MAGNETOBIOMECHANICS OF SMART SUSPENSIONS IN BLOOD

E. YU. Taran, O. O. Melnyk

Faculty of Mechanics and Mathematics,
Kyiv Taras Shevchenko National University,
Volodymyrska 64, 01033 Kiev, Ukraine

taran@univ.kiev.ua

Abstract:

The rheological equations for a dilute suspension of rigid axisymmetric elongated particles possessing a permanent magnetic moment with blood as a carrier fluid are derived within the frames of a structure-phenomenological approach. The V.K. Stokes fluid with couple stresses and the uniaxial dumbbell are used as the rheological and hydrodynamic models of blood and suspended particles, respectively. The influence of an external magnetic field on a rheological behaviour of the suspension under study is examined.

Introduction

The possibility of control over the rheological properties of a dilute suspension in blood of rigid elongated particles possessing permanent magnetic moment with the use of external magnetic field is under study on the base of constitutive equations obtained in the paper.

Suspensions in blood of magnetically sensitive particles can arise on the addition of particles of medical substances formed on the base of magnetic carriers to blood [1, 2]. In particular, such suspensions arise on addition of ferro- or ferrimagnetic micro- and nanoparticles coated with either polysaccharides or proteins designed for diagnostic or hyperthermic treatment of cancer.

On introducing suspended particles into blood, one should know the answers to the following questions:

- what is the dynamic of suspended particles in blood flows in vessels and channels of devices used in blood research?
- what is the total influence of additions to blood on its mechanical-rheological properties?
- what is the influence of additions on a blood flow?

In order to carry out investigations in this area the equation of motion of rigid axially symmetric suspended particles possessing permanent magnetic moment in gradient flows of blood as well as the rheological equation for a dilute suspension of such particles in blood are derived in the paper within the frames of the *structure-phenomenological approach* proposed by Shmakov and Taran [3] and developed by Taran in [4].

The modeling used within the framework of the structure-phenomenological approach [3, 4] is *multiscale*. In *the first scale level of modelling*, blood as liquid medium is modeled by the classic continuum, and the V.K. Stokes fluid with couple stresses [5] is used as a phenomenological rheological model of blood.

In *the second scale level of modelling*, we assume that the characteristic size of suspended particles is much greater than the characteristic size of red blood cells. This allows us to consider the interaction of blood with suspended particles as with hydrodynamic bodies. As a hydrodynamic model of suspended particles, an uniaxial dumbbell is utilized in the present paper.

In our studies of dilute suspensions with blood as the carrier fluid, we assume, on the other hand, that the characteristic size of suspended particles is much smaller than the characteristic size of the suspension's macroflow region. In *the third scale level of modelling*, this assumption makes it possible to model the considered suspension with the use of the structure continuum with two internal microparameters, namely, the unit vector characterizing the orientation of suspended particles and the vector characterizing the relative angular velocity of the particles with respect to the carrier fluid. In this level of modelling, the rheological equation for stress in the suspension is postulated phenomenologically, and phenomenological rheological constants appearing in the constructed rheological equation are evaluated analytically using the results obtained in the second level of modelling, that is, in the structural part of theory.

The possibility of control over the rheological properties of a dilute suspension in blood of rigid elongated particles possessing a permanent magnetic moment with the use of an external magnetic field is explored. In order to do this, the obtained rheological equation is used to examine the rheological behaviour of the considered suspension in a simple shearing flow in the presence of a steady external transverse magnetic field.

ISSN: 1727-1983

The rheological model of dilute suspension in blood of magnetically sensitive rigid elongated particles

Human blood possesses a complex structure and consequently is a non-Newtonian fluid from the point of view of fluid mechanics [6]. In *the first scale level of modelling*, we assume that the characteristic size of blood microstructure elements, namely, red blood cells, is significantly smaller than the characteristic size of suspended particles in blood. According to such an assumption, we model blood using the classic continuum and use phenomenological rheological model that takes into account the characteristic features of blood's microstructure. In this paper, the V.K. Stokes fluid with couple stresses [5] is used as a rheological model of blood. This model was used earlier for description of blood flow in [7].

