drugs to affected organs and so on.

Suspension in blood also arise outside of the human body, for example, on addition of polymeric beads containing a fine magnetic colloid encapsulated in the inner core of the polymeric matrix [1] for the improvement of biochemical/biomedical analyses of blood. Suspensions in blood arise too in devices for dialysis of blood.

While solving medical problems through the use of suspensions in blood, the possible consequences of biomechanical intervention into the human body should be remembered. In particular, it is necessary to study the influence of the addition of suspended particles on the viscosity of blood. In this paper, the simplest – spherical – form of suspended particles is considered, and also the analytical expression and numerical values for the effective viscosity of a dilute suspension of microspheres in blood as a suspension carrier fluid are obtained.

The rheological model of blood as the carrier fluid of a suspension

We assume in this paper that the radius of the suspended particles is significantly larger than the characteristic size of blood microstructure elements – red blood cells, platelets and white blood cells. This allows one to consider the interaction of blood with suspended particles as a hydrodynamic interaction of a liquid continuum with bodies suspended in it.

As usual in suspension rheology, the flow of the carrier fluid of the suspension – blood – around the suspended particles is considered within the Stokes approximation.

While choosing the continual rheological model of blood it is necessary to be knowledgeable about the rheological peculiarities of blood in gradient flows, its structural features, and also how the structure of blood influences its behaviour as a liquid medium.

In accordance with [2], blood behaves differently depending on the characteristic size of the flow region. Particulaly, in large vessels it behaves as the Newtonian fluid and in small vessels its behaviour is non-Newtonian.

The total volume of red blood cells is approximately 50 times more than the total volume of other formed elements of blood – platelets and white blood cells [2], therefore the rheological behaviour of blood is determined by the concentration and mechanical properties of red blood cells only [2].

As in any concentrated suspension, the high concentration of red blood cells – approximately 46% - in human blood causes neighboring red blood cells to change the spinning of each other in gradient flows of blood. Therefore, each red blood cell’s own angular velocity in gradient flows of blood differs from the regional angular velocity of the elementary blood volume that they occupy. This fact explaines the choice of the Cowin polar fluid [5] in the present paper, as in papers [3, 4], for rheological modeling of blood.

The phenomenological rheological model of the Cowin polar fluid [5] is one of the structural continuum models [6]. In order to account for the influence of the elements of fluid microstructure on the stress state in the fluid, it is assumed in the Cowin model [5] that the fluid particles, found in an elementary volume which is moving with the translational velocity v_i and rotating with the regional angular velocity $\omega_k = \frac{1}{2}\varepsilon_{klr} v_{r,l}$, may rotate furthermore with the angular velocity Ω_k around the center of the elementary volume. This means that the particles of the medium may have their own angular characteristics that differ from the angular velocity of the elementary volume as a whole. It is also asumed that a force couple is acting between the fluid particles. In this case, the effect of one part of the fluid on another part adjacent to it is characterised not only by the surface forces (viscous stresses) but also surface momentums (couple stresses). The rheological equations of state of the Cowin polar fluid are

$$\tau_{ij} = -p\delta_{ij} + 2\mu d_{ij} - 2kH_{ij}, \tag{1}$$

$$\Lambda_{ij} = \alpha\delta_{ij}\Psi_{rr} + (\beta+\gamma)\Psi_{ij} + (\beta-\gamma)\Psi_{ji}, \tag{2}$$

where τ_{ij} is the viscous stress tensor; Λ_{ij} is the couple stress tensor; d_{ij} is the strain rate tensor, $d_{ij} = (1/2)(v_{i,j} + v_{j,i})$; $v_{i,j}$ is the velocity gradient tensor; $H_{ij} = \varepsilon_{mij}(\Omega_m - \omega_m)$; ε_{mij} is the Levi-Civita tensor; Ψ_{ij} is the gradient of the fluid particles' own angular velocity Ω_m, $\Psi_{ij} = \Omega_{i,j}$; $\mu, k, \alpha, \beta, \gamma$ - are rheological constants; the comma in the indices denotes differentiation in the direction of the axis denoted by the index which follows the comma.

Considering the elementary flows in the papers [5, 7], it was obtained that the effective viscosity of the Cowin polar fluid (Eqs. (1), (2)) does not depend on the flow's kinematic characteristics, but is determined only by the flow's geometry and the rheological constants of the model defined by Eqs. (1), (2). So, the effective viscosity of the polar fluid (Eqs. (1), (2)) in the Couette flow is defined by the formula

$$\mu_a^{(0)} = \frac{\mu}{1-(N_0 l_0/h)\,\mathrm{th}(N_0 l_0/h)}, \tag{3}$$

where h is one-half of the width of the channel in the Couette flow; N_0 and l_0 are determined by the formulas

$$N_0 = \sqrt{\frac{k}{\mu+k}}, \quad l_0 = \sqrt{\frac{\beta+\gamma}{\mu}};$$

$\mathrm{th}\, z$ is the hyperbolic tangent.