The theory of V.K.Stokes [5] is the simplest generalization of the Newtonian theory of viscous fluids that takes into account the presence of couple stresses in gradient flows of fluids. The viscous and couple stresses τ_{ij} and M_{ij} in rheological model of V.K.Stokes [5] are completely defined by the field of velocity v_i

$$\tau_{(ij)} = -p\delta_{ij} + 2\mu d_{ij}, \tag{1}$$

$$\mu_{ij} = 4\eta K_{ij} + 4\eta' K_{ji}. \tag{2}$$

In rheological Eqs. (1) and (2), $\tau_{(ij)}$ is the symmetric part of stress tensor τ_{ij}; μ_{ij} is the deviatoric part of the couple stress tensor M_{ij}; p is the pressure; μ, η, η' are rheological constants; d_{ij} is the strain rate tensor, $d_{ij} = (1/2)\left(v_{i,j} + v_{j,i}\right)$; K_{ij} is the gradient tensor of angular microrotational velocities of the fluid particles that is determined as the gradient of the vorticity vector $\omega_i = (1/2)\varepsilon_{irs} v_{s,r}$, $K_{ij} = \omega_{j,i}$; δ_{ij}, ε_{ijk} are the Kronecker and Levi-Civita symbols.

The assumption that the characteristic size of the suspended particles is significantly larger than the characteristic size of red blood cells, together with the modelling of blood by the classic continuum allow us to consider the interaction of blood with suspended particles as with hydrodynamic bodies in *the second scale level of modelling*.

As a hydrodynamic model of suspended particles, a uniaxial dumbbell with axis L is utilized. According to [8], the friction coefficient ξ of dumbbell beads in the V.K.Stokes fluid (Eqs. (1) and (2)) does not depend on the flow around the beads:

$$\xi = \xi_N\left(1+B\right),$$

$$B = \frac{2+\nu}{r_0^2 + \left(2+\nu\right) r_0}. \tag{3}$$

In Eq. (3), ξ_N is the friction coefficient of a dumbbell bead of radius r in slow translational motion in the Newtonian fluid with dynamic viscosity μ, $\xi_N = 6\pi\mu r$; $\nu = \eta'/\eta$; $r_0 = r/l_0$, here $l_0 = \eta/\mu$.

In order to model blood by the V.K.Stokes fluid (Eqs. (1) and (2)), the material characteristic length l_0 is evaluated as function of haematocrit value C_b of blood on the base of results obtained in [7] and [9] (Table 1).

Table 1. The dependence of parameter l_0 on the haematocrit value C_b of blood.

$l_0 \cdot 10^6$, m	6.666	8.000	14.815
C_b,%	6	13	40

It is assumed in the paper that the suspended particles modeled by uniaxial dumbbells are magnetically sensitive, namely, they possess a permanent magnetic moment $p_i = Pn_i$, where P is the value of permanent magnetic moment; n_i is the unit vector characterizing orientation of an axially symmetric suspended particle as well as the orientation of its dumbbell model in the laboratory coordinate system. It is also assumed that the suspension is diluted to the extent that the interaction between the magnetic fields of the suspended particles, as well as the hydrodynamic interaction between them is not taken into account.

In the presence of the external magnetic field H_i, the dynamics of suspended dumbbell particles in gradient flows of the considered suspension in blood is defined by the hydrodynamic forces acting on the beads of the dumbbell

$$f_i^{(k)} = \xi\left[\left(-1\right)^k \frac{L}{2}\left(v_{i,j} n_j - \dot{n}_i\right) - v_{0i}\right], \tag{4}$$

$$\left(k = \overline{1,2}\right)$$

with angular momentum

$$M_i^{(h)} = (1/2) L^2 \xi \varepsilon_{ijk} n_j \left(d_{ks} n_s - N_k\right), \tag{5}$$

and also by the magnetic momentum

$$M_i^{(m)} = P\varepsilon_{ilk} n_k H_l. \tag{6}$$

In Eqs. (4)–(6), n_i is the unit vector characterizing orientation of suspended dumbbell particle; N_i is the vector characterizing angular velocity of suspended particle with respect to the carrier fluid, $N_i = \dot{n}_i - \omega_{ik} n_k$; here, the dot over n_i denotes the local time derivation and ω_{ik} is the velocity vortex tensor, $\omega_{ik} = (1/2)\left(v_{i,k} - v_{k,i}\right)$; v_{0i} is the migration velocity of a dumbbell with respect to the carrier fluid.