According to [5], the parameters N_0 and l_0 vary within the limits $0 \le N_0 \le 1$, $l_0 \ge 0$. At $N_0 = 0$, the rheological model of a polar fluid becomes a rheological model of the Newtonian fluid with the viscosity μ [5]. From Eq. (3), in this case, it is determined that $\mu_a^{(0)} = \mu$.

The parameter l_0, which has a dimension of length, is linked, according to [5], with the characteristic size of the microstructure elements of real microstructure fluids that are modeled by the polar fluid (Eqs. (1), (2)). The analysis of Eq. (3) shows that, while $0 < N_0 \le 1$, the influence of the rotational viscosity k of the polar fluid on the effective viscosity $\mu_a^{(0)}$ only takes place at finite values of $2h/l_0$, i.e. in relatively narrow channels of the Couette flow of the polar fluid. In the opposite case, i.e. at $h/l_0 \to \infty$, the influence of rotational viscosity k of the polar fluid (Eqs. (1), (2)) on its effective viscosity $\mu_a^{(0)}$ is absent; in this case, it follows from Eq. (3) that $\mu_a^{(0)} = \mu$, i.e. the polar fluid (Eqs. (1), (2)) behaves as the Newtonian fluid with the viscosity μ. This analysis demonstrates the similarity of rheological behaviour of the Cowin polar fluid at $0 < N_0 \le 1$ in narrow and wide channels and the rheological behaviour of blood in small and large blood vessels respectively.

The constitutive equations (1), (2) of the Cowin polar fluid were used in the papers [3, 4] for the rheological modeling of blood. The comparison in [4] of the velocity profiles of the polar fluid and blood in the Poiseuille flows, with the use of experimental data obtained in [8] allowed to obtain the values of parameters N_0, l_0 of the Cowin polar fluid for the rheological modeling of blood at the different haematocrit values C_b (Table1).

The effective viscosity of a dilute suspension of beads in blood

The study of a dilute suspension of beads of the same radius possessing zero buoyancy in the Cowin polar fluid (Eqs.(1), (2)) in [9] allowed to obtaine the expression for the effective viscosity μ_a of such suspension:

$$\mu_a = \mu\left(1 + 2{,}5cF(N_0; 2a/l_0)\right), \tag{4}$$

where c is the volume concentration of suspended beads, a is the radius of suspended beads;

$$F(N_0, 2a/l_0) = \frac{3N_0 K_{3/2}\left((2a/l_0)N_0\right)}{(2a/l_0)K_{5/2}\left((2a/l_0)N_0\right)}. \tag{5}$$

In (5 $K_{3/2}(z)$ and $K_{5/2}(z)$ are the functions of MacDonald of half-integer order.

The effective viscosity μ_a defined by Eqs. (4), (5) was obtained in [9] using the assumptions of the Einsteinian theory [10] of dilute suspensions:

1) rigid spherical suspended particles have the same dimensions;

2) the diameter d of suspended spherical particles is much smaller than the characteristic dimension $\bar{l}$ of the suspension macroflow region but is much greater than the characteristic dimension l of microstructural elements of the carrier fluid;

$$l << d << \bar{l};$$

3) no-slip condition is fulfilled on the surface of the suspended particles;

4) the motion of the suspension's carrier fluid with respect to the suspended particles is slow;

5) the volume concentration of suspended particles is small; the suspension is assumed to be diluted;

6) suspended particles possess zero buoyancy.

The use of Eqs. (1), (2) in this paper for rheological modeling of blood as a suspension carrier fluid requires the fulfillment of the assumptions 1 – 6 for the considered suspension of spherical particles in blood.

ISSN: 1727-1983

The assumptions 1, 2, 4 – 6 are not specific, they can be used for a suspension in blood as well as for a suspension with a low-molecular carrier fluid. But the fulfillment of condition 3 for a suspension in blood is not evident, since blood as a carrier fluid of the suspension is itself a suspension of its formed elements. But in spite of that, according to [4], no-slip condition for blood is also fulfilled. The comparison in [4] of different boundary conditions on the surface flowed around by blood, that was modeled by the Cowin polar fluid (Eqs. (1), (2)), showed that the results of theoretical calculations and experiments have the best coincidence at the fulfillment of no-slip condition.

The functions of MacDonald of half-integer order $K_{3/2}(z)$ and $K_{5/2}(z)$ are expressed in terms of elementary functions [11]. It allows us to obtain the effective viscosity μ_a of the considered dilute suspension in blood defined by Eqs. (4), (5) in a form suitable for analysis and calculations

$$\mu_a = \mu\left(1+\frac{5}{2}c\frac{N_0^2\left(2a/l_0\right)^2+3N_0\left(2a/l_0\right)+3}{N_0^2\left(\left(2a/l_0\right)^2-3\right)+3N_0\left(2a/l_0\right)\left(1-N_0^2\right)+3}\right). \quad (6)$$

The evaluation of parameters N_0 and l_0 of the polar fluid (Eqs. (1), (2)) in [4] while modeling blood flows allows to investigate the influence of the polar properties of blood on the effective viscosity of a dilute suspension of beads in it using Eq. (6).