The use of Eqs. (4)–(6) in the equations of the motion of the dumbbell particles, that without regard for its moment of inertia take the form

$$f_i^{(1)} + f_i^{(2)} = 0, \qquad M_i^{(h)} + M_i^{(m)} = 0,$$

allows us to obtain

$$v_{0i} = 0, \tag{7}$$

$$\dot{n}_i = \omega_{ik} n_k + d_{ik} n_k - d_{km} n_k n_m n_i +$$

ISSN: 1727-1983 © 2005 IFMBE

$$+\frac{P}{W}\left(H_i - n_k H_k n_i\right), \qquad (8)$$

where W is the rotational friction coefficient of a uniaxial dumbbell in the carrier fluid of the suspension, $W = (1/2)\xi L^2$.

According to Eq. (7), a migration of suspended particles modeled by uniaxial dumbbells with respect to the carrier fluid is absent. The rotational motion of suspended particles is defined by the constitutive Eq. (8) for the unit vector n_i characterizing the orientation of suspended particles.

In the frames of this structure theory, that is, in *the second scale level of modelling*, we also obtain the rate of mechanical energy dissipation per unit volume of the suspension

$$\Phi = \Phi_0 + n_0\Phi_p = \Phi_0 + n_0\frac{\xi L^2}{2}\times$$
$$\times\left(\langle N_i N_i\rangle - 2d_{ij}\langle N_i n_j\rangle + d_{ij}d_{ik}\langle n_j n_k\rangle\right) \qquad (9)$$

where Φ_0 is the rate of the mechanical energy dissipation per unit volume of the carrier fluid of the suspension in the absence of suspended particles; n_0 is the number of suspended particles per unit volume of the suspension; Φ_p is the rate of mechanical energy dissipation while flowing around the two beads of the dumbbell,

$$\Phi_p = \sum_{k=1}^{2}\xi\langle U_i^{(k)}U_i^{(k)}\rangle;$$

the angular brackets $\langle\ \rangle$ denote averaging with the use of the distribution function F of angular positions of suspended particles, which satisfies the equation

$$\frac{\partial F}{\partial t} + \frac{\partial}{\partial n_i}\left(F\,\dot n_i\right) = 0. \qquad (10)$$

In *the third scale level of modelling*, we assume that the dimensions of the suspended particles are significantly smaller than the characteristic dimension of the suspension's macroflow region. According to the structure-phenomenological approach [3, 4] used in the paper, such an assumption allows us to model considered suspension using the structural continuum with two internal microparameters, namely, n_i and N_i, characterizing the orientation of suspended particles and their relative angular velocity. The rheological equation for stress in the suspension is postulated phenomenologically by the expression

$$T_{ij} = t_{ij} + n_0\langle\sigma_{ij}\rangle, \qquad (11)$$

where t_{ij} is the stress tensor in the carrier fluid of the suspension in the absence of the suspended particles; $n_0\langle\sigma_{ij}\rangle$ is the stress caused by the presence of n_0 suspended particles per unit volume of the suspension. It follows from Eq. (9), that is, from the expression for the rate of mechanical energy dissipation per unit volume of the suspension obtained in the frames of the structure theory, that tensor σ_{ij} in Eq. (11) must depend on the variables d_{km}, n_l, N_s, that is,

$$\sigma_{ij} = \sigma_{ij}\left(d_{km}; n_l; N_s\right). \qquad (12)$$

The final choice of arguments for the functional relation (12) is determined by the structure of terms in Eq. (9) and by the symmetry of a uniaxial dumbbell with respect to the midpoint of it:

$$\sigma_{ij} = \sigma_{ij}\left(d_{km}; n_l n_p; N_s n_q\right). \qquad (13)$$

Furthermore, it follows from Eq. (9) that σ_{ij} has to be the polynomial function of its arguments, linear over d_{km} and N_i:

$$\sigma_{ij} = \left(a_0 + a_1 d_{km}n_k n_m\right)\delta_{ij} + \left(a_2 + a_3 d_{km}n_k n_m\right)n_i n_j +$$
$$+a_4 d_{ij} + a_5 d_{ik}n_k n_j +$$
$$+a_6 d_{jk}n_k n_i + a_7 n_i N_j + a_8 n_j N_i, \qquad (14)$$

where $a_i\left(i=\overline{0,8}\right)$ are constant phenomenological coefficients.

The phenomenological coefficients $a_i\left(i=\overline{0,8}\right)$ in Eqs. (11) and (14) are found from comparing the rate of mechanical energy dissipation per unit volume of the suspension defined by Eq. (9) that was determined in the structural part of theory, with the rate determined in the same way as in [4]

$$\Phi = \Phi_0 + n_0\langle\sigma_{ij}\rangle d_{ij} + n_0\left\langle N_i\varepsilon_{ijk}n_j M_k^{(h)}\right\rangle$$

within the framework of the structure-phenomenological approach. As a consequence of this, we obtain the rheological equation of a dilute suspension with blood as the carrier fluid

$$T_{ij} = t_{ij} + \frac{1}{2}n_0\xi L^2\left(d_{ik}\langle n_k n_j\rangle - \langle n_j N_i\rangle\right). \quad (15)$$

It is demonstrated in [10] that such an equation as Eq. (8) has the stationary solution $\left(\dot n_i = 0,\ N_i = -\omega_{ik}n_k\right)$ for steady-state shear flows of a suspension in the presence of the steady-state magnetic field H_i. This means that the suspended dumbbell particles of the considered suspension in blood acquire the same stationary orientation defined by the constitutive equation

$$W\left(v_{i,k}n_k - d_{km}n_k n_m n_i\right) + P\left(H_i - n_k H_k n_i\right) = 0 \quad (16)$$

under conditions of steady-state gradient flows of the suspension in the presence of the steady-state magnetic field H_i. The rheological equation for stress (Eq. (15)) in such an anisotropic suspension takes the form

$$T_{ij} = \tau_{ij} + nWv_{i,k}n_k n_j. \qquad (17)$$

W is a single rheological parameter in Eqs. (16) and (17), which characterizes the interaction of suspended particles with blood modeled by the V.K.Stokes fluid with couple stresses (Eqs. (1) and (2)). According to Eq. (3), taking into account the couple stresses arising in blood leads to an increase in the rotational friction coefficient of the suspended dumbbell particles:

$W = W_N (1+B)$, as compared with its value $W_N = (1/2)\xi_N L^2$ in a suspension with the Newtonian model of blood as the suspension carrier fluid.

In order to account for the elongation of the suspended particles we evaluate analytically the radius r of the dumbbell beads through the axial ratio of an ellipsoid of revolution from the equality $W_N = W_{NE}$. Here, W_{NE} is the rotational friction coefficient of the ellipsoid of revolution with the axis of symmetry $2a = L$ and the equatorial diameter $2b$ that is equivalent to the dumbbell as a hydrodynamic model of suspended particles in the Newtonian carrier fluid with the viscosity μ. The radius r of the beads of such a dumbbell is defined by the expression

$$r = \frac{2L\left(\bar{p}^4 - 1\right)}{2\bar{p}^4}\left(\frac{2\bar{p}^2 - 1}{2\bar{p}\sqrt{\bar{p}^2 - 1}}\ln\frac{\bar{p} + \sqrt{\bar{p}^2 - 1}}{\bar{p} - \sqrt{\bar{p}^2 - 1}} - 1\right)^{-1},$$

where $\bar{p} = a/b$. It should be noted that the ellipsoid of revolution is the most commonly encountered hydrodynamic model of suspended particles in suspensions with the Newtonian carrier fluid.

Magnetorheological behaviour of dilute suspensions in blood of magnetically sensitive rigid elongated particles

The use of the obtained rheological equations (16), (17) enables us to investigate the effect of the external magnetic field and the couple stresses arising in gradient flows of blood on the rheological characteristics of dilute suspension in blood of rigid magnetically sensitive particles studied in this paper. In order to do this, a steady simple shear flow