First of all, according to Eq. (6), in the limiting case $c = 0$, i.e. in the absence of suspended particles in the suspension, the carrier fluid – blood – modeled by a polar fluid behaves as the Newtonian fluid with the viscosity μ. Such a result corresponds with real behaviour of blood in large blood vessels [2]. This means that Eq. (6) determines the effective viscosity of a dilute suspension of beads in blood precisely in large blood vessels.

Secondly, the analysis of Eq. (6) also reveals that the increase of a/l_0 leads to the disappearence of the influence of rotational viscosity of blood k at $0 < N_0 \le 1$ on the suspension's effective viscosity. In such a limiting case, Eq. (6) takes the form

$$\mu_a = \mu\left(1+2,5c\right),$$

i.e. the effective viscosity of a dilute suspension of beads in blood is determined by the Einstein formula [10].

It is obvious from Eq. (6) that the influence of the rotational viscosity k of blood as a carrier fluid of the considered suspension on the effective suspension viscosity μ_a is revealed at finite values of the ratio $2a/l_0$, i.e. at a comparatively small size of suspended spherical particles.

The equation (6) is used in the paper for finding the numerical values of the characteristic viscosity

$$\left[\mu_a\right] = \frac{\mu_a - \mu}{\mu c}$$

of the suspension. The results of the calculation of $\left[\mu_a\right]$ for the considered suspension in blood at the different values of radius a of suspended particles and haematocrit values C_b of blood as a carrier fluid of the suspension are given in Table 1.

Table 1: Numerical values of the characteristic viscosity $\left[\mu_a\right]$ of dilute suspension of beads in blood.

C_b,%	N_0	$l_0 \cdot 10^6$, m	$\left[\mu_a\right]$			
			1	2	3	4
5	0,5021	8,475	2,8385	2,8071	2,7808	2,7586
10	0,5316	12,968	2,9952	2,9543	2,9193	2,8891
20	0,5501	16,597	3,1111	3,0649	3,0246	2,9893
30	0,5547	20,526	3,1963	3,1492	3,1072	3,0699
40	0,5569	23,462	3,2486	3,2019	3,1599	3,1219

The Table 1 columns 1 – 4 for the characteristic viscosity $\left[\mu_a\right]$ of the suspension correspond to the four values of radius a of suspended particles: $a = 3.5\cdot10^{-5}$ m, $4\cdot10^{-5}$ m, $4.5\cdot10^{-5}$ m, $5\cdot10^{-5}$ m. Such values of radius a of suspended spherical particles are significantly greater than the effective radius of red blood cells, which ranges from $2.56\cdot10^{-6}$ m to $2.88\cdot10^{-6}$ m considering that the red blood cells' volume ranges from 70µm^3 to 100µm^3 [2]. Such a choice of radius of suspended spherical particles ensures correctness of using the Einstein theory [10] to rheological study of dilute suspension in blood.

Conclusions

The analysis of the analytical expression for the effective viscisity μ_a of a dilute suspension of beads in blood (Eq. (3.3)) and the numerical values for the characteristic viscosity $\left[\mu_a\right]$ of the considered suspension shows that blood with suspended beads 70 - 100 microns in diameter reveals its non-Newtonian, i.e. polar, properties even in those gradient flows in which blood behaves as the Newtonian fluid in the absence of suspended particles. Among such flows are blood flows in middle-sized and large vessels or in channels of most apparatuses outside the human body.

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The obtained numerical values of the characteristic viscosity $[\mu_a]$ of the considered suspension also show that taking into account the polar properties of blood as a carrier fluid of the suspension leads to the increase of the suspension's characteristic viscosity in comparison with a dilute suspension with the Newtonian model of blood. In particular, the characteristic viscosity $[\mu_a]$ is increased from the well known Einstein value 2.5 [10] for a dilute suspension of beads with the Newtonian carrier fluid to the values listed in Table 1, which were obtained in the present paper while modeling blood as a carrier fluid of the suspension by the Cowin polar fluid (Eqs. (2.1), (2.2)) for different values of haematocrit values C_b and different values of radius a of the suspended beads.

The studies carried out in the present paper expand the range of uses of the Cowin polar fluid as a rheological model of blood. The Cowin polar fluid should be used to model blood as a carrier fluid of a dilute suspension of rigid microspheres even in middle-sized and large blood vessels or in channels of most apparatuses in cases when blood exhibits properties of the polar fluid while interacting with suspended particles.

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ISSN: 1727-1983