$$v_x = 0, \quad v_y = Kx, \quad v_z = 0 \qquad (K = const) \qquad (18)$$

of such a suspension is considered in a cross magnetic field

$$H_x = H, \quad H_y = H_z = 0 \qquad (H = const). \quad (19)$$

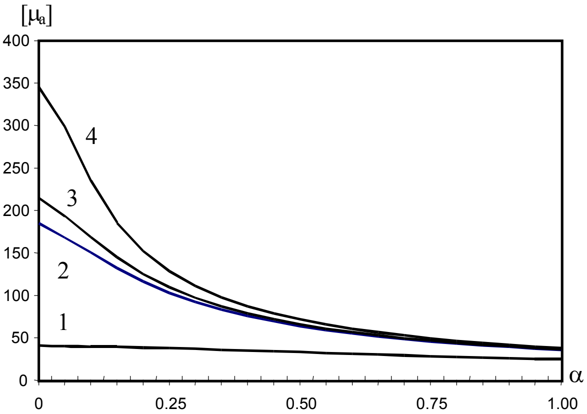
The calculations show that the considered suspension in blood reveals non-Newtonian dependences of the effective suspension viscosity μ_a and a non-zero difference of normal stresses σ_1

$$\mu_a \equiv \frac{T_{xy} + T_{yx}}{2K} = \mu + n_0 W_N \frac{\sqrt{1 + 4\alpha^2 (1+B)^2} - 1}{4\alpha^2 (1+B)}, \qquad (20)$$

$$\sigma_1 \equiv T_{yy} - T_{zz} = n_0 K W_N \frac{\left(\sqrt{1 + 4\alpha^2 (1+B)^2} - 1\right)^{3/2}}{2\sqrt{2}\alpha^2 (1+B)} \qquad (21)$$

on the parameter $\alpha = KW_N/(PH)$. In Eqs. (20) and (21), B is defined by the Eq. (3).

The use of Eqs. (20) and (21) makes it possible to calculate the numerical values of the characteistic viscosity of the suspension $[\mu_a] = (\mu_a - \mu)/\mu V$ and σ_1^*/K as functions of α (Figs. 1 and 2), where $\sigma_1^* = \sigma_1/\mu V$; V is the volume concentration of suspended particles, $V = (4/3)n_0 \pi a b^2$; the length of the suspended particles is $L = 4\cdot 10^{-5}$ m with axial ratios of suspended particles $\bar{p} = 3, 5, 10$ at haematocrit values of blood $C_b = 6\%, 13\%, 40\%$ respectively.



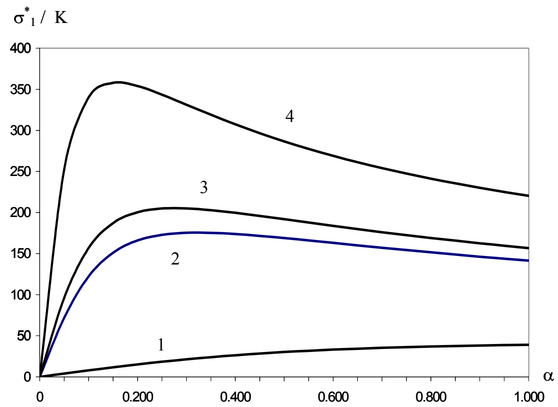


Figure 1: The dependences of $[\mu_a]$ and σ_1^*/K on α at $\bar{p} = 10$; curves 1 correspond to the suspension with the Newtonian carrier fluid of the viscosity μ; curves 2 – 4 correspond to the suspension in blood with the haematocrit values $C_b = 6\%, 13\%, 40\%$ respectively.

The numerical evaluation of μ_a demonstrates that the enhancement of the shear rate K of the flow (Eq. (18)) at the fixed strength H of the magnetic field (Eq. (19)) leads to the pseudoplastic decrease of the effective suspension viscosity μ_a (Figs. 1 and 2). The pseudoplastic decrease of the effective viscosity μ_a of the suspension and variation of the first difference of normal stresses σ_1 are due to the variation of the orientation of suspended particles under the combined action of hydrodynamic forces and the external magnetic field.

 ISSN: 1727-1983 © 2005 IFMBE

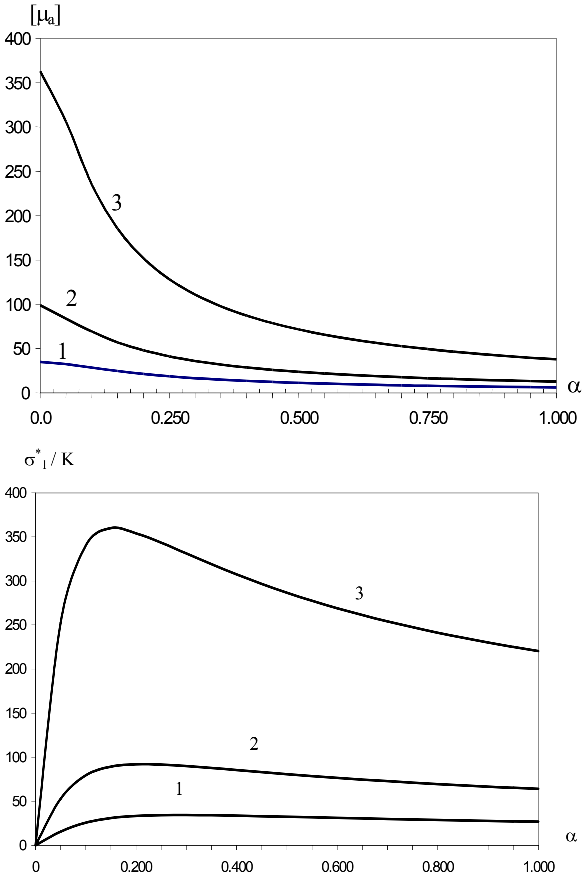


Figure 2: The dependences of $\left[\mu_a\right]$ and σ_1^*/K on α at C_b=40%; curves 1 – 3 correspond to $\overline{p} = 3,5,10$ respectively.

The calculations also show that the effective viscosity μ_a of the considered suspension in blood and the first difference of normal stresses σ_1 in it are augmented when increasing the haematocrit value C_b of blood as the carrier fluid of the suspension holding K and H fixed (curves 2-4 on Fig. 1) and due to increasing values of $\overline{p}$ (curves 1-3 on Fig. 2).

Conclusions

The use of the magnetorheological model of dilute suspension in blood developed in this paper shows that the increase of the strength H of the magnetic field (Eq. (19)) at the constant shear rate K of the flow (Eq. (18)) leads to the increase of the effective viscosity μ_a of a dilute suspension of magnetically sensitive elongated particles in blood and also to the decrease of the first difference σ_1 of normal stresses. In such a manner the change of H may be used as a control factor of the rheological properties of a suspension formed on addition to blood of magnetically sensitive elongated particles, and therefore may be used to control the blood flow in vessels.

References

[1] HAFELY, U. et al, (Ed), (1997): 'Scientific and clinical applications of magnetic carriers', (Plenum Press, New York)

[2] GILLIES, G. T., RITTER, R. C., BROADDUS, W. C.et al, (1994): 'Magnetic manipulation instrumentation for medical physics research', *Rev. Sci. Instrum.*, **65**, pp. 533-562

[3] SHMAKOV, Yu. I. and TARAN, E. Yu. (1970): 'Structure-continual approach in rheology of polymeric materials', *Inzh.- Fiz. Zhurn.*, **18**, No. 6, pp. 1019-1024, (in Russian)

[4] TARAN, E. Yu. (1977): 'Rheological equation of state of dilute suspensions of rigid dumbbells with beads at the endpoints', *Prikl. Mekhanika*, **13**, No. 4, pp. 110-115, (in Russian)

[5] STOKES, V. K. (1966): 'Couple stresses in fluids', *Phys.Fluids*, **9**, pp. 1709-1715

[6] LEVTOV, V. A., REGIRER, S. A. and SHADRINA, N. Kh. (1982): '*Rheology of blood'*, (Meditsina, Moscow), (in Russian)

[7] VALANIS, K. S. and SUN, C. T. (1969): 'Poiseuille flow of a fluid with couple stresses with application to blood flow', *Biorheology*, **65**, No. 2, pp. 85-97

[8] STOKES, V. K. (1971): 'Effects of couple stresses in fluids on the creeping flow past a sphere', *Phys. Fluids*, **14**, pp. 1580-1582

[9] BUGLIARELLO, G., KAPUR, C. and HSIAO, G. (1965): 'The profile velocity and other characteristics of blood flow in a non-uniform shear field', *Proc. of the 4th Inter. Congr. on Rheol.*, (Interscience, New York), pp. 351-370

[10] TARAN, E. Yu.(1978): 'Influence of electric field on rheological behaviour of dilute suspension of dipole dumbbells in visco-elastic fluid of Oldroyd', *Mekhanika Polimerov*, No. 3, pp. 519-524, (in Russian)

ISSN: 1727-1983 © 2005 IFMBE